PBS Lesson Series

Math: Grade 7, Lesson 11, Determine Unit Rates with Ratios of Fractions

Lesson Focus: Find unit rates with ratios of fractions, and use them to solve multi-step problems.
Practice Focus: Students will focus on practicing finding unit rates in order to use them to solve multi-step problems.
Objective: Find a unit rate involving unit fractions, find and apply a unit rate involving fractions, and solve problems using unit rates
Key Vocabulary: Unit rate, ratio
TN Standards: 7.RP.A.1 and 7.RP.A.3
Teacher Materials:

- Paper or white board
- Pen/pencil/marker
- Prepared copies of the examples (to save time)
- Student Practice Packet

Student Materials:

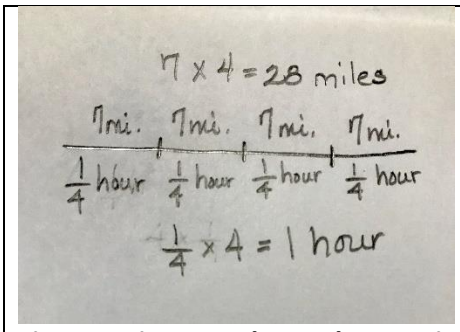
- Paper and a pencil, and a surface to write on

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 7th graders out there, though all children are welcome to tune in. This lesson is the eleventh in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about find unit rates with ratios of fractions, and use them to solve multi-step problems! Before we get started, to participate fully in our lesson today, you will need:** • **Paper, a pencil and a surface to write on and the optional student packet** **Ok, let's begin!**	Students get materials ready for the lesson.

Intro (3 minutes) **Let's start by looking at a situation in which we can use rates to solve a problem!** **Allison and her pen pal Yuki planted bean seeds at the same time. Allison and Yuki video chat about their seedlings. Allison says her plant has grown 2.5 inches in 5 days. Yuki says her plant has grown 5.5 centimeters in 4 days.** **Allison's plant grew 2.5 inches in 5 days. I wonder how many inches did it grow per day?** [pause] **We can create a ratio that will help us think about this situation.** $$\frac{2.5\ inches}{5\ days}$$ **Let's create a unit rate!** **Remember that a unit rate is a rate with a denominator of 1. The denominator is the bottom number of the fraction** $$\frac{2.5\ inches \div 5}{5\ days \div 5}$$ **We divide the denominator by 5 so it becomes "1 day". To ensure that we are creating an equivalent ratio, we must also divide the numerator by 5.** **This gives us the unit rate:** $$\frac{0.5}{1}$$	Student watches and listens. They think about how to change rates to unit rates.
What does this new ratio mean in the context of the problem? [Pause] **That's right! This means that Allison's plant grew 0.5 inch each day.** $$\frac{0.5\ inch}{1\ day}$$ **Let's think through another problem.** **What if Yuki says her plant has grown 5.5 cm in 4 days? How many centimeters did Yuki's plant grow per day?** [pause] **Let's create a rate.** $$\frac{5.5\ centimeters}{4\ days}$$ **How do we then create a unit rate?** [Pause] **That's right! We divide both numerator and denominator by 4.**	Student thinks about what it means in the context of the problem.

$$\frac{5.5\ cm \div 4}{4\ days \div 4}$$ $$\frac{1.375}{1}$$ **What does this mean?** [Pause] **That's right! This means that Yuki's plant grew 1.375 cm each day.** **What if we wanted to compare which plant grew more?** [pause] **Do you think it is Allison's or Yuki's?** [pause] **Make a guess and write it down! Now let's do the math!** **How can we compare inches and cm?** [Pause] **That's right! We need them to be the same unit. We must convert one to the other. Do you know how many cm are in an inch?** [Pause] **That's it! We know that 1 inch = 2.54 cm. So this means** $$\frac{1inch}{2} = \frac{2.54\ cm}{2}$$ **So,** **½ inch = 1.27 cm!** **0.5 inch = 1.27 cm. This means that Allison's plant grew 1.27 cm per day.** **Yuki's plant is growing faster since it grew 1.375 cm each day and 1.375 is greater than 1.27! How many of you guessed that? Great Job!**	
Teacher Model (10 minutes) **Now that we have reminded ourselves of how to use ratios and rates to solve problems, let's do some more examples! Let's find talk about finding a unit rate. What is a unit rate?** [Pause] **Remember that a unit rate is a rate with a denominator of 1.** Objective 1: Find a unit rate involving unit fractions **Sergio is training for a triathlon. His target speed on his bicycle is 25 miles per hour. He recorded a distance of 7 miles at 15 minutes. Did he achieve his target speed for the first 7 miles of his ride?** [Pause] **What do we know?** [Pause]	Students use unit rate with simple fractions. Student thinks about what they remember about unit rates. Student thinks about how to set up this problem to be solved.

