Math: Grade 7, Lesson 13, Describe Proportional Relationships: Constant of Proportionality

Lesson Focus: This lesson will focus of the use of the constant of proportionality and how to write equations that represent proportional relationships. Focus will then shift to using those equations to solve problems involving proportional relationships.
Practice Focus: Students will focus on using the constant of proportionality in order to write equations and use them to solve problems.
Objective: The objective of this lesson is to use the constant of proportionality in order to write equations and then use them to in order to solve problems.
Key Vocabulary: Unit rate, Ratio, Constant of Proportionality, Equivalent
TN Standards: 7.RP.A.2
Teacher Materials:

- Paper or white board
- Pen/pencil/marker
- Prepared copies of the examples (to save time)
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 7th graders out there, though all children are welcome to tune in. This lesson is the thirteenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about using the constant of proportionality to write equations and then use them to solve problems. Before we get started, to participate fully in our lesson today, you will need:** • **Paper, a pencil and a surface to write on and the optional student packet**	Students get materials ready for the lesson.

Ok, let's begin!	
<u>Intro</u> (5 minutes) **Have you ever run a race before? It can be quite tiring and fun at the same time! Well Jamal is about to run a very big race and needs our help!** [Teacher displays problem on the board.] **Jamal can run 1 mile in 5.05 minutes. If Jamal maintains this pace during a 5-Kilometer (5k) race, He expects to break the course record of 15.25 minutes. Is Jamal's expectation reasonable? Explain.** **Wow this is a big race for Jamal so I don't want to make a mistake but certainly want to help him! What if we let m equal the number of miles in a 5k race. Do you remember how many km are in a mile?** [pause] **That's it! There are 1.6 km in 1 mile. What have we already worked with this week that could help us solve this problem? Yes! Proportions!** [Teacher shows conversion work on the board.] $$\frac{\mathbf{1\ mile}}{\mathbf{1.6\ Km}} = \frac{\mathbf{m\ Miles}}{\mathbf{5\ Km}}$$ **We can solve this equation for m. How can we isolate the variable, m?** [pause] **Yes! Since the 5 is being divided, we can eliminate the 5 in the denominator by multiplying. Let's multiply both sides by 5.** $$\mathbf{5 \cdot \frac{1}{1.6} = \frac{m}{5} \cdot 5}$$ $$\mathbf{\frac{5}{1.6} = m}$$ $$\mathbf{3.125 = m}$$ **What does this mean?** [pause] **That's it! There are 3.125 miles in a 5k race.** [Teacher writes 3.125 miles on the board.] **The problem says that Jamal can run 1 mile in 5.05 minutes. Wow that is fast! Are you that fast?** [pause] **If the total race is 3.125 miles long we need to figure out if Jamal can break the course record of 15.25 minutes. Let's set up a proportion!** [Teacher will show work below.] $$\frac{\mathbf{1\ mile}}{\mathbf{5.05\ minutes}} = \frac{\mathbf{x\ miles}}{\mathbf{15.25\ minutes}}$$	Student watches and listens. They think about ratios, unit rates, and constant of proportionality.

