PBS Lesson Series

Math: Grade 8, Lesson 19, Volume of Cylinders

Lesson Focus: Using formulas to find the volume of real-world objects shaped like cylinders.
Practice Focus: Students will focus on using formulas to find the volume of real-world objects shaped like cylinders.
Objective: Students will use formulas to find the real-world volume of objects with a focus on cylindrical shapes using attributes like radius, diameter and circumference to achieve this objective.
Students will discern information needed to construct the formula to find volume and ignore extraneous information.
Key Vocabulary: Volume, cylinder, radius, diameter, circumference
TN Standards: 8.G.C.7
Teacher Materials:

- Whiteboard & Markers
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator or calculator app is useful but not necessary

**Note: This may not be included in every lesson. In round 1, there were very few notes to the teacher delivering. If you have them they go here.*

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all children are welcome to tune in. This lesson is the nineteenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about using formulas to find the volume of real-world objects shaped like cylinders! Before we get started, to participate fully in our lesson today, you will need:** • **Paper and a pencil, and a surface to write on** • **Calculator or calculator app is useful but not necessary**	Students get materials ready for the lesson.

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Ok, let's begin!	
Intro (2 min) **Cylinders are everywhere. Take a look around your kitchen. Do you have any cans of soup? How about a container of oatmeal that's round? If you do, those are cylinders!** **A cylinder has two parallel congruent circular bases. The bases are connected by a curved surface.** [Draw a cylinder on the whiteboard to illustrate.] **Today, we are going to be finding the volume of a cylinder. The volume of something is the number of cubic units needed to fill a given space. To find volume in a cylinder, we'll need to be able multiply the area of the base by the height.** **There are a few other terms you should keep in mind while we work some problems with cylinders. You may have already learned about things like circumference, diameter and radius from studying circles.** [Draw a circle.] **Let's take a quick look at a circle and refresh your memory. Do you recall that the circumference is the distance around a circle? Then, the radius is a line segment with one endpoint at the center of the circle, and one endpoint on the circle** [Draw the radius]**. Finally, the diameter is a line segment that passes through the center of the circle, with endpoints of either side** [Draw the diameter]**. As you can see, the diameter is also twice the radius.** **We use pi or π to express the ratio of the circumference of a circle to the diameter of the circle,** [Draw the pi symbol on the board as equal to C/D.] **but it also has application with cylinders, too. Remember, we will use 3.14 to approximate the value of pi.** **You find the volume, V, of a cylinder by multiplying the height, h, by the area of the base, B:** [Teacher speak and write the formula.] ***V = Bh*** **This formula likely looks familiar to you as it's also used to find the volume of a prism. Both have two parallel, congruent bases, but the bases are polygons for prisms, and circles for a cylinder. Since the base of a cylinder is a circle, $B = \pi r^2$** [Write the formula.] **So, $V = Bh$ or $V = \pi r^2 h$** **Now that you understand what to look for when finding the volume of a cylinder, let's try some application problems together!**	Students review the components of the equation for the volume of a cylinder. Students then see the volume in symbolic form as an equation.
Teacher Model (10-12 min)	

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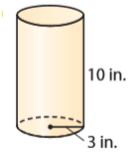
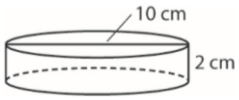
Objective 1: Using formulas to find the volume of cylinder. Example 1 **Be sure to keep in mind these steps when using the formula for the volume of a cylinder: Substitute, simplify, and multiply. We'll take a look at this step-by-step in the following problem.** **Let's solve for the volume of the following cylinder** [Write or show the cylinder on the board, with measurements.]  **As you can see, the radius is 3 inches, and height is 10 inches. First step, we're going to use substitution in our formula: So, volume equals 3.14 times 3 squared for the radius times 10 for the height.** [Teacher write the problem on the board and explain while moving through the steps.] $V = \pi r^2 h$ $\approx 3.14 \cdot 3^2 \cdot 10$ $\approx 3.14 \cdot 9 \cdot 10$ $\approx 282.6 \text{ in}^3$ **Notice we simplified 3 squared into 9, then we multiplied to find our answer, which was 282.6. So the volume is about 282.6 inches cubed. Why cubed? Remember when we said that volume was the number of cubic units needed to fill a given space? Why did we say the volume was "about" 282.6 inches cubed? Did you notice that we used this symbol ≈ ?** [Point to the approximate symbol in the formula written on the board.] **This means approximate, or almost equal to. This is because the value substituted for pi is approximate, so the answer must also be approximate.** **What if you want a formula for the volume of a cylinder that involves the diameter *d* instead of the radius *r*? How would you rewrite it? If you recall, we said diameter was across the entire circle, so it is twice the radius, so $2r = d$ and $r = \frac{d}{2}$** [Write this on the board while speaking.] **You can use this to figure out the radius. Or, if we then use $\frac{d}{2}$ in place of *r*, we would have the following:** $V = \pi \left(\frac{d}{2}\right)^2 h$ Objective 2: Use formulas to find the volume of a cylinder in a	Objective #1: Students represent and solve for the volume of a cylinder by describing the volume formula V = Bh of a cylinder in terms of its base area and height

