PBS Lesson Series

Math: Grade 8, Lesson 17, The Converse of the Pythagorean Theorem

Lesson Focus: Using the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.
Practice Focus: Students will focus on using the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.
Objective: Students will understand the converse of the Pythagorean Theorem as a reversal of a conditional of "if-then" statement. Students will use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.
Key Vocabulary: Pythagorean Triple, If-Then statement, conditional statement, converse, hypotenuse, legs, square root, right triangle, right angle
TN Standards: 8.G.B.5
Teacher Materials:

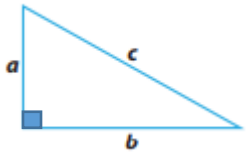
- Whiteboard and Markers
- Calculator
- Large scale model of right triangles recommended
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator or calculator app strongly recommended

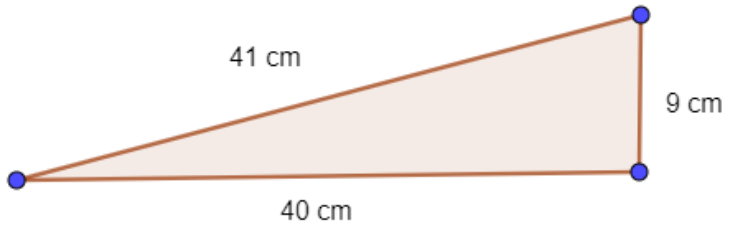
**Note: Students will need a calculator to determine square roots of numbers. You will also want to have a large scale drawing of a right triangle to model the scenarios prepared ahead of time.*

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all children are welcome to tune in. This lesson is the seventeenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about using the Converse of the Pythagorean Theorem in mathematics! Before we get started, to participate fully in our lesson today, you will need:**	Students get materials ready for the lesson.

• Paper and a pencil, and a surface to write on • Calculator or calculator app recommended **Ok, let's begin!**	
Intro (3-5 min) **Let's get warmed up. In Lesson 16, we explored the Pythagorean Theorem that told us this:** [Write or show from yesterday and read aloud.] ***In a right triangle, the square of the length of the hypotenuse (c) is equal to the sum of the squares of the lengths of the legs (a and b).*** $a^2 + b^2 = c^2$  **Many times, we end up having to round some of our calculations, but there are certain sets of right triangle side lengths that are all integers. Remember that an integer is a whole number (not a fractional number) that can be positive, negative, or zero. In this case, since these values are all measurements, they will be positive whole numbers. This is called a Pythagorean Triple. Write this definition down while I read it aloud.** [Write or show and read aloud.] ***A Pythagorean Triple is a set of positive integers, a, b, and c that fits the Pythagorean Theorem.*** **This means that there are some combinations of measures that are integers and also form a right triangle. Other triangles might have integer side lengths, but that doesn't mean that the triangle is a right triangle.** **Let's make a list. You write some of these down with me.** **The most commonly used Pythagorean Triple is 3, 4, and 5. So, let's check it out to see if these values fit. Which one of the measures should be the hypotenuse in this set?** [Pause] **Right! It should be the longest length if we think this is a right triangle. Let's test it out. Follow along with me.** [Write and read aloud.] **If we substitute in the values for a, b, and c, the final mathematical statement should be true. Let's see.**	Students listen to the brief history of Pythagoras and write down the Pythagorean Theorem along with a sketch. Students listen to and write down the definition of a Pythagorean Triple.

