Math: Grade 8, Lesson 20, Volume of Cones

Lesson Focus: Using formulas to find the volume of real-world objects shaped like cones.
Practice Focus: Students will focus on using formulas to find the volume of real-world objects shaped like cones.
Objective: Students will use formulas to find the real-world volume of objects with a focus on cone shapes using attributes like radius, diameter and circumference to achieve this objective.
Students will use formulas and reasoning to determine the differences and similarities between cones and cylinders.
Key Vocabulary: Volume, cone, radius, diameter, circumference, slant height
TN Standards: 8.G.C.7
Teacher Materials:

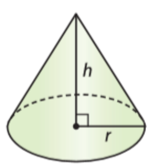
- Whiteboard & Markers
- Student Practice Packet

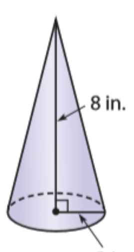
Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator or calculator app is useful but not necessary

Notes: You will draw one or more cones for these problems, so you may also want to have a large scale drawing of cone(s) to model the scenarios prepared ahead of time.

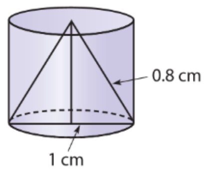
Teacher Do	Student Do
<u>Opening</u> (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all children are welcome to tune in. This lesson is the twentieth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about using formulas to find the volume of real-world objects shaped like cones! Before we get started, to participate fully in our lesson today, you will need:** • **Paper and a pencil, and a surface to write on** • **Calculator or calculator app is useful but not necessary**	Students get materials ready for the lesson.

Ok, let's begin!	
<u>Intro</u> (3 min) **Whether you know it or not, you are probably very familiar with conical shapes. Lots of real-world items are shaped like a cone – funnels, party hats, and, of course, ice cream cones!** **In Lesson 19, we found the volume of cylinders. This lesson is focused on cones, and there are a lot of similarities. A cone has one circular base and a curved lateral surface. A cylinder also has a curved lateral surface, but it contains two congruent, circular bases rather than just one.** **Another similarity is in the measurements required to find volume in a cone. Much like cylinders, you need to know the area of the base or its radius, and the height of the cone.** **One difference, however, is the height of a cone versus a cylinder – a cone actually has two aspects that are referred to as height. The height of a cone is the length of the segment from the vertex perpendicular to the center of the base. Recall that the vertex is a special point in an object, usually where two or more lines or edges meet**. [Draw the cone on the board and point out height of the cone.]  **A cone also has a slant height. This is the length of the segment from the vertex to any point on the circumference of the circular base** [Point to the slant height on the cone.] **The slant height is used to find the lateral surface area of the cone, so it is not necessary to find the volume of the cone.** **So, how do you find the volume of a cone? The volume *V* of a cone with radius r is one third the area of the base *B* times the height *h*. Here is the formula, be sure to write these down because we will be using them later:** [Write equations on the board.] $V = \frac{1}{3}Bh$ **or** $V = \frac{1}{3}\pi r^2 h$ **Notice the similarity to the formula we learned for cylinders in Lesson 19, which, you may recall, looks like this:** [Write formula on the board.] $V = Bh$ **or** $V = \pi r^2 h$	Students review the components of the equation for the volume of a cylinder. Students then see the volume of a cone is related, but only $\frac{1}{3}$ as much.