We know that Sergio want to ride his bike at a rate of 25 miles per hour. **We know that Sergio goes 7 miles in 15 minutes.** **What are we trying to find out? What is the problem asking us to do?** [Pause] **The problem is asking if Sergio is going 25 miles per hour in the first 15 minutes of his trip!** **How do we do that? We see that we are given how far Sergio rides in 15 minutes, but the problem asks about miles per hour.** [Pause] **Maybe we should convert 15 minutes to hours?** [Pause] **Yes!** **What is the relationship between 15 minutes and 1 hour?** [Pause] **Since there are 60 minutes in an hour, and 15 minutes out of 60 minutes is 15/60 or $\frac{1}{3}$.** [Pause] **We know that 15 minutes is $\frac{1}{4}$ of an hour.** **How can we use a unit rate to describe Sergio's cycling speed?** [pause] **First let's write a ratio. Remember that a ratio is a comparison of two values by division.** [Pause] $$\frac{7\ miles}{15\ minutes}$$ **Can you draw a diagram to show how the distance Sergio bikes is related to the time he bikes?** [Pause and draw this diagram. As you draw, explain…] **Since we are given $\frac{1}{4}$ of the hour, we can think about what this would look like for the entire hour. How many one-fourths are in 1?** [Pause] [Pause] **Yes! There are 4. We can label each $\frac{1}{4}$ hour with 7 miles. Since we have 4 sections, 7 x 4 = 28.**	Student thinks about the information given in the problem. Student thinks about what the problem is asking them to do. Student thinks about converting 15 minutes to a part of an hour.



Student draws a diagram.

Since 7x4 is 28, we know that Sergio's speed was 28 miles per hour.

Can you make a table to find a unit rate? [Pause]
[Create the table as shown. As you draw the table, explain…]
We are given distance in miles and time in minutes. We have converted the minutes to hours already. So our table would look like this:

Miles	**7**	
Hour	$\frac{1}{4}$	

Student creates a table.

We know that $\frac{1}{4}$ hour x 4 is going to give us a whole hour. Since we multiplied our time by 4, we must multiply our distance by 4.

Miles	**7 x 4**	**28**
Hour	$\frac{1}{4}$ **x 4**	**1**

What does this mean?
[Pause]

Student interprets the answer.

His speed was 28 miles per hour!
Sergio achieved his goal!

Objective 2: Find and apply a unit rate involving fractions.
We just did a problem where we had to convert minutes to hours. Let's look at a problem where we are given time as a fraction of an hour. Let's think about this situation!
Bronwyn mow the lawn every other weekend. She can mow 12,000 square feet in $\frac{2}{3}$ hour. The lawn is 36,000 square feet. How long does it take her to mow the entire lawn? [pause]

Students find the unit rate using a complex fraction and use the unit rate to solve the problem.

