PBS Lesson Series

Math: Grade 8, Lesson 16, The Pythagorean Theorem

Lesson Focus: Using the Pythagorean Theorem to determine missing side-lengths in a right triangle.
Practice Focus: Students will focus on using models and diagrams to represent and solve (find missing side-lengths) right triangles using the Pythagorean Theorem.
Objective: Students will use models and diagrams to represent and solve right triangles using the Pythagorean Theorem in two dimensions. Students will use models and diagrams to represent and solve right triangles using the Pythagorean Theorem in three dimensions.
Key Vocabulary: hypotenuse, legs, theorem, square root, right triangle, right angle
TN Standards: 8.G.B.5
Teacher Materials:

- Whiteboard & Markers
- Calculator
- Large scale model of right triangles recommended
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator or calculator app strongly recommended

**Note: Students will need a calculator to determine square roots of numbers. You will also want to have a large scale drawing of a right triangle to model the scenarios prepared ahead of time.*

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all children are welcome to tune in. This lesson is the sixteenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about using the Pythagorean Theorem in mathematics! Before we get started, to participate fully in our lesson today, you will need:** • Paper and a pencil, and a surface to write on • Calculator or calculator app recommended **Ok, let's begin!**	Students get materials ready for the lesson.

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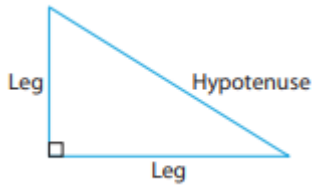
Intro (2 min) **You may have heard about the name of the Pythagorean Theorem we are going to work with today, but did you know about the Theorem's namesake? Around 2500 years ago, a Greek philosopher named Pythagoras lived on the island called Samos in the North Aegean Sea. We don't have too many details about his early life, but at some point, he traveled around and ended up in southern Italy where he founded a school. Pythagoras was credited with many mathematical and scientific discoveries, and his work influenced other scientists you may have heard of such as Nicolaus Copernicus and Isaac Newton.** **Today, we are going to explore the Pythagorean Theorem. There is historical evidence that the concept behind this theorem was known before Pythagoras' time, but he formalized it this way: Write this down on your notes with me.** [Write or show and read the theorem and representation.] ***In a right triangle, the square of the length of the hypotenuse (c) is equal to the sum of the squares of the lengths of the legs (a and b).*** $a^2 + b^2 = c^2$ **Did you get that written down?** [Pause] **Did you notice a couple vocabulary words that we might need to explore?** [Pause] **Let's look at a picture of a right triangle and explore the vocabulary together. You sketch it along with me.** [Draw and label the model.]  **What do you notice about the hypotenuse?** [Pause] **Right! It's the longest of the three sides, and it's across from the right angle in the triangle. The other sides are called legs, and we can label one of them *a* and the other one *b* when we are working on problems.** **Now, that you've got the vocabulary and model, let's try some application problems together.**	Students listen to the brief history of Pythagoras and write down the Pythagorean Theorem along with a sketch.

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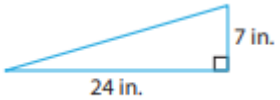
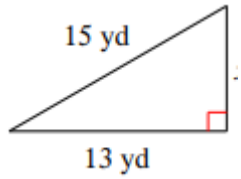
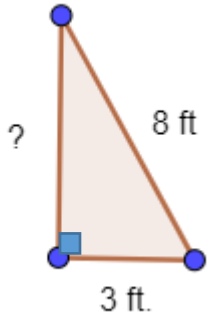
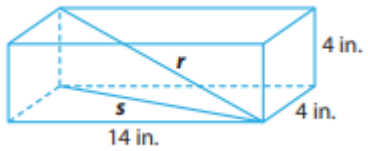
Teacher Model (10-12 min) Objective 1: Students will use models and diagrams to represent and solve right triangles in two dimensions using the Pythagorean Theorem. [Example 1] **Let's start with some triangles with missing side lengths. Sketch this example.** [Draw, label, and speak the diagram.]  **Now, we want to find the length of the side of the triangle that's not labeled. However, we need to decide if this missing side length is a leg or a hypotenuse. Look at the diagram and refer back to the notes you made in the beginning.** [Pause] **If you said that the missing side length is the hypotenuse, then you are correct! It is the side opposite of the right angle in the triangle, and it should be the longest side length.** **What are the other two lengths of 7 and 24 inches?** [Pause] **If you said those were leg lengths, you are correct!** **Does it matter which one is *a* or which one is *b*?** [Pause] **No, because we are adding the squares of the sides, and addition is a commutative operation – meaning we can add in any order.** **From the order of operations, you will also remember that we multiply first, which is what we are doing when we square numbers.** **We know that the calculation can be made using the algebraic equation $a^2 + b^2 = c^2$ where *a* and *b* are leg lengths, and *c* is the hypotenuse length.** **So, we can substitute in the equation to solve like this: Write along with me.** [Write and read aloud.] $a^2 + b^2 = c^2$ $7^2 + 24^2 = c^2$ **Squaring numbers means to multiply the number by itself. By order of operations, we do this first before we add the terms. Like this:** [Continue writing and reading aloud.] $49 + 576 = c^2$ $625 = c^2$ **To solve it from here, you must take the square root of each side. Remember that taking a square root of a number is finding a number that you can multiply by itself to get the number you started with. Sometimes this is not a whole**	Objective #1: Students will draw sketches of right triangles with missing side lengths and will identify if the missing length is a hypotenuse or a leg measurement. Students will use the Pythagorean Theorem to solve for the measure of the missing side length in 2 dimensions.