If we were to find the volume of a cone, then turn it into a cylinder with the same base and height, could we use the cone volume to find the cylinder volume? [Pause] **Yes, we could! How would we do that?** [Pause] **That's right, we would need to multiply it by 3, because the cone's volume is only one third of the product of the new cylinder's base area and height.** **You may also recall from the cylinder lesson that sometimes we are provided with the measurement for diameter instead of radius. As long as you remember that diameter is twice the radius, you will be able to determine the radius and use the formula you've learned: [Write the following on the board while saying aloud.]** $\boldsymbol{r = \frac{d}{2}}$ **Now that you understand what to look for when finding the volume of a cone, let's try some application problems together!**	
<u>Teacher Model</u> (10-12 min) Objective 1: Using formulas to find the volume of cylinder. Example 1 **Recall that during our lesson on finding the volume of a cylinder, we mentioned some steps that are very helpful when solving the formula for volume: Substitute, simplify, and multiply. These will also be useful when solving for volume in cones.** **Let's find the volume of this cone:** [Write or show the cone on the board, with measurements.]  **First, let's start with our formula and substitute the actual measurements for height and radius. You follow along and write this down with me:** [Write formula on board.] $\boldsymbol{V = \frac{1}{3}\pi r^2 h}$ $\boldsymbol{V = \frac{1}{3} \cdot 3.14 \cdot 2^2 \cdot 8}$	Objective #1: Students represent and solve for the volume of a cone by describing the volume formula, $V = \frac{1}{3}Bh$ as related to the volume of a cylinder in terms of its base area and height, but only $\frac{1}{3}$ as much.

Next, we will simplify: [Write simplification on board.] $V = \frac{1}{3} \cdot 3.14 \cdot 2^2 \cdot 8$ $\approx \frac{1}{3} \cdot 3.14 \cdot 4 \cdot 8$ **Finally, let's multiply our numbers to find the solution:** [Write on board.] $\approx \frac{1}{3} \cdot 3.14 \cdot 4 \cdot 8$ $\approx 33.5\ \text{in}^3$ **So the volume is about 33.5 in^3. Recall from the cylinder lesson that our answer is cubed because volume is the number of cubic units needed to fill a given space. Also remember we use the approximation symbol** $\approx$ [Point to the approximate symbol in the formula written on the board.] **which means almost equal to, because the value substituted for pi is approximate, so the answer must also be approximate.**	
Example 2 **Let's work another problem. Listen to the details.** [Show the problem on the board while saying aloud.] **For her geography project, Karen built a clay model of a volcano in the shape of a cone. Her model has a diameter of 12 inches and a height of 8 inches. Find the volume of clay in her model to the nearest tenth. Use 3.14 for pi.** **Did you notice in this problem, we're given diameter, not radius?** [Pause] **But this won't be a problem for us, because we know that diameter is twice the radius, so we can easily figure out the radius:** [Write on the board while speaking.] $r = \frac{12}{2} = 6$ **in.** **Now that we know the radius is 6 inches, we can find the volume of clay in Karen's model. Let's start with our formula, making sure to substitute the problem's measurements and simplifying where necessary, so we can multiply and come up with our answer. You follow along and write this down with me:** [Talk through the formula while writing it on the board.] $V = \frac{1}{3} \pi r^2 h$ $\approx \frac{1}{3} \cdot 3.14 \cdot 6^2 \cdot 8$	Objective #1 continued: Example 2 Students use the formula for finding the volume of a cone from a description only without a diagram.

$\approx 3.14 \cdot 36 \cdot 8$ **≈ 301.44 in^3** **Since we are rounding to the nearest tenth, the volume of clay in Karen's volcano is about 301.4 inches cubed.** **You're really getting the hang of this! Let's look at a few other problems that involve both cones and cylinders. Remember we said there are a lot of similarities between the two? It's true, and if you already know how to find the volume of a cylinder, then you can most definitely find the volume of a cone!**	
Objective 2: Solve problems involving the volume of cylinders and cones. Example 1 **Here is our problem.** [Show the problem on the board while saying aloud.] **The area of the base of a cylinder is 45 square inches and its height is 10 inches. A cone has the same area for its base and the same height. What is the volume of the cone?** **The base area and height is the same for both the cylinder and the cone, so we can use this information to find the volume of the cone.** [Write formula on board.] **$V = \frac{1}{3}$ Bh** **$= \frac{1}{3} 45 \cdot 10$** **$= 150$ in^3** **The volume of the cone is 150 inches cubed. What if we also wanted to know the volume of the cylinder? Because the cylinder and cone have the same base and height, then you can use the same information to find the volume of the cylinder:** [Write the formula on the board.] **$V = Bh$** **$V = 45 \cdot 10$** **$= 450$ in^3** **Note that the volume for the cylinder is three times that of the cone. Therefore, this means that all cones have only $\frac{1}{3}$ the volume of their related cylinder.**	Objective 2: Example 1: Students construct viable arguments by making conjectures and building a logical progression of statements. Students explore ways to find the volume of a cylinder, working from descriptions then relating the result to finding the volume of the related cone which is only $\frac{1}{3}$ as much.
Example 2 **Let's take a look at one more problem, and this one's a little tricky, so pay close attention**. [Write or show problem on	Objective 2 continued: Example 2: Students construct viable arguments by making conjectures and building

