PBS Lesson Series

Math: Grade 7, Lesson 14, Graph proportional relationships

Lesson Focus: Use a graph to recognize proportionality
Practice Focus: Students will focus on practicing graphing relationships and analyzing graphs in order to determine proportionality.
Objective: Use graphing to recognize a proportional relationship. Interpret the graph of a proportional relationship. Recognize graphs of proportional relationships.
Key Vocabulary: x-value, x-coordinate, y-value, y-coordinate
TN Standards: 7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2c, 7.RP.A.2d
Teacher Materials:

- Paper or white board
- Pen/pencil/marker
- Graph paper
- Examples written/typed (to save time)
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Graph paper

**Note: Have all the axes drawn on your graphs before the lesson to save time.*

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 7th graders out there, though all children are welcome to tune in. This lesson is the fourteenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about using a graph to recognize proportionality in mathematics! Before we get started, to participate fully in our lesson today, you will need:** • **Paper and a pencil** • **Graph paper (If you do not have graph paper you can still participate!)** • **Surface to write on**	Students get materials ready for the lesson.

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Ok, let's begin!

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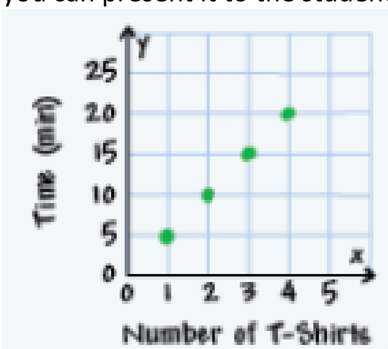
<u>Intro</u> (3 minutes)
Let's begin by looking at data on a graph.
This graph shows the time it takes Jacey to print t-shirts for her school's math club. [Create this graph before the lesson so you can present it to the students.]



Let's analyze this graph.

a. **Use the points on the graph to complete the table.** [As you complete the table point out the points on the graph, paying extra attention to distinguishing the x-values and the y-values.]
Notice that the axes are labeled. The x-axis is labeled "number of t-shirts". The y-axis is labeled "time in minutes". The first point we see is (1, 5). 1 is the x-value, so it is the number of t-shirts. Let's place 1 in our table on the x row. The y-value is 5, so it is the number of minutes. Let's place 5 in the table in the y-row. What does the point (1, 5) tell us? [Pause]
The point (1, 5) tells us that it takes Jacey 5 minutes to print 1 t-shirt.
Next we have the point (2, 10). It takes 10 minutes to print 2 t-shirts. Let's place that ordered pair in the table.
Then we have (3, 15). It takes 15 minutes to print 3 t-shirts.
Finally we have (4, 20). What does this ordered pair tell us? [Pause]
Good! It tells us that it takes Jacey 20 minutes to print 4 t-shirts.

Number of t-shirts (x)	**1**	**2**	**3**	**4**
Time in minutes (y)	**5**	**10**	**15**	**20**

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Student thinks about the graph. They notice any patterns.

Students think about how to organize the data from the graph in a table representation.