Let's create a ratio that tell us how many square feet per hour Bronwyn mows. What would that ratio look like? [Pause] **Since she mows 12000 square feet per hour, the ratio would look like this:**

Student thinks about how to create the ratio.

$$\frac{\mathbf{12000}\ \boldsymbol{square\ feet}}{\frac{\mathbf{2}}{\mathbf{3}}\ \boldsymbol{hour}}$$ [Pause]	Student thinks about how to change the ratio to a unit rate.
We need a unit rate. Remember a unit rate is a ratio that has a denominator of 1 and the denominator is the number on the bottom of the fraction. What can we multiply by $\frac{2}{3}$ to make it be 1? [Pause]	
Did you say $\frac{3}{2}$? Yes! $\frac{3}{2}$. How can we keep our ratio (remember that it is a fraction) equivalent? [Pause]	
Yes! Remember that we must also multiply the numerator by $\frac{3}{2}$. Good job! [Pause]**Now our ratio looks like this. Our numerator and our denominator are both multiplied by $\frac{3}{2}$.** $$\frac{\mathbf{12000}\ \boldsymbol{x}\ \frac{\mathbf{3}}{\mathbf{2}}}{\frac{\mathbf{2}}{\mathbf{3}}\ \boldsymbol{x}\ \frac{\mathbf{3}}{\mathbf{2}}}$$ [Pause] **12000 times $\frac{3}{2}$ can be thought of as 12000 times 3, then divide by 2. So this becomes 36000 divided by 2 or 18000. $\frac{2}{3}$ times $\frac{3}{2}$ is 1. This give us…** $$\frac{\mathbf{18000}}{\mathbf{1}}$$	Student interprets the answer.
Now we have a UNIT RATIO. Remember that a unit ratio is a ratio whose denominator is 1. What does this mean in our problem? [Pause] **It means that we now have the time in 1 whole hour!**	
What is our answer? [Pause]	
This means that Bronwyn can mow 18,000 square feet in 1 hour. **Remember that the lawn is 36,000 square feet. How long will it take Bronwyn to mow the entire lawn?** [pause] **We know that 36,000 divided by 18,000 is 2, so it will take her 2 hours!**	Student thinks about how to create a table.

We could also think about this in a table.
Let's create a table for this problem.

Area (sq ft)	12000	18000	36000
Time (hours)	2/3	6/6	2

So, Bronwyn can mow the entire lawn in 2 hours!
Notice that when we used a fraction, we had a numerator of 12000 and a denominator of $\frac{2}{3}$. Where do you see these numbers in the table? Yes! The area is 12000 and the time is $\frac{2}{3}$.
Notice in the table that we multiply the area and the time by $\frac{3}{2}$. When we used the ratio, we also multiplied our numerator and denominator by $\frac{2}{3}$.

This is two ways to approach this problem. Both ways are correct! Which way do you like better? [Pause]

Sometimes I like to use a ratio that looks like a fraction, and other times I like to use a table.

Objective 3: Solve problems using unit rate
You are doing great! Let's look at another one!
Now let's look at a problem where the numerator and the denominator in the ratio are both fractions!

Objective 3: Solve problems using unit rate
Student solves a problem using the unit rate.

Let's look at a map. Omar knows that his friend Chris lives 3/5 mile away. How far is the school from Omar's house?

Student studies the map and thinks about what information is given.

What do you notice about the map? What information are we given?
[Pause]

We can see from the map that the distance from Omar's house to Chris' house is $\frac{3}{4}$ inch. The distance from Omar's house to school is 2 inches. [Pause] **Let's think about the scale of the map. We know that Omar's house is $\frac{3}{5}$ mile from Chris' house. We see on the map that the distance from Omar's house to Chris' house is marked as $\frac{3}{4}$ inch, so $\frac{3}{4}$ inch on the map represents $\frac{3}{5}$ actual miles.** **How can we determine the actual length of the 2 inch distance from Omar's house to the school?** [Pause] **Using what we know we can set up a ratio.** **We know that** $$\frac{\frac{3}{5}\ miles}{\frac{3}{4}\ miles}$$ **How can we write this as a unit rate?** **How can we change $\frac{3}{4}$ to 1?** [Pause] **Yes! We can multiply it by $\frac{4}{3}$, because $\frac{3}{4}$ x $\frac{4}{3}$ is 1!** $$\frac{\frac{3}{5}x\frac{4}{3}}{\frac{3}{4}x\frac{4}{3}}$$ [Pause] **This becomes** $$\frac{\frac{4}{5}}{1}$$ **What does this mean?** [Pause] **It means that 1 inch on the map represents $\frac{4}{5}$ miles.** **How does that help us determine the distance from Omar's house to the school?** [Pause] **Now we know the scale. We know that 1 inch is $\frac{4}{5}$ miles. How can we determine the actual distance if the map says it is 2 inches?** [Pause]	Student thinks about the scale and what it means. Student creates a unit rate. Student interprets the unit rate. Student thinks about how we can use the unit rate to determine the 2-inch distance. Student interprets the answer.