real-world context. Example 1 **Let's take a look at another problem. This one will involve a drum, which is also a cylindrical object. Did you know that the world's largest bass drum is used by the Longhorn Band at the University of Texas at Austin? It's even got a name – they call it Big Bertha, and it is definitely a very big drum, with a diameter of 8 feet and height of 4.5 feet. Let's find the volume of that drum, and then round the answer to the nearest tenth.** **What information do we need to find volume?** [Pause] **That's right, we need the radius and the height. What are the numbers I mentioned?** [Pause] **Yes, 4.5 feet was the height, and 8 feet was the diameter.** [Teacher writes these measurements on the board.] **But we need to the find the radius:** [Write this on the board while speaking.] $r = \frac{d}{2} = \frac{8}{2} = 4$**ft** **We used 8 feet for the diameter, which simplified into four feet. Since the diameter is twice the radius, this makes perfect sense!** **Now, let's substitute the rest of the information we have about Big Bertha:** [Write this on the board while speaking.] $V = \pi r^2 h$ $\approx 3.14 \cdot 4^2 \cdot 4.5$ $\approx 3.14 \cdot 16 \cdot 4.5$ ≈ 226.0 **ft**^{3} **We multiplied pi times the radius times the height, to get 226.08. We also said we wanted to round the answer to the nearest tenth, so our answer is 226.1 feet cubed.** Example 2 **Always pay attention to the information provided in the problem, because it can be easy to confuse diameter and radius. Make sure you are using the right numbers from the information provided. Let's take a look at another example of this in action.** [Read and show the word problem written on the board.] **A Japanese odaiko is a very large drum that is made by hollowing out a section of a tree trunk. A museum in Japan has three odaikos of similar size carved from a single tree trunk. The largest measures about 2.7 meters in both diameter and length, and weighs about 4.5 metric tons. Using**	Objective 2: Example 1: Students construct viable arguments by making conjectures and building a logical progression of statements. Students explore ways to find the volume of a cylinder, working from descriptions or diagrams. Objective 2: Example 2: Students represent the volume in symbolic form as an equation and use measurements that include extraneous information to construct the formula, then solve.

the volume formula for a cylinder, approximate the volume of the drum to the nearest tenth. **What information are we given here?** [Pause] **That's right, we're told that diameter and length are both 2.7 meters. Are we given any other information?** [Pause] **Yes, the 4.5 metric tons. Will we use that number?** [Pause] **No? Why not?** [Pause] **Right, because the measurement describes the weight of the drum, which is not related to volume, therefore it's extraneous, or extra, information. It's a good idea to circle all the numbers and units of measurement in a real-world application problem like this, to better understand what you are working with before you begin calculating.** **But we still have a problem – we need to find the radius of the base, and we were given the diameter. We can solve that in the same way we did with the last problem:** [Write this on the board while speaking.] $r = \frac{d}{2} = \frac{2.7}{2} = 1.35$ **m** **When we divide 2.7 by 2, we find that the radius is 1.35 meters. Now we can solve for the volume.** [Write this on the board while speaking.] $V = \pi r^2 h$ $\approx 3.14 \cdot 1.35^2 \cdot 2.7$ $\approx 3.14 \cdot 1.8225 \cdot 2.7$ $\approx 15.45\ m^3$ **Since the problem says to round to the nearest tenth, then our answer becomes 15.5 meters cubed, or 15.5m^3** [Write on board.] **We've learned several tactics that can be used when working on these types of problems, and I want to recap them before we move on. Jot these down, and keep them in mind while we are working some more problems:** • **Remember: Substitute, Simplify, and Modify. When you follow these steps, you'll be able to work out the formula and find the volume for a cylinder.** • **Volume is $V = \pi r^2 h$, so you need the radius of the cylinder to solve. Sometimes you may only be provided with the diameter or you may have additional information that's not needed. Circle pertinent numbers and units of measurement to help you sort through the information.** • **Your answer is an approximate number, because pi is**	