board while reading aloud.] **The tip of a sharpened pencil is shaped like a cone. How much of the pencil is lost after the tip is formed? Use the measurements in the picture to solve the problem.**  **As you can see, the tip of this pencil was formed out of a cylinder with a height of 0.8 centimeters and a diameter of 1 centimeter.** **First, let's find the volume of our cylinder. Write down the steps and follow along with me.** [Write on board as you talk through the formula.] $V = \pi r^2 h$ $\approx 3.14 \cdot 0.5^2 \cdot 0.8$ $\approx 3.14 \cdot 1 \cdot 0.8$ $\approx 0.63\ cm^3$ **The cylinder had a volume of approximately 0.63 centimeters cubed. Now we can find the volume of the cone.** [Write on board as you talk through the formula.] $V_{cone} = \frac{1}{3} V_{cylinder}$ $V_{cone} = \frac{1}{3} \cdot 0.63$ $V_{cone} = 0.21\ cm^3$ **The volume of the cone is 0.21 centimeters cubed. To find out how much volume was lost, subtract the volume of the cone from the volume of the cylinder.** [Pause] **Did you get an answer?** [Pause] **That's right, it's 0.42 centimeters cubed.** Tying the learning together: **Before we move on, let's recap a few things we've learned about finding the volume of cones. Jot these down and keep them in mind while we work on a few more problems:** [Show or write the list on the board, and read each aloud.] • **As with a cylinder, you need the radius and the height to determine volume. If only diameter is provided, remember that the diameter is twice the radius.** • **As with a cylinder, the volume of a cone will be an**	a logical progression of statements. Students explore ways to find the volume of a cylinder, working from descriptions and a diagram then relating the result to finding the volume of the related cone which is only $\frac{1}{3}$ as much. Tying the learning together: Students will listen to the teacher do a think aloud about the steps and processes we have learned about finding the volume of a cylinder and cone.

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approximate number, because pi is an approximate number.

- **The volume of a cone is one third the product of the area of the base and height of the cone.**
- **If you have a cone and cylinder that are congruent (same area of base, same height) and you know the volume of the cone, you can multiply it by 3 to find the volume of the cylinder.**

OK, let's move on to some more problems!

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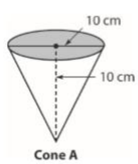
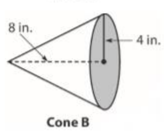
<u>Guided Practice</u> (10-12 min)
[I do]
You've probably noticed that we've been looking at a lot of diagrams of cones, as we did in the previous lesson, with diagrams of cylinders. Reading a diagram accurately is important when solving problems, so let's take a closer look at these two: [Draw or show the diagrams for Cone A and B on the board.]





Notice that the diameter and the radius of the base are shown by a solid line. [Point to solid lines on Cone A and B.] **The height of the cone is shown by a dashed line.** [Point to dashed lines on Cones A and B.]