$a^2 + b^2 = c^2$ **$3^2 + 4^2 = 5^2$** **9 + 16 = 25** **25 = 25** **So, yes! This creates a true statement! So, since 3, 4, and 5 are integers, this set of side measures is a Pythagorean Triple.** **Let's try another one. What happens if we change the side lengths all by adding one so now we have 4, 5, and 6? Let's see. Follow-along with me.** [Write and read aloud.] **If we use 6 as the hypotenuse, we get these statements.** **$a^2 + b^2 = c^2$** **$4^2 + 5^2 = 6^2$** **16 + 25 = 36** **41 = 36** **Well, now, this isn't true, is it?** [Pause] **41 is not equal to 36. Therefore, 4, 5, and 6 are not a Pythagorean Triple, and in fact, although they do form a triangle, it isn't a right triangle as we will see in our lesson today.** **There are many other Pythagorean Triples. We can discover some as we work through the Pythagorean Theorem, but for today, let's just write them down together on our list. Remember, the set 3, 4, 5 was a Pythagorean Triple.** [Write and read aloud.] **3, 4, 5** **5, 12, 13** **7, 24, 25** **8, 15, 17** **Any multiple of these sets will also be a Pythagorean Triple by the Multiplication Property of Equality. So as an example, 6, 8, 10** [Write and read aloud.] **is also a Pythagorean Triple because it is a multiple of the 3, 4, 5 set when you multiply each term by 2.** **Are you feeling warmed up?** [Pause] **Great! Let's get started on today's lesson.**	
<u>Teacher Model</u> (10-12 min) Objective 1: Students will understand the converse of the Pythagorean Theorem as a reversal of a conditional of "if-then" statement. **When we started the lesson today, we wrote down the Pythagorean Theorem and if you notice, it starts out with the**	Objective #1: Students will think about and write down the Pythagorean Theorem and its converse understanding that a

assumption that you have a right triangle. Look at what you wrote down. [Pause] **We can actually rewrite this as an "If-Then" statement or a conditional statement. Follow along with me.** [Write and read aloud.] **IF you have a right triangle, THEN the square of the length of the hypotenuse equals the sum of the squares of the length of the legs.** **Now, in mathematical terms, the CONVERSE of this "If-Then" statement or conditional statement is just reversing what part goes with the "If" and with the "Then".** **Let's write the CONVERSE of the Pythagorean Theorem down together.** [Write and read aloud.] **The Converse of the Pythagorean Theorem says:** **"IF the square of the length of the hypotenuse equals the sum of the square of the length of the legs, THEN the triangle is a right triangle."** **Do you see the difference in the two statements?** **In this case, the converse statement is just as true as the original theorem. This isn't always the case, but we can prove that both statements are true in this instance.** **However, in today's lesson, we are going to move ahead with the understanding that the Converse of the Pythagorean Theorem is true, and our focus is going to be on using the Converse to determine if side lengths of a triangle form a right triangle or not.**	converse statement is reversing a conditional or "if-then" statement.
Objective 2: Students will use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle. [Example 1] **For our first example, let's determine if the given side lengths of a triangle will create a right triangle or not. Remember that by the Converse of the Pythagorean Theorem, IF the sum of the squares of the leg lengths is equal to the square of the length of the hypotenuse, THEN we will have a right triangle. In our warm-up, we already looked at something similar.** **Let's sketch a triangle and label the side lengths. Draw along with me.** [Draw and label the triangle.]	Objective 2: Students will use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle. They will determine the length to use as the hypotenuse and the other lengths as legs to substitute in to test.



Now, we want to know if this triangle is actually a right triangle. So, we will test it using the Converse of the Pythagorean Theorem.

First, let's decide which values are the legs and which value is the hypotenuse. [Pause]
In this case, we would say that the longest length would be hypotenuse if this is a right triangle. [Point to the side labeled 41 cm.] **The other two lengths** [Point to the two other sides.] **are the legs.**

We will substitute in and see if the resulting statement is true. Follow along with me [Write and read aloud as you go.]

$$a^2 + b^2 = c^2$$

Substitute in 9 and 40 for the leg measures and 41 for the hypotenuse measure.

$$9^2 + 40^2 = 41^2$$

Now simplify both sides and determine if the resulting statement is true or false. You can use your calculator or calculator app to help with the arithmetic if you like.

$$81 + 1600 = 1681$$
$$1681 = 1681 \text{ (true)}$$

We can we can see that this statement is true. Therefore, a triangle with side lengths 9, 40, and 41 IS a right triangle.

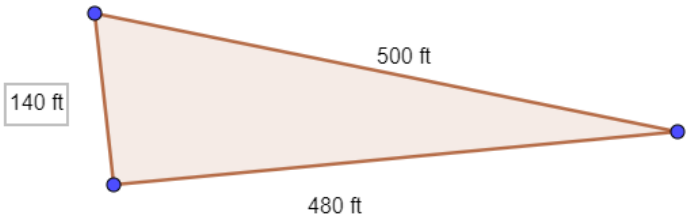
Bonus question! Think back to our introduction. What else can we say about 9, 40, and 41? [Pause]
Did you say that 9, 40 and 41 are a Pythagorean Triple? Then you're right because 9, 40, and 41 are integers that create a right triangle. Great work!