If 1 inch is $\frac{4}{5}$ mile, then 2 inches must be $\frac{4}{5}$ miles x 2 inches. $\frac{4}{5}$ x 2 is $\frac{8}{5}$ or $1\frac{3}{5}$ miles!

It is $1\frac{3}{5}$ miles from Omar's house to the school!

Good job!

We have looked at using ratios to represent problems. We have looked at creating unit rates to help us think about the situations in a different way. I think you are ready to try some on your own!

Guided Practice (14 minutes)

Remember Sergio? Let's think about that problem again

[I do]

Sergio increase his target speed to 30 miles per hour. How many more miles does Sergio need to ride in $\frac{1}{4}$ hour to achieve his target goal?

[Pause]

Student recalls the Sergio problem.

I am going to set up a table. You can do it whatever way you choose!

[Pause]

Student chooses a strategy.

Miles		**30**
Hour	$\frac{1}{4}$	**1**

Student creates a unit rate.

Why are we dividing by 4? [Pause]

1 divided by 4 is $\frac{1}{4}$. So if we divide 1 by 4 to get $\frac{1}{4}$, we can divide 30 by 4 to get the missing value!

[Pause]

Student interprets the answer.

Miles	**7.5**	**30**
Hour	$\frac{1}{4}$	**1**

What does this mean? [Pause]

This means that Sergio must ride 7.5 miles in $\frac{1}{4}$ hour to achieve this target speed, so he needs to ride an additional 0.5mile per $\frac{1}{4}$ hour.

Student recalls the Bronwyn problem.

Good job! [We do] **Let's think about the earlier problem with Bronwyn. Every other weekend Bronwyn's brother Daniel mows the lawn. He can mow 15,000 square feet in $\frac{3}{4}$ hour. Who mows the lawn in less time?** **Recall that Bronwyn mows the 36,000 square feet in 2 hours. How long does it take Daniel to mow 36,000 square feet?** [Pause] **Let's set up a ratio.** $$\frac{15000 sq\ ft}{\frac{3}{4} hour}$$ **Does your ratio look like this?** [Pause] **Good!** **How can we create a unit ratio?** [Pause] **We can multiply $\frac{3}{4}$ by $\frac{4}{3}$ to make the denominator 1.** $$\frac{15000 x \frac{4}{3}}{\frac{3}{4} x \frac{4}{3}}$$ [Pause] **15000 times $\frac{4}{3}$ can be thought of as 15000 times 4 then divide by 3. This becomes 60000 divided by 3 or 20000. $\frac{3}{4}$ times $\frac{4}{3}$ is 1, so we have…** [Pause] $$\frac{20000}{1}$$ **What does this mean?** [Pause] **This means that Daniel can mow 20,000 square feet in 1 hour. So who mows the fastest?** [Pause] **Bronwyn mows the lawn at a rate of 18,000 square feet per hour. Daniel mows the lawn at a rate of 20,000 square feet per hour. Daniel mows faster!** **Well done!**	Student sets up a ratio. Student creates a unit rate. Student interprets the answer.