Does the diagram of Cone A show the diameter, or the radius? [Pause] **That's right! It is the diameter, because the endpoints are along the circumference of the base. So that means Cone B must be showing the radius. But what is the radius of Cone A, how do we find that?** [Pause] **Remember that the diameter is twice the radius. A simple formula shows this:** [Write the formula on the board.]
$d = 2r$ **or** $r = \frac{d}{2}$
Knowing this, what is the radius of Cone A? [Pause] **Yes, it is 5 centimeters!**

Now, let's find the area of the base for Cone A. Try to follow

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<td>Students use the volume formula for a cone given a diagram of a cone when either the radius or diameter of the base is known.</td>
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along with me and write down the steps of the problem. Remember that the formula for finding the area of the base is as follows: [Write the formula on the board.] $B = \pi r^2$ **Let's substitute the information we know, and find the area of the base:** [Write the problem on the board as you say it aloud.] $B = 3.14 \cdot 5^2$ $= 3.15 \cdot 25$ $= 78.5 \text{ cm}^3$ **Now we know that our base area is 78.5 centimeters cubed, we can easily find the volume of Cone A. Let's use our formula:** [Write the problem on the board as you say it aloud.] $V = \frac{1}{3} Bh$ $= \frac{1}{3} \cdot 78.5 \cdot 10$ $= \frac{1}{3} \cdot 785$ $= 261.666 \approx 261.7 \text{ cm}^3$ **Did you get the answer too?** [Pause] **The volume for Cone A, rounded to the nearest tenth, is 261.7 centimeters cubed.** **Since we already know the height and radius, let's go ahead and find the volume for Cone B. Get your pencil and paper ready! Do you have the formula from the previous problems?** [Pause] **Good! Here we go!** [Write the problem on the board as you say it aloud.] $V = \frac{1}{3} \pi r^2 h$ $\approx \frac{1}{3} \cdot 3.14 \cdot 4^2 \cdot 8$ $\approx \frac{1}{3} \cdot 3.14 \cdot 16 \cdot 8$ $\approx \frac{1}{3} \cdot 401.92$ $\approx 133.97333 \text{ cm}^3$ **If we round to the nearest tenth, our answer is 134 centimeters cubed.** **Let's try another. Get your pencil and paper ready, we'll do this one together.** [We do] [Write or show the problem on the board and read it aloud.] **Lucas makes models of cones to explore how changing**	Students apply their knowledge of using the volume of a cone formula to solve a real-world application

dimensions affect volume. Cone A is 10 centimeters high and its base has a diameter of 4 centimeters. Cone B is twice as tall with a height of 20 centimeters and a diameter of 4 centimeters. Cone C is the same height as Cone A, 10 centimeters, but the diameter of its base is 8 centimeters. **That's a lot of information! It may be helpful to make a table to organize all of the measurements. If you recall from our previous lesson on cylinders, we did this when we were working with three candles with different heights and volumes. Here's our chart**: [Write or show chart on the board.]	problem.

Cone Size	Radius	Height	Volume
Cone A			
Cone B			
Cone C			

Let's fill in the information we've been provided in the problem. We know that Cone A has a height of 10 centimeters, so we can add that. [Circle this information in the problem, then add 10 cm in the height column for Cone A.] **We also know that the diameter for Cone A is 4 centimeters** [Circle this information in the problem.]**, but we need the radius for our table. How do we find that?** [Pause] **Did you remember that the diameter is twice the radius? So what would the radius be for Cone A?** [Pause] **Yes, that's right, it would be 2 centimeters! We'll put that measurement in our table.** [Add 2 cm in the radius column for Cone A.]

Now, what else does our problem tell us? Again, we have the height and diameter for Cone B. [Circle these in the problem.] **So let's fill in what we know in our table.** [Add 20 cm in the height column for Cone B.] **Again, since diameter is twice the radius, we can add 2 centimeters in Cone B's radius column.** [Add 2 cm to the radius column for Cone B.] **We also know that Cone C is the same height as Cone A** [Circle this in the problem.] **so let's add that to our table** [Add 10 cm to the height column for Cone C.] **and once again, we will take half the diameter, which the problem tells us is 8 centimeters** [Circle this in the problem.]**, so we can then add 4 centimeters for Cone C** [Add 4 cm to the radius column for Cone C.]
Does your table look like this? [Pause to allow student to look at table.] **Good! Now we can find the volume for all three.**

Cone Size	Radius	Height	Volume
Cone A	2 cm	10 cm	
Cone B	2 cm	20 cm	
Cone C	4 cm	10 cm	