How can we solve for x? That's right! We can use the inverse operation for division! What operation is that? [Pause] **That's it multiplication! Let's multiply both sides by 15.25** $$\mathbf{15.25 \cdot \frac{1}{5.05} = \frac{x}{15.25} \cdot 15.25}$$ **That gives us 15.25 divided by 5.05 to be the value of x.** $$\mathbf{\frac{15.25}{5.05} = x}$$ **Let's put this in our calculator. 15.25 divided by 5.05 gives us 3.02.** $$\mathbf{3.02 = x}$$ **What does 3.02 mean in the context of the problem?** [Pause] **You got it! At Jamal's pace he can run 3.02 miles in 15.25 minutes. Since this is just shy of the 3.125 race he won't quite break the course record at that pace.** **We just used the constant of proportionality to help us solve a real world problem to help Jamal! Wouldn't you like to learn more about it? Let's do it!**	
<u>Teacher Model</u> (10 minutes) **The constant of proportionality is the constant multiple that relates proportional quantities x and y. It is the value of the ratio $\frac{y}{x}$ and is represented by k in the equation y = k (x).** **That's a lot of variables. Here is another example of how to calculate the constant of proportionality and how to write an equation. Later on we will use it in order to solve problems.** Objective 1: Calculating the Constant of Proportionality and Writing an Equation Using the Constant of Proportionality [Teacher reads problem aloud and displays on board.] **A sponge is an example of a filter feeder. It takes in food by filtering water through its body. The sponge maintains a constant flow of water through its body. What is the constant of proportionality if the sponge passes 8 liters in 10 hours and 12 liters in 15 hours?** **It may be helpful to make a table in order to organize the information. You can do this too!** [Teacher displays table.] **Go**	Objective 1: Students learn how to apply the definition of the constant of proportionality and learn how to write an equation using it.

ahead and copy this table down as we will use it throughout the problem. [Pause]

Student copies down table

Liters	Hours	$\frac{liters}{hours}$

From the problem, the sponge can pass 8 liters of water in 10 hours so add this information to the table. [Teacher adds 8 liters and 10 hours to their table.]

Student thinks about the information given in the problem.

Liters	hours	$\frac{liters}{hours}$
8 liters	**10 hours**	

Student adds information to their table.

Next from the problem the sponge passes 12 liters in 15 hours so add that to the table as well. [Teacher adds 12 liters and 15 hours to their table.]

Liters	Hours	$\frac{liters}{hours}$
8 liters	**10 hours**	
12 liters	**15 hours**	

Student adds information to their table

Now we can start calculating the constant of proportionality. We are going to place the liters in the numerators of the ratio and the corresponding time in the denominator of the ratio. We will have the two ratios of $\frac{\mathbf{8\ liters}}{\mathbf{10\ hours}}\ \boldsymbol{and}\ \frac{\mathbf{12\ liters}}{\mathbf{15\ hours}}$**.** [Teacher writes ratios on the board.]

Do you know what $\frac{\mathbf{8}}{\mathbf{10}}$ **is equal to?** [pause] **That's right** $\frac{8}{10} = \frac{4}{5}$ **by dividing both the top and bottom by 2. Similarly do you know what** $\frac{\mathbf{12}}{\mathbf{15}}$ **is equal to?** [pause] **You got it again** $\frac{\mathbf{12}}{\mathbf{15}} = \frac{\mathbf{4}}{\mathbf{5}}$ **by dividing top and bottom by 3.** [Teacher writes work shown with ratios on board.]

Since both ratios equal $\frac{\mathbf{4}}{\mathbf{5}}$ **the relationship is proportional and the constant of proportionality is** $\frac{\mathbf{4}}{\mathbf{5}} = \mathbf{0.8}$ [Teacher writes 0.8 on the board.]

We can finish this problem by putting the constant of proportionality in our table and writing an equation. [Teacher places constant of proportionality in table and writes the equation on the board.]

Liters	Hours	$\frac{liters}{hours}$
8 liters	**10 hours**	$\frac{4}{5}$ = 0.8
12 liters	**15 hours**	$\frac{4}{5}$ = 0.8

Student adds information to their table

Our equation for this constant of proportionality would be; y = 0.8(x) oy y = 4/5 x where x is the number of liters filtered and y is the amount of time.

Objective 2: Using the Constant of Proportionality to solve Problems

Objective 2: Students use the constant of proportionality to solve problems.

Now that we know how to find the constant of proportionality, let's look at some examples of how we can use it to solve problems

[Teacher puts the problem on the board and reads it aloud.]