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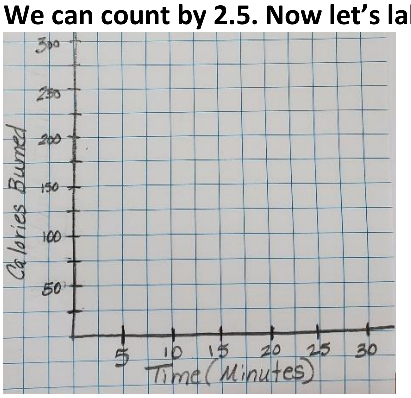
Now that we have our data organized in a graph and in a table, let's decide if these quantities are proportional. Do you remember how to do that? [Pause] **Yes! We can create ratios from the table and determine if the ratios are equivalent! Let's use the ratio of time to number of t-shirts. This means that the time will be our numerator and the number of t-shirts will be our denominator.** **What are the ratios?** [Pause] $$\frac{5}{1}, \frac{10}{2}, \frac{15}{3}, \frac{20}{4}$$ **Are these ratios equivalent? Do they have the same unit rate?** [Pause] **Let's see!** $$\frac{5}{1} = 5, \frac{10}{2} = 5, \frac{15}{3} = 5, \frac{20}{4} = 5$$ **Yes! Every ratio has a unit rate of 5. The ratios are equivalent. This means that this relationship is proportional.** **b. Now let's look at the graph a little closer. Start at (1, 5). As you move from one point to the next on the graph, how does the x-coordinate change?** [Follow each point on the graph as you say the following.] **The x-value increases by one each time.** **How does the y-value change?** [Start at (1, 5) again. Follow each point on the graph as you say the following.] **The y-value increases by 5 each time.** **Does this agree with the pattern we see in our table?** [Pause] **Yes it does! As the number of t-shirts (the x-value) increases by 1, the time (the y-value) increases by 5.** **What do you think this means?** [Pause] **It takes 5 minutes to print each t-shirt! Good job!** **This is the type of thinking we are going to be using in the lesson. Are you ready to get started? Let's go!**	Student creates ratios from the table, then converts them to unit rates. Student analyzes each point in the graph, and they think about how each variable changes from one point to the next point. Student thinks about the relationship between the graph and the table.
<u>Teacher Model</u> (13 minutes) Objective 1: Use graphing to recognize a proportional relationship **Tanya exercised for 30 minutes. She noted the calories burned at three times during her workout. How can Tanya**	Objective 1: Students will use a graph to recognize proportional reasoning. This will lead to a better understanding of slope later.

use this information to find how many calories she burned after 15 minutes of exercise?

Time (minutes)	**10**	**20**	**30**
Calories burned	**95**	**190**	**285**

What do we know?
[Pause]
We know the number of calories burned after 10 minutes, 20 minutes and 30 minutes.
[Pause]
What are we trying to find?
[Pause]
We want to know how many calories Tanya burned after 15 minutes. That is not given in the information.
How can we use the information we are given to find the number of calories burned after 15 minutes?
[Pause]
Maybe it will help to look at the data organized a different way. How can we represent this data in a different way?
[Pause]
Yes! We can graph this data.
[Have the axes drawn but not labeled. Explain what you are doing as you create the graph.]
We have an x-axis and a y-axis drawn.
What should we label them? [Pause]
The x-axis will represent the time (in minutes) [Label the x-axis.]
The y-axis will represent the number of calories burned.
[Label the y-axis.]
What about the scale? Do we have room to graph 285 calories if we count by ones?
[Pause]
No, so we need to think of some other scale. [Pause]
Let's count by 25. What about the x-axis?
[Pause]
We can count by 2.5. Now let's label our scale.



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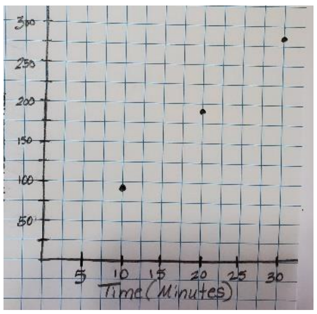
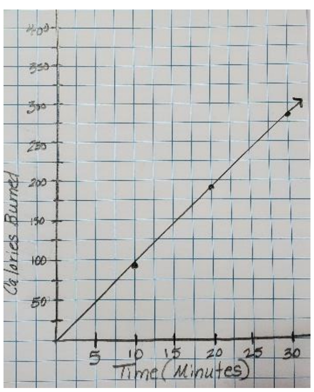
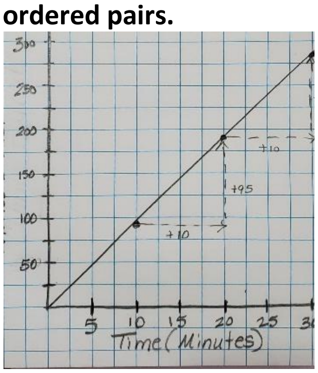
Now that we have our graph drawn and labeled, let's plot the data.
[As you graph the ordered pairs, say the following.]
First we have (10, 95).
Remember the x-value tells us to go right 10. The y-value tells us to go up 95. Then we place the point at the intersection of those numbers.
Next we graph (20, 190). Again the x-value tells us to go right 20, and the y-value tells us to go up 190.
The last ordered pair, (30, 285) tells us to go right 30 and up 285.