First, let's work on Cone A. Recall your formula for volume, and try to work these along with me. If you have one, you can use your calculator as well. [Write the formula on the board and talk through as you are working the problem. Pause before writing the answer to give students the opportunity to complete on their own.] $\boldsymbol{V = \frac{1}{3}\pi r^2 h}$ $\approx \frac{1}{3} \cdot 3.14 \cdot 2^2 \cdot 10$ $\approx \frac{1}{3} \cdot 3.14 \cdot 4 \cdot 10$ $\approx \frac{1}{3} \cdot 125.6$ $\approx 41.8666667 \text{ cm}^3$ **If we round to the nearest hundredth, we will have 41.87 centimeters cubed. Now, let's work out the volume for Cone B.** [Write the formula on the board and talk through as you are working the problem. Pause before writing the answer to give students the opportunity to complete on their own.] $\boldsymbol{V = \frac{1}{3}\pi r^2 h}$ $\approx \frac{1}{3} \cdot 3.14 \cdot 2^2 \cdot 20$ $\approx \frac{1}{3} \cdot 3.14 \cdot 4 \cdot 20$ $\approx \frac{1}{3} \cdot 251.2$ $\approx 83.7333333 \text{ cm}^3$ **If we round to the nearest hundredth, we will have 83.73 centimeters cubed. Now, let's work out the volume for Cone C.** [Write the formula on the board and talk through as you are working the problem. Pause before writing the answer to give students the opportunity to complete on their own.] $\boldsymbol{V = \frac{1}{3}\pi r^2 h}$ $\approx \frac{1}{3} \cdot 3.14 \cdot 4^2 \cdot 10$ $\approx \frac{1}{3} \cdot 3.14 \cdot 16 \cdot 10$ $\approx \frac{1}{3} \cdot 502.4$ $\approx 167.466667 \text{ cm}^3$ **If we round to the nearest hundredth, we will have 167.47 centimeters cubed.**	

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Great, we've found the volumes for all three! So let's add them to our table: [Add volume answers for all three cones into the volume column.]

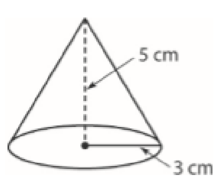
Cone Size	**Radius**	**Height**	**Volume**
Cone A	**2 cm**	**10 cm**	**41.87 cm^3**
Cone B	**2 cm**	**20 cm**	**83.73 cm^3**
Cone C	**4 cm**	**10 cm**	**167.47 cm^3**

Now that we have this information, we can better evaluate the size, shape and volume of our cones. For instance, how much greater is the volume of Cone B than Cone A? Let's find out: [Write the following on the board.]
83.73 cm^3 – 41.87 cm^3

See if you can get the answer, and round to the nearest tenth. [Pause, allowing time for student to solve the problem.] **Did you get 41.9 centimeters cubed?** [Pause] **Great! So now we know that Cone B's volume is approximately 41.9 cubic centimeters more than Cone A's volume.**

Which cone has the greatest volume? [Pause] **Yes, it's Cone C, but did you notice that it is 10 centimeters less in height than Cone B? Yet, it has significantly more volume. Would the volume of a cone increase more when you double the height of the original, or when you double its radius?** [Pause] **The volume increases when you double the radius, because the radius is squared.**

[You do]
Now let's work on another problem, and this time, I'd like you to use your knowledge of cones and cylinders to find the volume of this cone. Get out your pencil and paper, I'd like you work this one on your own if you can. Take a look at this cone, and find the volume: [Write or show the diagram on the board.]



Make sure you know whether this is showing the diameter or the radius, and take note of the measurements. [Pause, allowing time for student to assess the diagram.] **If you are having trouble getting started, here's the formula you can use to find the volume:** [Write the formula on the board.]

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Students apply their knowledge for finding the volume of a cone using their previous knowledge to determine what information has been given.