an approximate number. **OK, let's move on to some more problems!**	
Guided Practice (10-12 min) [I do] **Let's use the information from the diagram to find the area of the base and the volume of a cylinder. Pay attention to each step, and try to follow along using your own paper and pencil, to solve the problems.** [Draw the cylinder on the board including measurements.]  **First, let's take a look at the drawing. What numbers do we see here?** [Pause] **We have 10 centimeters, and 2 centimeters** [Point to the numbers while speaking.] **Is 10 centimeters the diameter, or the radius?** [Pause] **See how the line goes all the way across, with the endpoints resting on either side of the base? That means we have been provided the diameter. What is 2 centimeters a measurement of?** [Pause] **That's right, it's the height, or some problems may refer to it as length.** **You may recall the formula for finding the base of a cylinder is:** [Write the formula on the board while speaking aloud.] $B = \pi r^2$ **Let's substitute the values we know: [**Write and speak the formula.] $B = 3.14 \cdot (5)^2$ **Then, we can simplify:** $B = 3.14 \cdot 25$ **Finally, let's solve:** $B = 78.5 \text{ cm}^2$ **Using this information, now we can find the volume of the cylinder. Recall that the formula for volume of a cylinder is as follows:** [Write and speak the formula.] $V = Bh$ **or** $V = \pi r^2 h$ **Let's substitute the values we already know:** [Write and speak the formula.] $V = 78.5 \cdot 2$ **We determined that the area of the base was 78.5 centimeters squared. The height was 2 centimeters. There's really nothing to simply, so now we multiply 78.5 times 2 to**	Students use the volume formula given a diagram of a simple cylinder.

solve:
***V* = 157 cm^3**

Let's try another. Get your pencil and paper ready, we'll do this one together.

[We do]
You're given the following candles as a gift: [Show or draw the illustration of the candles on the board.]

You want to find out which has the most wax by volume. We've got a lot of dimensions here, so let's make a table to keep track of all of our information. Write this down with me: [Write the table on the board.]

Candle Type	Radius	Height	Volume
Tall			
Medium			
Short			

What information can we fill in right away? [Pause] **That's right, we're given the height on all three candles, so let's go ahead and fill that in:**

Candle Type	Radius	Height	Volume
Tall		**10 in.**	
Medium		**6 in.**	
Short		**4 in.**	

Why can't we fill in the radius right now? [Pause] **Yes, we were provided initially with the diameter, so we'll have to figure out the radius for each of these first. Do you recall the formula to find radius?** [Pause] **Did you write down this:** [Teacher writes and says the formula.]
$r = \frac{d}{2}$

So let's solve for each of the candles. First, let's find the radius for the tall candle: [Write and speak.]

$r = \frac{d}{2} = \frac{4}{2} =$ **2 in.**

Students apply their knowledge of using the volume of a cylinder formula to solve a real-world application problem.

Notice you can simplify to 2 inches. Next, the Medium Candle, and simplify: [Write and speak.]
$r = \frac{d}{2} = \frac{6}{2} = 3$ **in.**

Finally, the Short Candle: [Write and speak.]
$r = \frac{d}{2} = \frac{8}{2} = 4$ **in.**

Now, we can add our new information to the table. [Pause] Did you add it? You should have something like this:

Candle Type	Radius	Height	Volume
Tall	2 in.	10 in.	
Medium	3 in.	6 in.	
Short	4 in.	4 in.	

Now, we'll solve for volume for each of the candles. Let's start with the tall candle. First, you substitute the information you have into the formula. You can simplify that two squared to 4, then multiply pi times 4 times 10. [Speak and write the steps of the formula.]
$V = \pi r^2 h$
$\approx 3.14 \cdot 2^2 \cdot 10$
$\approx 3.14 \cdot 4 \cdot 10$

Did you get the answer? [Pause] **If your answer was 125.6, congratulations, that's correct**! [Write the answer on the board.]
$\approx 125.6 \text{ in}^3$

OK, let's do the same thing with the Medium Candle. Remember, substitute, simplify, multiply. Try to follow along as I write, and come up with the answer. [Write the formula on the board, pausing between each step to give students time to answer.]
$V = \pi r^2 h$
$\approx 3.14 \cdot 3^2 \cdot 6$
$\approx 3.14 \cdot 9 \cdot 6$
What's the answer? [Pause] **Did you get 169.56? That's right!** [Write on board.]
$\approx 169.56 \text{ in}^3$

Finally, we'll find the volume for the short candle. Follow the steps we've completed for the other two: [Teacher write the formulas on the board]
$V = \pi r^2 h$
$\approx 3.14 \cdot 2^2 \cdot 4$
$\approx 3.14 \cdot 4 \cdot 4$

What's the answer? [Pause] **Did you get 200.96? That's right!** [Write on board.]