If 1 inch is equal to 2.54 centimeters. How many centimeters is an 18-inch ruler? [Pause]

First we need to find the constant of proportionality in order to write an equation. Luckily for us the problem tells us the constant of proportionality. For every 1 inch there are 2.54 centimeters. That means that 2.54 is the constant of proportionality.

Student thinks about how to create the ratio.

Now we need to write our equation. [Teacher writes equation on the board.] **y = 2.54 (x) where Y is the number of centimeters and x is the number of inches.**

Student thinks about how to write an equation

The problem goes on to ask; how many centimeters are in an 18 inch ruler? Using our equations we can solve in order to find the number of centimeters. [Teacher writes solving process below.]

$$\boldsymbol{y = 2.54\,x}$$
$$\boldsymbol{y = 2.54\,(18\,inches)}$$
$$\boldsymbol{y = 45.72}$$

There are 45.72 inches in an 18-inch ruler.

You are starting to get the hang of this! Let's move on then I think you will be ready to start trying parts on your own and eventually do the whole problem on your own! You can do it!

Guided Practice (14 minutes) **Since it is spring time I thought it would be nice to have flowers around my house! I recently went to the florist and she had this problem for me. Check it out.** [I do] [Teacher posts and reads problem aloud.] **A florist sells a dozen roses for $35.40. She sells individual roses for the same unit cost. Write an equation to represent the relationship between the number of roses, x, and the total cost of the roses, y. How much would 18 roses cost?**	
Work along with me. First we need to calculate how much each rose costs which is the constant of proportionality. The problem says that for a dozen roses it cost $35.40. How many roses are in a dozen? [pause] **There are 12 in a dozen. That means we have a ratio of** $\frac{\$35.40}{12\ Roses}$. **Let's write that down.** [Teacher writes ratio on the board]. **We need to calculate the constant of proportionality in order to write our equation. This means that we need to divide the numerator and denominator by 12.** [Teacher shows work below on the board] $$\frac{\$35.40}{12\ Roses} \div \frac{12}{12} = \frac{\$2.95}{1\ rose}$$ **From this work we can see that it cost $2.95 per rose which is the constant of proportionality. The problem asks us to write an equation to represent the relationship between the number or roses, x, and the cost, y.** [Teacher writes the equation below on the board while saying it aloud.] **y = 2.95 (x) is our equation.** **Lastly, the problem asks for a price of 18 roses. Since x represents the number of roses I am going to substitute 18 for x.** [Teacher shows work below on the board.] $$y = 2.95\ (x)$$ $$y = 2.95\ (18)$$ $$y = \$53.10$$ **From the equation and simplifying I was able to calculate that it would cost $53.10 to purchase 18 roses.**	Student thinks about a strategy. Student answers prompt

On this next one I want you to help me as I go through it ok? [Pause] **Don't be worried you can do it!** [We do] **Do you enjoy baseball? How about hotdogs? Maybe spending time outside with friends? You may just enjoy this next problem then!** [Teacher posts and reads aloud the problem.] **The manager of a concession stand estimates that she needs 3 hot dogs for every 5 people who attend a baseball game. If 1,200 people attend the game, how many hot dogs should the manager order?**	Student think about strategies to use and what the first step for this problem will be but also understand why it is the first step.
From our previous examples what would be our first step? [Pause] **Right! We need to calculate the constant of proportionality. Why do we need this though?** [Pause] **Yes. We need it so that we can write our equation and then use that equation to solve the problem.**	
The problem tells us that the manager needs 3 hot dogs for every 5 people. That sounds like a ratio to me! Can you write a ratio of hotdogs to people for me? [Pause] **Good job. Your ratio should look like mine on the board.** [Teacher writes ratio below on the board.] $$\frac{3\ hotdogs}{5\ people}$$	Student writes a ratio.
Can we reduce the fraction $\frac{3}{5}$? [Pause] **The answer is no. What can we do?** [Pause] **That's it! We could change it to a decimal, 0.6, or we could just leave it the way it is. Recall from our previous example that we use the constant of proportionality to form an equation that we are able to use to solve problems. We usually make those equations in the form of y = k(x) where k is the constant of proportionality.**	Student responds to prompt.
What do you think our constant of proportionality is for this problem? [Pause] **Got it! It's the $\frac{3}{5}$, or 0.6, that we calculated just a minute ago.**	Student responds to prompt
Now can you write the equation for me? [Pause] **My equation looks like this.** [Teacher writes equation on the board.] $y = \frac{3}{5}$**(x). That's right it could also write it like y= 0.6x. What does x represent in this equation?** [Pause] **What**	Student writes an equation Student responds to prompt

