PBS Lesson Series

Math: Grade 7, Lesson 12, Understand Proportional Relationships: Equivalent Ratios

Lesson Focus: Determine whether quantities are proportional by testing for equivalent ratios
Practice Focus: Students will focus on practicing finding equivalent ratios in order to solve problems.
Objective: Recognize a Proportional Relationship. Decide whether quantities are proportional. Use proportions to solve problems; 3: Use proportions to solve problems.
Key Vocabulary: ratio, unit rate, proportion, proportional relationship, square, area, perimeter, proportion, inverse operation
TN Standards: 7.RP.A.1, 7.RP.A.2a, 7.RP.A.2b, 7.RP.A.3
Teacher Materials:

- Paper or white board
- Pen/pencil/marker
- The examples prepared (to save time)
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on

**Note: This lesson may run long, so be sure to have all problems written/typed, all drawings and tables created before the lesson begins to save time.*

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 7th graders out there, though all children are welcome to tune in. This lesson is the twelfth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about Determine whether quantities are proportional by testing for equivalent ratios! Before we get started, to participate fully in our lesson today, you will need:** • **Paper, a pencil, and a surface to write on** **Ok, let's begin!**	Students get materials ready for the lesson.
Intro (4 minutes) **Today we are going to continue our discussion of ratios. Recall that a ratio is a comparison of two quantities using division. We can write ratios as fractions. Also remember that a unit rate is a ratio where the denominator is 1. Today we are going to add another concept to**	Student thinks about what they already know about ratios and unit rate.

our understanding of ratios! We are going to talk about proportions!
Do you know what a proportion is?
[Pause]
Yes? Great! You will like this lesson!
No? Great! That means today you get to learn something new!

Let's begin by thinking about weight. What is weight?
[Pause]

Weight is a measure of force affected by gravity. The Moon's gravity is less than the Earth's gravity, so objects weigh less on the Moon than on Earth.

Student thinks about the problem.

A rabbit that weighs 4.5 pounds on Earth will weigh only 0.75 pounds on the Moon.
A dog that weighs 90 pounds on Earth will weigh only 15 pounds on the Moon.
Think about any pattern that you see here.
[Pause]

Student tries to find a pattern.

Using what we know, let's determine how much a cat will weigh on the Moon. An average cat may weigh about 8 pounds on Earth. How much do you think it will weigh on the Moon? Make a guess that you know is too high. [Pause]
Make a guess that you know is too low. [Pause]

Student makes a high guess and a low guess.

Let's see if our answer is between your high guess and your low guess!
What information are we given?
[Pause]

Student determines what information they are given.

We know that a rabbit that weighs 4.5 pounds on Earth will weigh only 0.75 pounds on the Moon.
A dog that weighs 90 pounds on Earth will weigh only 15 pounds on the Moon. We know that our cat weighs 8 pounds on Earth.
[Pause]
How can we organize this information?
[Pause]
I'm going to make a table. Do you HAVE to organize with a table?
[Pause]
No! You may choose to organize the information a different way.

	Rabbit	**Dog**	**Cat**
Weight on Earth (lb.)	**4.5**	**90**	**8**
Weight on the Moon (lb.)	**0.75**	**15**	**?**

What have we learned that might help us find the cat's weight on the Moon?
[Pause]

Student thinks about how to use prior knowledge to solve this problem.

Yes! Think about any pattern that you noticed. Is there a relationship between weight on Earth and weight on the Moon for?
[Pause]
Do you notice anything similar about this relationship with the rabbit AND the dog?
[Pause]

Student notices similarities between the data.

How can we determine a relationship?
[Pause]
YES! We can find a unit rate. How do we find a unit rate?
[Pause]

Student recalls how to find a unit rate.

Recall that a unit rate is a ratio whose denominator is 1. We can make ratios and find unit rates!
Do you remember how to do that?
[Pause]
Let's look at our table.