[You do] **Let's try another one! This time you try this own your own, then we will do it together. I know you can do it!** [Give student time to do the problem] **Recall our map problem! Sonoma bikes 5 miles to Paige's house. On a map, they measure that distance as $\frac{5}{6}$ cm. The same map shows that the mall is $3\frac{1}{2}$ cm from Paige's house. What is the actual distance between Paige's house and the mall?** **How can we write a ratio?** [Pause] $$\frac{\frac{5}{6}cm}{5\ miles}$$ [Pause] **How can we rewrite this so we know how many cm per mile?** [Pause] **We need the denominator to be 1. In other words, we need a unit rate. How do we do that?** [Pause] $$\frac{\frac{5}{6} \div 5}{5 \div 5}$$ [Pause] $$\frac{\frac{5}{6}\ x \frac{1}{5}}{1}$$ $$\frac{\frac{1}{6}}{1}$$ **What does this mean?** [Pause] **This means that each cm on the map represents 1 mile. So, if it is $3\frac{1}{2}$ cm from the mall to Paige's house, how far is it actually?** [Pause] $$\frac{3\frac{1}{2}}{\frac{1}{6}}$$ [Pause]	Student recalls the map problem. Student writes a ratio. Student creates a unit rate. Student interprets the answer.

I like to use improper fractions, so this becomes $\frac{7}{2}\ x\ 6$ **Or 21.** [Pause]	
What does this mean? [Pause]	
This means that it is actually 21 miles from the mall to Paige's house!	Student interprets the answer.
<u>Additional problem (if needed)</u> **Pam and Jake drive trucks for a produce company. One day Pam drove 102 miles in** $1\frac{1}{2}$ **hours. Jake drove 54 miles in** $\frac{3}{4}$ **hour. Who drove at a faster rate?**	
What do we know? [Pause] **We know that Pam drove 102 miles. We know it took her** $1\frac{1}{2}$ **hours. We know that Jake drove 54 miles in** $\frac{3}{4}$ **hour.** [Pause]	Student thinks about what information is given in the problem.
What are we trying to find? [Pause] **We are trying to find out who drove faster.**	Student determines what the problem is asking.
How can we find out who drove faster? What is another way to ask this question? [Pause] **Yes! We could find their speeds. We could find their speeds in miles per hour. Say that out loud! "Miles per hour". That phrase tells us how to write the ratio!**	Student thinks about what is mean to ask who drove faster.
$\frac{Miles}{hour}$	Student thinks about speed as a ratio.
Let's first look at Pam's speed. She drove 102 miles in $1\frac{1}{2}$ **hours. What will her ratio look like?** [Pause] $\frac{102 Miles}{1\frac{1}{2} hours}$	Student writes a ratio to represent Pam's speed.
What can we do to change this to a unit rate? [Pause]	

We need to know how far she went in 1 hour. We need miles per 1 hour. **How can we change $1\frac{1}{2}$ to 1 hour?** [Pause] **Let's first change $1\frac{1}{2}$ to an improper fraction.** **$1\frac{1}{2}$ is $\frac{3}{2}$. How can we change $\frac{3}{2}$ to 1?** [Pause] **Yes! We can multiply by $\frac{2}{3}$!** **Our ratio becomes:** $$\frac{102\ x\frac{2}{3}}{\frac{3}{2}x\frac{2}{3}}$$	Student changes the ratio to a unit rate.
This is $\frac{68}{1}$ **What does this mean?** [Pause] **This means that Pam drove 68 miles in 1 hour or 68 miles per hour.** **Now let's look at Jake's speed.** **How can we write a ratio to represent Jake's speed?** [Pause] $$\frac{54miles}{\frac{3}{4}hour}$$	Student interprets the unit rate. Student writes a ratio to represent Jake's speed.
How can we change this to a unit rate? [Pause] **Good! We can multiply the $\frac{3}{4}$ by $\frac{4}{3}$ to make it 1 hour.** [Pause] $$\frac{54\ x\frac{4}{3}}{\frac{3}{4}x\frac{4}{3}}$$ [Pause] **This becomes:** $$\frac{72}{1}$$	Student changes the ratio to a unit rate.
What does this mean? [Pause] **This means that Jake drove 72 miles in 1 hour or 72 miles per hour.** **So, who drove faster?** [Pause] **Jake drove faster!**	Student interprets the meaning of the unit rate. Student answers the question.

Independent Practice (1 min) **Great work, 7th grade! Today, we reviewed how to find unit rates with ratios of fractions, and use them to solve multi-step problems! You sure did a great job! After the video, you will have some problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) • **I enjoyed reviewing finding unit rates with ratios of fractions, and use them to solve multi-step problems with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	