Is this relationship linear? Can we connect these points with a line? Yes we can!



Let's use our graph to find the constant of proportionality. Find the differences between the coordinates of any two ordered pairs.



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Student plots the order pairs, paying special attention to how to graph the x-value and the y-value.

Student thinks about how to determine if this is linear.

Student draws a line through the data points, so they know it is linear.

Student uses the graph to find the constant of proportionality.

Student thinks about how to use the constant of proportionality to answer the question.

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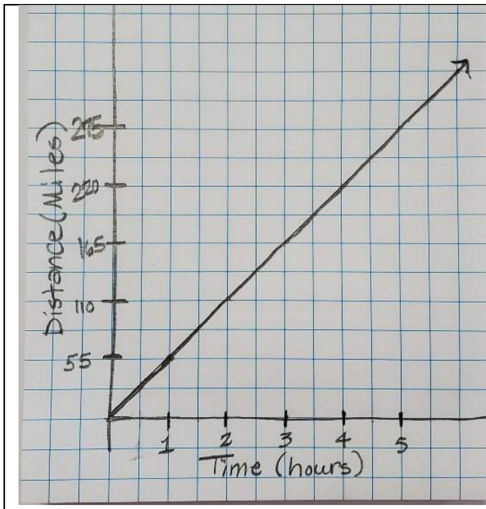
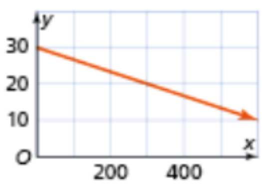
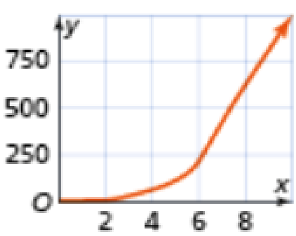
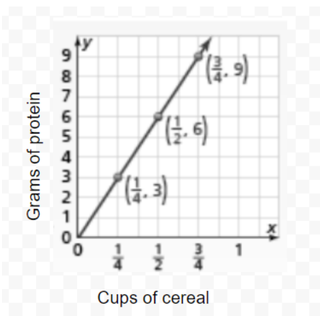
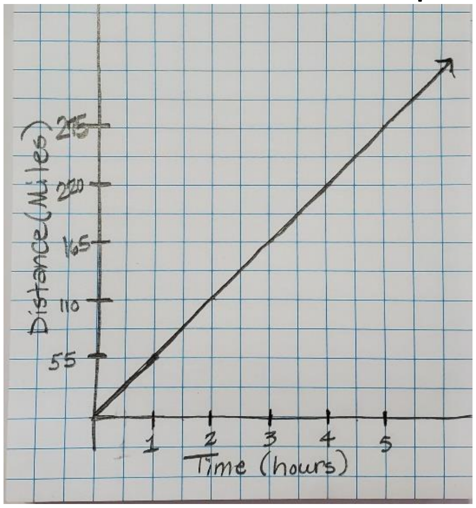
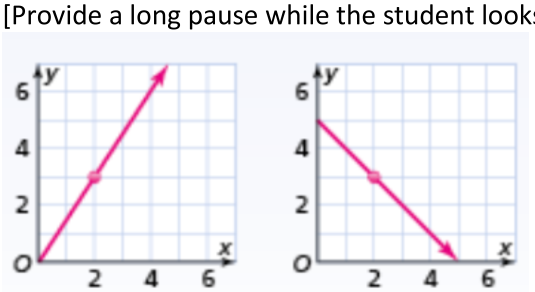
So the constant of proportionality is $\frac{95}{10}$ or 9.5. How can we use this to find the number of calories Tanya burns in 15 minutes? [Pause] **We can use a proportion.** $$\frac{c}{15}=\frac{95}{10}$$ **Let's solve this proportion. We can multiply both sides by 15 to isolate the number of calories burned, c.** $$15\cdot\frac{c}{15}=\frac{95}{10}\cdot 15$$ $$c=142.5$$ **This means that Tanya will burn 142.5 calories in 15 minutes.** **We could also use what we learned about the equation. We know that y = (constant of proportionality)x.** **So we could create an equation:** **y = 9.5x or using our variables: c = 9.5t, where t is the time in minutes and c is the number of calories burned.** **So if c = 9.5t, and we want to know the number of calories burned, c, in 15 minutes, we can substitute the 15 for time, t.** **c = 9.5 (15)** **c = 142.5** **Tanya will burn 142.5 calories in 15 minutes! Notice that whether we use the proportion** [Refer back to the proportion from earlier in the problem.] **or whether we use c = 9.5t, we are still multiplying the constant of proportionality by 15!** **We used the data given in a table to graph, now let's start with a graph to solve this next problem.** Objective 2: Interpret the graph of a proportional relationship **The graph shows a proportional relationship between the distance and the amount of time Mr. Brown drives. Let's take a close look at the graph.** [Have the graph prepared and show it to the students as you talk about it.]	Student sets up a proportion and solves to find the calories. Student uses y = mx to find the number of calories. Student notices that both strategies involve multiplying the constant of proportionality by 15. Objective 2: Interpret the graph of a proportional relationship: The student will use a graph to find the constant of proportionality. The student will notice that when the x-value is 1, the ordered pair represents the unit rate for the problem. The student will use the constant of proportionality to solve a problem. This reasoning will lead to a deeper understanding of interpreting the slope later on.