	Rabbit	**Dog**	**Cat**
Weight on Earth (lb.)	**4.5**	**90**	**8**
Weight on the Moon (lb.)	**0.75**	**15**	**?**

We can set up a ratio with weight on Earth to weight on the Moon.
For the rabbit:

Student sets up a ratio.

$$\frac{4.5}{0.75}$$

How can we make a unit rate? [Pause]
We must make the denominator be 1. We can use the fact that a ratio is a comparison by division! We divide both the denominator and the numerator by 0.75 so the denominator becomes 1. That will create a unit rate. We can divide.
4.5 divided by 0.75 is 6, so the unit rate will look like

Student creates a unit rate.

$$\frac{6}{1}$$

Now let's look at a ratio for the dog's weight.

$$\frac{90}{15}$$

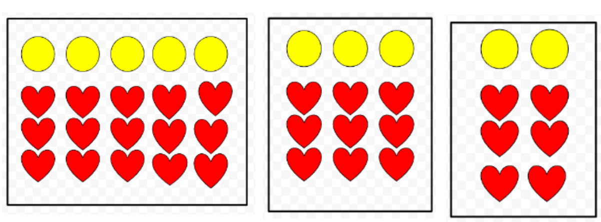
How can we make a unit rate? [Pause]
We can use division!
90 divided by 15 is 6. So the unit rate will look like

$$\frac{6}{1}$$

What do you notice? [Pause]
The weight of the rabbit and the weight of the dog have the same unit rate!

Student notices that the unit rates are equivalent.

WOW! What does this mean?
[Pause]

This means that an object's weight on Earth is 6 times the object's weight on the Moon. OR we could say that an object's weight on the Moon is $\frac{1}{6}$ that object's weight on Earth. **How can that help us determine the cat's weight on the Moon?** [Pause] **Remember that the cat weighs 8 pounds on Earth.** [Pause] **We can find $\frac{1}{6}$ of 8!** **$\frac{1}{6}$ times 8** $\frac{1}{6}x8$ $\frac{8}{6}$ $\frac{4}{3}$ $1\frac{1}{3}$ **So the cat will weigh $1\frac{1}{3}$ pounds on the Moon!** **Good job!** **We can say that since the unit rates for the weights of the dog and the rabbit are the same, they have a PROPORTIONAL RELATIONSHIP. What does that mean? That is what we are learning today! Here we go!**	Student interprets the results, and thinks about how to use this to answer the question. Student finds $\frac{1}{6}$ of 8. Student thinks about the meaning of proportional relationship.
Teacher Model (13 minutes) Objective 1: Recognize a Proportional Relationship **Do you like playing games on your phone? What is your favorite game?** [Pause] **On Sarah's favorite mobile game, she is awarded game lives when she finds gold coins. This is what she sees when she finds gold coins. The coins are circles and the lives are hearts:**  **What do you notice? Do you see a pattern?** [Pause] **How many lives does Sarah get for each gold coin?** [Pause] **Yes! Sarah gets 3 lives for each gold coin.**	The student will learn to recognize a proportional relationship. This will enable them to set up proportions and use the proportions to solve problems. It will also later provide an insight into understanding slope. Student tries to find a pattern.

How can we write these as ratios? [Pause]
Let's organize our information with a table.
[As you are creating the table, explain…]
For 5 coins, she gets 15 lives.
For 3 coins, she gets 9 lives.
For 2 coins, she gets 6 lives.
Now, let's set up ratios of number of coins to number of lives.
This will be 5 fifteenths. This can be rewritten as one-third.
3 ninths can be rewritten as one-third.
2 sixths can be rewritten as one-third.

Number of Coins	**5**	**3**	**2**
Number of Lives	**15**	**9**	**6**
Ratio of Coins to Lives	$\frac{5}{15}=\frac{1}{3}$	$\frac{3}{9}=\frac{1}{3}$	$\frac{2}{6}=\frac{1}{3}$

What do you notice? [Pause]
All three ratios have the same unit rate!
Notice that

$$\frac{5}{15}=\frac{3}{9}=\frac{2}{6}=\frac{1}{3}$$

Recall that these are called "equivalent fractions".
This relationship is a PROPORTIONAL RELATIONSHIP!
Two quantities are in a proportional relationship if all the ratios that relate the quantities are equivalent.
Each ratio of $\frac{coins}{lives}$ is equivalent to $\frac{1}{3}$. The number of games lives awarded is proportional to the number of gold coins found!

Objective 2: Decide whether quantities are proportional
Next let's think about the relationship between area of a square and the length of a side. Recall that a square has side lengths that are all equal.