number, so you might want your calculator or calculator app to help you. $\sqrt{625} = \sqrt{c^2}$ ***25 = c*** **So, we see that the length of the hypotenuse is 25 inches. Does that make sense given the diagram and the definition of a hypotenuse?** [Pause] **It seems to because the value is larger than the other two measurements, and the missing side length was the hypotenuse.** [Example 2] **Now, let's try another one. Sketch this model and label it with me.** [Draw, label, and speak the diagram.]  **Which side length is missing this time? The hypotenuse or a leg?** [Pause] **If you said "leg", you are absolutely correct! Remember, this missing length can be labeled as *a* OR *b*.** **Which measurement is the hypotenuse?** [Pause] **If you said 15 yd, you are correct again! It is the side opposite of the right angle. So, we can substitute in the equation to solve like this: Write along with me.** [Write and speak.] $a^2 + b^2 = c^2$ $13^2 + b^2 = 15^2$ $169 + b^2 = 225$ $169-169 + b^2 = 225 -169$ $b^2 = 56$ **To solve it from here, you must take the square root of each side. You will want your calculator or calculator app to help you. Round your answer to the nearest tenth.** $\sqrt{b^2} = \sqrt{56}$ ***b = 7.5*** **So, the missing side length, rounded to the nearest tenth, is 7.5 yds.** [Example 3] **Let's look at a problem with a little context next. Listen to the problem as I read it aloud.** [Show and read the problem aloud.] **A painter has an 8 foot ladder that is being used to paint a room in a house. For safety reasons, the ladder must be**	Objective #1 (cont): Students will draw sketches of right triangles with missing side lengths and will identify if the missing length is a hypotenuse or a leg measurement. Students will use the Pythagorean Theorem to solve for the measure of the missing side length in 2 dimensions. Objective #1 (cont): Students will draw sketches of right triangles with missing side lengths and will identify if the missing length is a hypotenuse or a leg measurement. Students will use the Pythagorean Theorem to solve for the measure of the missing side length in 2 dimensions.

placed at least 3 feet away from the base of the wall. To the nearest tenth of a foot, how high will the ladder reach on the wall? Assume that the wall and the floor meet at a right angle. **Did you sketch a picture? Does yours look something like this?** [Pause. Show diagram.]  **So, we need to find the missing side length which is how high up on the wall that the ladder will come. In this case, we need a leg length. So, we can solve it this way. Follow along with me.** [Write and speak.] $a^2 + b^2 = c^2$ $a^2 + 3^2 = 8^2$ $a^2 + 9 = 64$ $a^2 + 9 - 9 = 64 - 9$ $a^2 = 55$ **To solve it from here, you must take the square root of each side. You will want your calculator or calculator app to help you. Round your answer to the nearest tenth.** $\sqrt{a^2} = \sqrt{55}$ $b = 7.4$ **So, the missing side length, rounded to the nearest tenth, is 7.4 ft.**	
Objective 2: Students will use models and diagrams to represent and solve right triangles using the Pythagorean Theorem in three dimensions **Let's move this up one more level and see what the Pythagorean Theorem looks like in three dimensions.** **I'll read the problem aloud and show you the model. You follow along and sketch the model.** [Show and read the problem aloud.] **Tina ordered a replacement part for her desk. It was shipped in a box that measures 4 in. by 4 in. by 14 in. What is the**	Objective #2: Students will draw sketches and diagrams to represent 3 dimensional problems (shipping boxes) to solve contextual problems looking for the interior diagonal length of the box using the Pythagorean Theorem to calculate the measurement.

greatest length in whole inches that the part could have been?

Let's look at the diagram of this box. [Show the diagram.]



Now, this one can be tricky because we have to think about what measurements look like INSIDE of a shipping box. In this case, we want to find the measurement of the length labeled *r* because that is the corner to corner length that would give us the maximum size we could fit in the box.