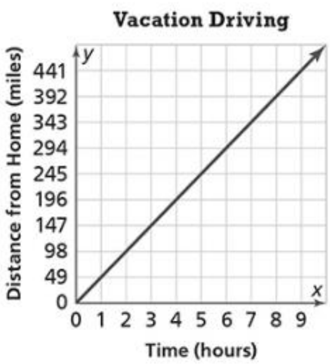


What do you notice about the graph? [Pause] **The graph is linear and passes through the origin. What does this tell us about the situation?** [Pause] **It is a proportional relationship!** **Let's think about what each of these points tells us about the problem: (0, 0), (1, 55) and (5, 275).** [Pause] **What does the point (0, 0) tell us about the problem?** [Pause] **It tell us that at time 0, Mr. Brown had traveled 0 miles. Does that make sense?** [Pause] **Yes!** **What does (1, 55) tell us about the problem?** [Pause] **This point tells us that at 1 hour Mr. Brown had traveled 55 miles. What else does this point represent?** [Pause] **Think about speed.** [Pause] **Yes! This point tells us that Mr. Brown was traveling 55 miles in 1 hour or 55 miles per hour! When the x-value is 1, the y-value is the constant of proportionality!**	Student notices that the graph passes through the origin and that it is linear. Student recalls that this means it is a proportional relationship. Student thinks about what each point means in the context of the problem.
What about (5, 275)? [Pause] **This tells us that when Mr. Brown had driven for 5 hours, he had traveled a distance of 275 miles. Does this agree with our constant of proportionality, 55? How can we know?** [Pause]	Student thinks about the constant of proportionality.
We can write the ratio $\frac{275}{5}$. Can we change this to a unit rate? [Pause] **Yes!** $\frac{275}{5} = 55$	Student realizes that they can use any point on the graph to find the constant of proportionality.

We know our constant of proportionality. Let's write an equation that represents Mr. Brown's trip. **y = 55x (where x is the time and y is the miles traveled)** **We have created a graph from data, we have analyzed a graph to find data. Now let's practice determining if a graph represents a proportional relationship.**	
Objective 3: Recognize graphs of proportional relationships **Explain why each graph does or does not represent a proportional relationship.**	Objective 3: The student will recognize graphs of proportional relationships by noticing that they pass through the origin and they are linear.
 **This graph does represent a proportional relationship, because it is linear and it passes through the origin.**	Student recognizes that this graph meets both requirements of being a proportional relationship.
 **What do you think about this one?** [Pause] **It is linear! Does it pass through the origin?** [Pause] **No, it does not pass through the origin. So, although it is linear, it is does NOT represent a proportional relationship because it does not pass through the origin.**	Student recognizes that this graph does not pass through the origin, therefore it is not a proportional relationship.
What do you think about this one? [Pause]  **This graph does pass through the origin, but it is not linear (notice that it curves and is not straight).** [Pause] **So this graph does NOT represent a proportional relationship.** **Remember that graphs of proportional relationships will be linear and will pass through the origin.**	Student recognizes that this graph is not linear, therefore it is not a proportional relationship.