How do we find area of a square?
[Pause]
Yes! You multiply the length times the width, or you can find the length of the side squared.
We are going to compare the area and the side lengths of three squares and decide if the relationship between the area and the side length is proportional:

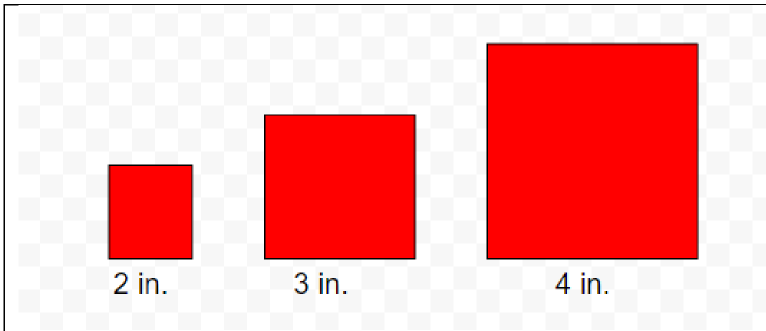
Student notices that there are 3 lives for every coin.

Student thinks about how to write the ratios.
Student creates a table.

Student notices that all three have the same unit rate.

Student will decide if a quantities are proportional. This will help students understand that proportional relationships form a pattern. Later they will see that this pattern allows them to see that a relationship is linear.

Student thinks about how to find area of a square.



One square has side lengths of 2 inches, one has side lengths of 3 inches, and the next has side lengths of 4 inches.
Let's make a table to organize the data.
[As you complete the table, explain…]
Let's record the side lengths: 2, 3 and 4
Now let's record the area. Recall that area of a square is the side length squared. So, 2 squared is 4 (because 2 times 2 is 4). 3 squared is 9 (because 3 times 3 is 9). 4 squared is 16 (because 4 times 4 is 16).
Now let's create ratios of area to side length. An area of 4 to a side length of 2 is four- halves or 2. An area of 9 to a side length of 3 is nine-thirds or 3. An area of 16 with a side length of 4 is sixteen-fourths or 4.

Student organizes the data in a table.

Side length	Area	$\frac{area}{side\ length}$
2	4	$\frac{4}{2}=\frac{2}{1}$
3	9	$\frac{9}{3}=\frac{3}{1}$
4	16	$\frac{16}{4}=\frac{4}{1}$

Student notices that the unit rates are not equivalent.

What do you notice? Do you see a pattern?
[Pause]

$$\frac{2}{1} \neq \frac{3}{1} \neq \frac{4}{1}$$

The unit rates are not equivalent. What does this mean? Is the relationship between the area and the side length proportional?
[Pause]

Student thinks about the definition of proportional relationship.

Relationships are proportional ONLY IF the ratios are equivalent. Since all the unit rates are different, the ratios are NOT equivalent. Therefore, the relationship between area and side length are NOT proportional.

What about the relationship between perimeter and side length? Is that relationship proportional?
[Pause]
How do we find the perimeter?
[Pause]

Student thinks about how to find the perimeter of a square.

To find the perimeter, you find the sum of the side lengths. Since our shapes are squares, all the side lengths are the same. We can use 4 times the side length.
Again, let's organize our data in a table.
[As you complete the table, explain…]
Our side lengths are 2, 3 and 4. The perimeter is 4 times the side length, so the square with side length of 2 has a perimeter of 4 times 2 or 8. The square with side length of 3 has a perimeter of 4 times 3 or 12. The square with side length of 4 has a perimeter of 4 times 4 or 16. Next let's find the ratios of perimeter to side length.
A perimeter of 8 to a side length of 2 is eight-halves or 4. A perimeter of 12 to a side length of 3 is twelve-thirds or 4. A perimeter of 16 to a side length of 4 is sixteen-fourths or 4.

Student thinks about how to create the table.

Side length	**perimeter**	$\frac{perimeter}{side\ length}$
2	**8**	$\frac{8}{2}=\frac{4}{1}$
3	**12**	$\frac{12}{3}=\frac{4}{1}$
4	**16**	$\frac{16}{4}=\frac{4}{1}$

Student notices that all the ratios are equivalent.

[pause]
What do you notice?
[pause]
Is the relationship between perimeter and side length proportional?
[pause]
Yes! $\frac{8}{2}=\frac{12}{3}=\frac{16}{4}=\frac{4}{1}$
Since all of the ratios are equivalent, this IS a proportional relationship!

Student thinks about the definition of proportional relationship, and they decide that this is a proportional relationship.