does y represent in this equation? [Pause] **The x variable represents the number of people who will attend the baseball game and the y variable is the number of hotdogs that will need to be ordered. It is really important to know what our variables mean!**	Student responds
The last part of the problem is using our equation in order to figure out how many hotdogs the manager needs to order if 1,200 people attend the game. Which variable represents the number of people in our equation $y = \frac{3}{5}(x)$**?** [Pause] **Did you say x?** [Pause] **You got it! Can you calculate how many hotdogs the manger needs? Its ok take your time!** [Pause] [The teacher solves the problem as shown below to reinforce students work.] $\boldsymbol{y = \frac{3}{5}(x)}$ $\boldsymbol{y = \frac{3}{5}(1200)}$ $\boldsymbol{y = 720}$	Student checks their answers
After working it out the manager needs 720 hotdogs! Does your answer look like mine? [pause] **Great! You do this next one all by yourself!** [You do] [The teacher reads problem and puts it up on the board.]	
A half dozen cupcakes cost \$15. What constant of proportionality relates the number of cupcakes and the cost? Write an equation that shows the relationship. How much would it cost for two dozen cupcakes?	Student thinks about what information is given in the problem.
What would you do first? [pause] **Find the constant of proportionality?** [pause] **Great! Go ahead and do it!** [pause]	Student determines what the problem is asking and how to proceed.
The constant of proportionality is going to be a price per cupcake. The problem says that it is \$15 per half dozen. How many cupcakes are in a half dozen? [pause] **6 you got it. What ratio did you write?** [pause] **Perfect it should look like this** $\frac{\$15}{6\ cupcakes}$**. Are you able to reduce that ratio?** [pause] **Awesome can you do it for me and then write it down?** [pause] **Here is the ratio that I wrote** $\frac{\$2.50}{1\ cupcake}$**. Does it look like yours?** [pause] **Great. Let's keep going.**	Student responds and writes the ratio.

If it cost \$2.50 per cupcake can you write an equation for me using this constant of proportionality? [pause] **Go for it!** [pause] **Make sure you explain what your variables represent!** [pause] **I wrote the equation y = 2.50 (x) where x is the number of cupcakes purchased and y is the total cost. Does that look similar to yours?** [pause] **Glad we are on the same page!** **Last part of the problem! You need to figure out how much it will cost to purchase 2 dozen cupcakes. How many cupcakes is that?** [pause] **24! Go ahead and finish the problem! You got it.** [Pause teacher after the pause will substitute 24 in for x and solve.] $y = \mathbf{2.50}\ (x)$ $y = \mathbf{2.50}\ (\mathbf{24})$ $y = \$\mathbf{60}$ **You got it! It would cost \$60 for two dozen cupcakes.** Additional problem (if needed) **If a car can travel 50 miles on 2.5 gallons of gas, what is the constant of proportionality per gallon? Write an equation using the constant of proportionality. How far could the car travel with 30 gallons of gas?**	Student writes equation and explains variables. Student uses equation to solve a problem
Independent Practice (1 min) **Great work, 7th grade! Today, we worked on how to use constants for proportionalities in order solve problems and write equations! You sure did a great job! After the video, you will have some problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **I enjoyed reviewing finding unit rates with ratios of fractions, and use them to solve multi-step problems with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	

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