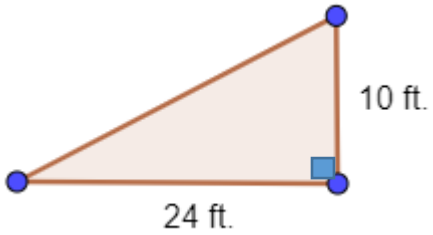
But first, we need to find the measurement of the length labeled *s* in the diagram. Because we have the dimensions of the box, we can determine the length of *s* using the Pythagorean Theorem. Now, is *s* going to be a hypotenuse or a leg length first? [Pause]
Did you see it? If *s* is our missing length, it is a hypotenuse of the triangle formed by the base of the box. [Point and trace along the triangle on the base. You can also use a marker to shade it in.]
So, the two leg lengths are 4 in. and 14 in. Let's solve for the hypotenuse *s*. Follow along with me. [Write and speak.]

$$4^2 + 14^2 = s^2$$
$$16 + 196 = s^2$$
$$212 = s^2$$

To solve it from here, you must take the square root of each side. You might want your calculator or calculator app to help you. Round your answer to the nearest hundredth.

$$\sqrt{212} = \sqrt{s^2}$$
$$14.56 = s$$

Now that we have the side length of *s*, we can find the value of *r*. But this time, what triangle are we looking at? Let's see. [Pause]
Did you see it? I'll trace it out for you. [Trace the interior triangle using side lengths *r* and *s* and the height of the box. You can also use a marker to shade it in.]
Now, this time, let's decide if *r* is the hypotenuse or a leg length. What do you think[Pause]
Right! In this triangle, *r* is the hypotenuse, and the measurement we found for *s* is a leg length. The height of the box is 4 inches. So, let's solve. Follow along with me.

[Write and speak.] $4^2 + 14.56^2 = r^2$ $16 + 212 = r^2$ $228 = r^2$ **To solve it from here, you must take the square root of each side. You will want your calculator or calculator app to help you. Round your answer to the nearest tenth.** $\sqrt{228} = \sqrt{r^2}$ $15.1 = r$ **Now, let's see how we need to answer the original question. It says, "What is the greatest length in WHOLE INCHES that the part could have been?"** **So, in this case, to respond to the question, we would say that the part could have been at most 15 inches long if we packed it in on the diagonal of the interior of the box.** **Great work! Now, let's practice a couple more.**	
Guided Practice (10-12 min) [I Do] **I'll walk you through another example. Look at the diagram, and make a sketch on your own paper.** [Pause and show the diagram.]  **Which side length is missing? The hypotenuse or a leg length?** [Pause] **Right! The hypotenuse is missing or what the Pythagorean Theorem calls *c* because it is the side opposite the right angle and appears to be the longest side length.** [Point to the blank side.] **The other measurements are legs.** [Point to the 10 ft. and 24 ft. measures] **We are ready to substitute in and solve. Follow along with me.** [Write and speak.] $a^2 + b^2 = c^2$ $10^2 + 24^2 = c^2$ $100 + 576 = c^2$ $676 = c^2$ **To solve it from here, you must take the square root of each side. You might want your calculator or calculator app to help you. Round your answer to the nearest tenth.** $\sqrt{676} = \sqrt{c^2}$	Students use their knowledge of identifying the missing side lengths in a right triangle and using the Pythagorean Theorem to calculate the measurement of the missing length in two and three dimensions.

$$26 = c$$

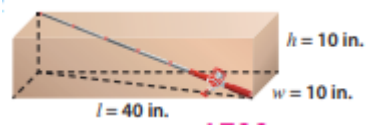
So, in this case, we calculate that the length of the hypotenuse of the right triangle is exactly 26 ft. long.

Now, let's look at one more together.

[We Do]
Let's work on another shipping box problem. I'll read this aloud, and you follow along. Make a sketch to model the problem. [Show and read the problem aloud.]

Mr. Luna wants to ship a fishing rod that is 42 inches long to a customer. He has a box that measures 10 inches by 10 inches by 40 inches. Will the fishing rod fit inside the box?

Here's a sketch of the problem. [Show the diagram.]



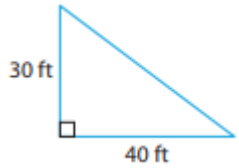
First, let's find the length of the diagonal across the bottom of the box. [Point and trace out the triangle on the bottom.] **Is this diagonal length a hypotenuse or a leg length?** [Pause] **Right! It's the hypotenuse that we are trying to find first. What are the lengths of the two legs?** [Point and trace the two leg lengths. You can also shade it in. Pause.] **Right! The lengths are 40 inches and 10 inches. So, now, let's use what we know to find that diagonal length. I'll get you started, and give you time to work. I'll substitute in the leg lengths first.**
[Write and speak.]