Objective 3: Use proportions to solve problems.
We have discussed proportional relationships. We said that there is a proportional relationship if the ratios are equivalent.
Now we will talk about a PROPORTION. A proportion is an equation that represent equal ratios. In other words, we have a proportional relationship with an unknown.
Let's use this table to think about proportions.
There is a proportional relationship between the number of wing beats per second for a hummingbird in flight. Have your ever seen a hummingbird flying? [pause]
The wings beat so fast you can barely see them!
The wing goes up and come down and goes back up, this is considered a wing beat. Let's consider the data in this table.

Objective 3: The student will use what they know about proportional relationships to solve problems. This reasoning will help with the understanding of slope later.

Seconds (x)	**2**	**7**	**10**
Wing beats (y)	**160**	**560**	**800**

Student thinks about the data in the table and how it is organized.

First let's verify that this is a proportional relationship. How do we determine a proportional relationship? [pause] **Yes! We can find the ratio of wing beats per second and see if all the rations are equivalent!** $$\frac{wing\ beats}{seconds}$$ $$\frac{160}{2}=\frac{560}{7}=\frac{800}{10}=\frac{80}{1}$$ **Since all three ratios are equivalent this IS a proportional relationship!** **Let's use this information to determine the number of wing beats in 60 seconds. How can we do that?** [Pause] **If we know that the wings beat 80 times per second, how many times will the wings beat in 60 seconds?** [Pause] **Let's set up a PROPORTION.** $$\frac{80}{1}=\frac{y}{60}$$ **Remember that y is the number of wing beats per second.** **We know that there are 80 wing beats in 1 second, so there are y wing beats in 60 seconds. How do we find the value of y, the number of wing beats?** [pause] **We can solve this equation using strategies we already know!** **If we want to isolate the y, what do we need to do?** [pause] **We need to get rid of 60. How can we do that?** [pause] **The 60 is being divided, so what is the inverse of division? The 60 is being divided, so what is the opposite of division?** [pause] **Multiplication! Let's multiply both sides by 60.** $$60\cdot\frac{80}{1}=\frac{y}{60}\cdot 60$$ **This becomes** $$\frac{4800}{1}=y$$ **What does this mean?** [pause] **This means that the hummingbird wings will beat 4800 times in 60 seconds! WOW! That is fast!** Tying the learning together:	Student thinks about how to determine a proportional relationship. Student determines that this is a proportional relationship because the ratios are equivalent. Student thinks about how to set up a proportion. Student thinks about how to solve the proportion. Student interprets the answer. Student thinks about the connection between ratios, proportional relationships and proportions.

Let's take a minute and reflect on all we have discussed today. We know that a ratio is a comparison of two quantities by division. We know that a proportional relationship exists if the ratios are equivalent. We can determine equivalency by find unit rates and seeing if they are the same. We discussed the definition of proportion. We said that a proportion is an equation that represents equal ratios. Now let's use what we have learned to practice.	
Guided Practice (10 minutes) **Let's think back to our problem with Sarah's mobile game.** **Let's do a similar problem. We will think about the problem together.** [I do] **Miles records the time it takes to download a variety of file types. How is the download time related to the file size? Explain.**	Student copies the table and thinks about how to complete it.

Type of Media	File Size (MB)	Download Time (seconds)	$\frac{download\ time}{File\ size}$
Document	1.25	25	
Song	3.6	72	
Video	6.25	125	

Complete the table and decide if there is a proportional relationship.
[pause]

[We do]

Student completes the table and notices a relationship.

Type of Media	File Size (MB)	Download Time (seconds)	$\frac{download\ time}{File\ size}$
Document	1.25	25	$\frac{25}{1.25} = 20$
Song	3.6	72	$\frac{72}{3.6} = 20$
Video	6.25	125	$\frac{125}{6.25} = 20$

Is this a proportional relationship?
[pause]
How do you know?
[pause]
The ratios are equivalent. They are have the same unit rate.
This means the download sizes are proportional!

Student notices that all the unit rates are the same, so it is proportional.

Now let's try another example!

[You do]

Student thinks about the information given.