We have looked at three examples of using data and graphs to solve proportional relationship problems. Let's practice what we have learned! Here we go!	
<u>Guided Practice</u> (11 minutes) **Let's work together on this one.** [I do] **Each $\frac{1}{4}$ cup serving of cereal has 3 grams of protein. How can you use this graph to determine whether the quantities are proportional and to find how many grams of protein are in 1 cup of cereal?**  **How can we use the graph to determine whether the relationship between cups of cereal and grams of protein is proportional?** [Pause] **The graph is linear and it passes through the origin, therefore it is a proportional relationship!** **Can we use the data given in the graph to determine the number of grams of protein in 1 cup of cereal?** [Pause] **We can find the constant of proportionality and create an equation! You try!** [Pause] **We can choose any point on the graph, create a ratio, and find the unit rate! Let's use $(\frac{1}{2}, 6)$.** $$\frac{6}{\frac{1}{2}}$$ **How many halves are in 6?** [Pause] **12! Good! So that means the unit rate is 12. Let's think about what this means in the context of our problem.** **The unit rate is the comparison of the number of grams of protein to the cups of cereal.** [Pause] $$\frac{12g}{1cup}$$ **So there are 12 grams of protein in 1 cup of cereal!**	Student thinks about the information given in the problem and how it relates to the given graph. Student thinks about how to use the graph to determine a proportional relationship. Student uses what they have learned to find the constant of proportionality. Student finds the unit rate and knows that this is the constant of proportionality. Student uses the constant of proportionality to solve the problem.

[We do] **Let's think back to Mr. Brown's trip.** 	Student recalls the Mr. Brown problem.
Suppose the graph of Mr. Brown's trip is extended. Find the ordered pair with an x-coordinate of 7. What does this point represent in this situation? [Pause] **We know that the constant of proportionality is 55 miles per hour. What does an x-coordinate of 7 mean?** [Pause] **The x-coordinates in this problem are the time in hours, so the 7 represents hours. How can we know how far Mr. Brown traveled in 7 hours?** [Pause] **We can use our equation we created!**	Student thinks about what the point represents.
y = 55x, where y is the distance traveled and x is time. [Pause] **Since we know the 7 is time, x = 7.** **y = 55(7)** **y = 385** **Mr. Brown traveled 385 miles in 7 hours!**	Student uses the equation to solve the problem.
[You do] **Last one! Look at these graphs, and determine if they represent proportional relationships. Explain how you know. I am going to give you a couple of minutes to do this one on your own.** [Provide a long pause while the student looks at both graphs.]	Student interprets the answer.
	Student looks at the graphs and determines if they represent proportional relationships.

The first graph DOES represent a proportional relationship. It is linear, and it passes through the origin. **The second graph does NOT represent a proportional relationship. It is linear, but it does NOT pass through the origin.** Additional Problems (if needed): **The graph shows a proportional relationship between a family's distance from home and the time spent driving.**  **a. What does the point (1, 49) represent?** **b. Write an equation that represents the proportional relationship.** [Pause while the student works the problem.] **a. The point (1, 49) represents the time at 1 hour and the distance of 49 miles from home. This means that the family has traveled 49 miles from home at 1 hour.** **b. The constant of proportionality can be found by looking at the point where the x-coordinate is 1. The constant of proportionality in this problem will be $\frac{49}{1}$ or 49. This is means the family is traveling 49 miles per hour!**	Student analyzes the graph. Student thinks about what the point means in the context of the problem, then uses the point to write an equation. Student compares his work to the teacher's work.
<u>Independent Practice</u> (1 minute) **Today we have looked at proportional relationships and used the constant of proportionality to solve problems.** **Great work, 7th grade! You sure did a great job! After the video, you will have some problems to practice on your own. Good luck and do your best!"**	
<u>Closing</u> (1 min)	

I enjoyed reviewing graphs of proportional relationships with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!	