$$a^2 + b^2 = c^2$$
$$40^2 + 10^2 = c^2$$

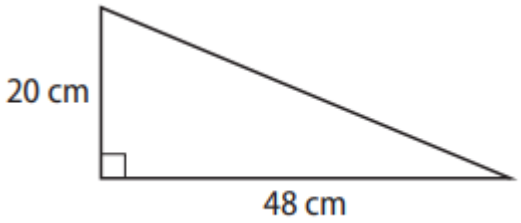
Now, you see if you can take it from here to find that diagonal length. [Pause here to allow time to work.]
Here's what I got. How about you? [Write and speak.]

$$1600 + 100 = c^2$$
$$1700 = c^2$$
$$\sqrt{1700} = \sqrt{c^2}$$
$$41.23 = c$$

Now, we want to use this length to find the measure of the inside diagonal. [Point and trace out the interior triangle. You can also shade it in.] **I'll get you started again, and see if you can answer the question about whether or not the fishing rod that is 24 inches long will fit in the box. I'll substitute in the leg lengths first. Don't forget that one leg length in this triangle is the height of the box or 10 inches, and the other leg is the length we just calculated.** [Write and speak.]

$a^2 + b^2 = c^2$ $10^2 + 41.23^2 = c^2$ **Now, you see if you can take it from here to find that diagonal length.** [Pause here to allow time to work.] **Here's what I got. How about you?** [Write and speak.] $100 + 1700 = c^2$ $1800 = c^2$ $\sqrt{1800} = \sqrt{c^2}$ $42.4 = c$ **So, does the fishing rod fit?** [Pause] **Yes! The box is just big enough to hold the rod! Great work! Now, you try one on your own.** [You Do] **Here's a simple diagram. Take a look and make a sketch to help you think about it. Decide if the missing side length is the hypotenuse or a leg length, and then use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth if needed.** [Show the diagram. The Pause to give students time to work.]  **Alright! How did you do? If you calculated that the missing side length is 50 feet, then you are correct! The length missing was the hypotenuse. Let me show you my calculations. You check yours against mine.** [Write or show and read aloud.] $a^2 + b^2 = c^2$ $30^2 + 40^2 = c^2$ $900 + 1600 = c^2$ $2500 = c^2$ $\sqrt{2500} = \sqrt{c^2}$ $50 = c$ **Great work!** Additional Problems (if needed):	

Here's another example. Look at the diagram, and make a sketch on your own paper. [Pause and show the diagram.]



Which side length is missing? The hypotenuse or a leg length? [Pause]
Right! The hypotenuse is missing or what the Pythagorean Theorem calls *c* because it is the side opposite the right angle and appears to be the longest side length. [Point to the blank side.] **The other measurements are legs.** [Point to the 20 cm and 48 cm measures.] **We are ready to substitute in and solve. Follow along with me.** [Write and speak.]

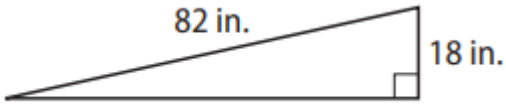
$$a^2 + b^2 = c^2$$
$$20^2 + 48 = c^2$$
$$400 + 2304 = c^2$$
$$2704 = c^2$$

To solve it from here, you must take the square root of each side. You will want your calculator or calculator app to help you. Round your answer to the nearest tenth.

$$\sqrt{2704} = \sqrt{c^2}$$
$$52 = c$$

So, in this case, we calculate that the length of the hypotenuse of the right triangle is exactly 52 cm long.

Here's one more example. Look at the diagram, and make a sketch on your own paper. [Pause and show the diagram.]



Which side length is missing? The hypotenuse or a leg length? [Pause]
Right! This time a leg length is missing. [Point to the blank side.] **The hypotenuse is the side opposite the right angle.** [Point to the 82 in. side.]. **The other side is another leg.** [Point to the 18 in. side.] **We are ready to substitute in and solve. Follow along with me, and remember that mathematically speaking, it doesn't make any difference which leg is labeled *a* or *b*. They are both leg measurements.** [Write and speak.]

$$18^2 + b^2 = 82^2$$
$$324 + b^2 = 6724$$

Take a minute and see if you can solve it from here. [Pause]

Check your work against mine. [Write and speak.] $324 - 324 + b^2 = 6724 - 324$ $b^2 = 6400$ **To solve it from here, remember you must take the square root of each side. You will want your calculator or calculator app to help you. Round your answer to the nearest tenth if needed.** $\sqrt{b^2} = \sqrt{6400}$ $b = 80$ **So, in this case, we calculate that the length of the missing leg of the right triangle is exactly 80 in. long.**	
Independent Practice (1 min.) **How are you feeling about using the Pythagorean Theorem to find missing side lengths? I hope your confidence will build with each problem! The Pythagorean Theorem has many applications in other settings, and we will look at some of those in the next few lessons. After the lesson, you will have some problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **Students, I enjoyed exploring the Pythagorean Theorem to find missing side lengths in a right triangle with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	

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