Ginny's favorite cookie recipe requires $1\frac{1}{2}$ cups of sugar to make 24 cookies. How much sugar does Ginny need to make 36 of these cookies? You try this one on your own! [Pause] **We can think of this ratio as the amount of sugar per number of cookies. The ratio will look like:** $$\frac{1\frac{1}{2}}{24}$$ **We know that this should be a proportional relationship (otherwise the cookies will be too sweet or not sweet enough!)** **So we can set up a proportion. Recall that a proportion is an equation with an unknown.** $$\frac{1\frac{1}{2}}{24} = \frac{x}{36}$$ **We know that it takes $1\frac{1}{2}$ cups of sugar for 24 cookies, so how much sugar, x, will it take for 36 cookies?** **Let's solve this equation!** $$36 \cdot \frac{1\frac{1}{2}}{24} = \frac{x}{36} \cdot 36$$ $$\frac{54}{24} = x$$ $$2\frac{1}{4} = x$$ **This means that Ginny will need $2\frac{1}{4}$ cups of sugar to make 36 of her favorite cookies!** **Good job!** **Want to try one more?** [Pause] **Aaron and Paisley sell lemonade at a school fair. The tables show the number of glasses they sold and the amount collected on 3 different days.** **A. Is there a proportional relationship between the number of glasses sold and the amount collected at the two lemonade stands?** **B. Who charges more for a glass of lemonade? Explain how you know.**	Student creates a ratio and sets up a proportion. Student solves the proportion and interprets the answer. Student compares their work to the teacher's work. Student thinks about the information given.

Aaron's Raspberry Lemonade

Number of Glasses	Amount Collected ($)
12	9.00
20	15.00
24	18.00

Paisley's Strawberry Lemonade

Number of Glasses	Amount Collected ($)
11	8.25

16	**12.00**
30	**22.50**

I will give you time to answer the questions.
[Pause]

Let's see how you did!
First we look at Aaron's table.

Number of Glasses	**Amount Collected ($)**	$\dfrac{\boldsymbol{amount\ collected}}{\boldsymbol{number\ of\ glasses}}$
12	**9.00**	$\dfrac{\mathbf{9}}{\mathbf{12}} = \mathbf{0.75}$
20	**15.00**	$\dfrac{\mathbf{15}}{\mathbf{20}} = \mathbf{0.75}$
24	**18.00**	$\dfrac{\mathbf{18}}{\mathbf{24}} = \mathbf{.75}$

What does this mean?

A. **There is a proportional relationship between the amount collected and the number of glasses sold for Aaron's data.**

Let's look at Paisley's table.

Number of Glasses	**Amount Collected ($)**	$\dfrac{\boldsymbol{amount\ collected}}{\boldsymbol{number\ of\ glasses}}$
11	**8.25**	$\dfrac{\mathbf{8.25}}{\mathbf{11}} = \mathbf{0.75}$
16	**12.00**	$\dfrac{\mathbf{12}}{\mathbf{16}} = \mathbf{0.75}$
30	**22.50**	$\dfrac{22.50}{30}$**=0.75**

What does this mean?

A. **There is a proportional relationship between the amount collected and the number of glasses sold for Paisley's data.**
B. **Who charges more for a glass of lemonade? They both charge $0.75! I know because this is the unit rate; the price per glass.**

Excellent job!

Additional Problems (if needed):

Use the table below. Do x and y have a proportional relationship? Explain how you know.

X	**2**	**3**	**5**	**8**

Student creates unit rates.

Student answers the questions, and explain their answers.

Student compares their work with the teacher's work.

Student analyzes the data in the table and finds the unit rate.

Student notices that the unit rates are not equivalent, therefore this is

<table>
<tr><td>

Y	**5**	**7.5**	**12.5**	**18**

I will give you time to answer the questions.
[Pause]
$\frac{5}{2}=\frac{7.5}{3}=\frac{12.5}{5}$**=2.5**
But $\frac{18}{8}=2.25$
This means that x and y do NOT have a proportional relationship.
</td><td>not a proportional relationship.</td></tr>
<tr><td>
<u>Independent Practice</u> (1 minute)
Great work 7th grade! Today, we reviewed Understanding Proportional Relationships: Equivalent Ratios. I hope you're seeing some connections to ratios, unit rates and proportions! You sure did a great job! After the video, you will have some problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.
[Teacher shows student practice page under document camera or camera zooms in on student practice page.]
Good luck and do your best!

You can find the independent practice problems on the state website!
</td><td></td></tr>
<tr><td>
<u>Closing </u>(1 min)

I enjoyed reviewing understanding Proportional Relationships: Equivalent Ratios with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!
</td><td></td></tr>
</table>