≈ **200.96 in^3**

Now we can complete our original table:

Candle Type	Radius	Height	Volume
Tall	**2 in.**	**10 in.**	**125.6 in^3**
Medium	**3 in.**	**6 in.**	**169.56 in^3**
Short	**4 in.**	**4 in.**	**200.96 in^3**

This information actually tells us a lot about the candles. Which one has the most wax by volume? [Pause] **That's right, it's the short candle. Which one has the least?** [Pause] **Yes, it's the tall candle. What if we wanted to determine the difference in volume between each candle, how would we go about doing that?** [Pause] **Could you subtract the volume of the tall candle from that of the short candle?** [Write on the board.]

200.96 – 125.6 =

[Pause] **Did you come up with 75.36?** [Pause and write in the answer.] **Good! Now you know how much more wax the short candle has than the tall candle.**

Let's do one more, and this time, I'd like you to try to work the problem with only a little help from me.

Students apply their knowledge for finding the volume of a cylinder using a base with a unit radius.

[You do]

Take a look at this cylinder. Let's find the area of the base and the volume.

Use the formula for finding the area of the base, substitute your values for what's been provided, simplify if needed, and then multiply the values.

[Pause]

If you are having problems, here's that formula again [Speak and write the formula on the board.]**:**

$B = \pi r^2$

[Pause] **Did you come up with this when you substitute the**

values? ***B* = 3.14 · 1^2** **When you multiplied, did you get 3.14 centimeters squared? Good!** **Now, work on the volume of the cylinder.** [Pause] **Remember your formulas – you can use either of these to find the volume of the cylinder, especially since you now know the area of the base:** ***V* = *Bh* or *V* = $\pi r^2 h$** [Pause] **Did you get 15.7 cm^3 as the volume? Great!** Additional Problems (if needed): Problem 1 **Since we have some time, let's do another problem. Listen carefully to the information, making notes if needed.** [Show problem on the board while reading aloud.] **Susan said the volume of a cylinder with a 3 inch diameter is two times the volume of a cylinder with the same height and a 1.5 inch radius. Is she right?** **You may have a guess, but let's look at the information closely. Cylinder A has a diameter of 3 inches. The height is not specified, so we'll assign one – to answer the question, the only criteria for height is that both cylinders are the same. So for Cylinder A:** [Speak and write the information on the board.] ***d* = 3 in** ***h* = 3 in** **Cylinder B has a radius of 1.5 inches, and a height of 3 inches. We know this because Cylinder B must have the same height as Cylinder A.** [Speak and write the information on the board.] **r = 1.5 in** **h = 3 in** **Right away we can notice the difference between the measurements provided for each cylinder. Remember the difference between diameter and radius?** [Pause] **I'm sure by this point, you know that Susan is wrong in her assessment of the cylinders, but what can we do to make sure?** [Pause] **That's right, we can find the radius for Cylinder A. If it's the same as Cylinder B, then the volumes would be equal.** [Speak through the formula as well as write on the board.] ***r* = $\frac{d}{2}$** ***r* = $\frac{3}{2}$ = 1.5 in**	

Now we can see that both cylinder volumes are in fact equal. Problem 2 **Let's do one more.** [Show the problem on the board and say it aloud.] **The cylindrical Giant Ocean Tank at the New England Aquarium in Boston is 24 feet deep and has a radius of 18.8 feet. Find the volume of the tank.** **We are told the tank is 24 feet deep, which is its height. We also have a radius of 18.8 feet, so let's find the volume!** [Speak and write the formulas on the board.] $V = \pi r^2 h$ $\approx 3.14 \cdot 18.8^2 \cdot 24$ **Remember to simplify as much as possible:** $\approx 3.14 \cdot 353.44 \cdot 24$ **What's the answer?** [Pause] **Did you get 26635.2 cubic feet? That's right! That is a big tank!** [Write answer on board.] $\approx 26635.2 \text{ ft}^3$	
Independent Practice (1 min) **You've done some great work in finding the area of the base of a cylinder as well as the volume of a cylinder! I hope you keep in mind the formulas and tactics that have been shared. After this lesson, you will have some problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **Students, I enjoyed using formulas to find the volume of real-world objects shaped like cylinders with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	