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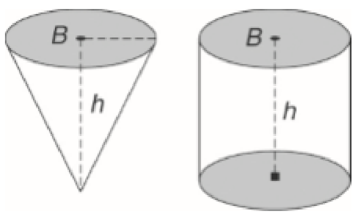
$\boldsymbol{V = \frac{1}{3}\pi r^2 h}$ [Pause again to give student time to substitute information and work the problem.] **Did you determine that the radius is 3 centimeters, and the height is 5 centimeters?** [Pause] **Good! Don't forget, use 3.14 for pi, and your answer will be approximate.** [Pause for student to work the problem.] **Did you get 47.1 centimeters cubed?** [Pause] **Great! You are doing very well!** Additional Problems (if needed): Problem 1 **Since we have some time, let's do another problem. Listen carefully to the information, making notes if needed.** [Write or show the problem on the board and say it aloud.] **A large traffic cone stands 28 inches in height and has a volume of 732.7 cubic inches. What is the diameter of the base of the cone?** **This one is a little different from what we've seen previously, because it's giving us the volume and asking us to solve for diameter. So how would we go about doing that?** [Pause] **Well, we can start with our formula, and substitute the information that we already know:** [Write or show the formula on the board.] $\boldsymbol{V = \frac{1}{3}\pi r^2 h}$ **Since we have the volume as well as the height, we can substitute those values into our formula like this:** $\mathbf{732.7 = \frac{1}{3} \cdot 3.14 \cdot \boldsymbol{r}^2 \cdot 28}$ **From here, we can simplify:** $\mathbf{732.7 = 29.3\boldsymbol{r}^2}$ $\mathbf{25 = \boldsymbol{r}^2}$ $\mathbf{5 = \boldsymbol{r}}$ **The question asks for diameter, and we know that the diameter is twice the radius, so we can solve this way:** [Write on the board while saying the steps aloud.] $\boldsymbol{d} \mathbf{= 2\boldsymbol{r} = 2(5) = 10}$ **in.** **Our diameter is 10 inches.** Problem 2 **Let's do one more problem. Listen to the details carefully, and consider what the problem is asking.** [Write or show the problem and say it aloud.] **Herb knows that the volume of a cone is one third that of a**	

cylinder with the same base and height. He reasons that a cone with the same height as a given cylinder but 3 times the radius should therefore have the same volume as the cylinder, since $\frac{1}{3} \cdot 3 = 1$. Is Herb correct?

To answer this, let's take another look at a cone and cylinder diagram [Draw or show the diagrams on the board]. **According to Herb, as long as both the cone and cylinder have the same height, but the cone has 3 times the radius of the cylinder, they will have the same volume.**



How is he coming to this conclusion? [Pause] **Most likely Herb remembers that the volume of a cone is one third the product of the base area and height of a cylinder. But does that make Herb's statement correct?** [Pause] **No, Herb is incorrect, but let's use a sample cone and cylinder to show why.**

Let's say we have a cylinder with a radius of 1 inch and a height of 5 inches. In order to have 3 times the radius, our cone would have a 3 inch radius and a 5 inch height. Let's start with the cylinder. If you recall at the beginning of the lesson, we provided the following formula to find the volume of a cylinder: [Write or show the formula on the board.]

$V = \pi r^2 h$

If we substitute in our sample measurements, we get the following:

$V \approx 3.14 \cdot 1^2 \cdot 5$

$\approx 3.14 \cdot 1 \cdot 5$

$\approx 15.7 \text{ in}^3$

Now let's get the volume for the cone: [Write the formula on the board and talk through as you are writing.]

$V \approx \frac{1}{3} \cdot 3.14 \cdot 3^2 \cdot 5$

$\approx \frac{1}{3} \cdot 3.14 \cdot 3 \cdot 28$

$\approx 87.9 \text{ in}^3$

The volume of the cone is three times that of the cylinder, because the radius is squared in the formula.

Independent Practice (1 min) **You've done some great work in finding the volume of a cone! I hope you keep in mind the formulas and tactics that have been shared.** **After this lesson, you will have some problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **Students, I enjoyed using formulas to find the volume of real-world objects shaped like cones with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	