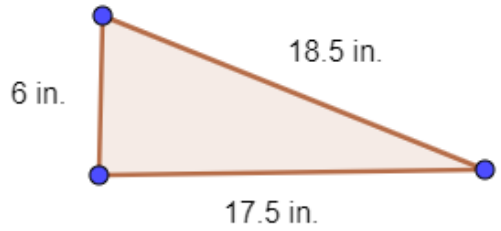
[Example 2]
Let's think about a second example. You can draw a sketch if you like. Here are the side length measures of a triangle. [Write the list and read aloud.]
8 inches, 10 inches, and 12 inches.

Which measurement should be the hypotenuse and which should be the leg measures? [Pause]

Objective 2 (cont): Students will use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle. They will determine

Did you decide that the hypotenuse would the 12 inch measure if it is a right triangle and the leg lengths should be 8 and 10? Great! Let's test this set out. **We will substitute in and see if the resulting statement is true. Follow along with me** [Write and read aloud as you go.] $a^2 + b^2 = c^2$ **Substitute in 8 and 10 for the leg measures and 12 for the hypotenuse measure.** $8^2 + 10^2 = 12^2$ **Now simplify both sides and determine if the resulting statement is true or false. You can use your calculator or calculator app to help with the arithmetic if you like.** $64 + 100 = 144$ $164 = 144$ **(false)** **We can we can see that this statement is FALSE. Therefore, a triangle with side lengths 8, 10, and 12 IS NOT a right triangle.** **Another bonus question for you: Is this a Pythagorean Triple?** [Pause] **Did you say no? Then you are correct! Even though these side lengths of a triangle are integers, they are not a Pythagorean Triple because they DO NOT form a right triangle.** [Example 3] **Let's think about this problem. I will read it aloud, and you can make a sketch of the situation.** [Show and read aloud.] **A blueprint for a triangular playground shows that the sides measure 480 ft., 140 ft., and 500 ft. Is the playground in the shape of a right triangle?** **Let me read it again while you finish your sketch.** [Show and read aloud.] **A blueprint for a triangular playground shows that the sides measure 480 ft., 140 ft., and 500 ft. Is the playground in the shape of a right triangle?** **Did you sketch a model something like this?** [Show the possible diagram.]	the length to use as the hypotenuse and the other lengths as legs to substitute in to test. Objective 2 (cont): Students will use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle. They will determine the length to use as the hypotenuse and the other lengths as legs to substitute in to test.

 **As before, let's use the Converse of the Pythagorean Theorem to help us answer the question about whether or not this is a right triangle.** **Like in the other two problems, we need to decide which length could be the hypotenuse and which lengths would be the leg measures if this is a right triangle.** [Pause] **Right! The leg lengths are the two shorter measures of 140 ft. and 480 ft. The hypotenuse measure would be the longest length or 500 ft. in this case. Now, we are ready to test to see if this is a right triangle.** **We will substitute in and see if the resulting statement is true. Follow along with me** [Write and read aloud as you go.] $a^2 + b^2 = c^2$ **Substitute in 140 and 480 for the leg measures and 500 for the hypotenuse measure.** $140^2 + 480^2 = 500^2$ **Now simplify both sides and determine if the resulting statement is true or false. You can use your calculator or calculator app to help with the arithmetic if you like.** $19,600 + 230,400 = 250,000$ $250,000 = 250,000$ **(true)** **We can we can see that this statement is TRUE. Therefore, a triangle with side lengths of 140 ft., 480 ft. and 500 ft IS a right triangle.** **Great work! Now, let's practice a couple more.**	
Guided Practice (10-12 min) I Do: **I'll walk you through another example. Look at the diagram, and make a sketch on your own paper.** [Pause and show the diagram.]	Students use their knowledge of using the Converse of the Pythagorean Theorem to determine if triangles are right triangles or not. Students follow along, then work along with the teacher, and then work independently on a problem.



Our question still is whether or not these side length measures will form a right triangle. First, we determine which length to use as the hypotenuse and which lengths to use as leg lengths for our test. What do you think? [Pause]
Right! 18.5 in. is the hypotenuse length for our test because it is the longest measurement. [Point at 18.5 in. in the diagram.]
The other measures, 6 in. [Point at 6 in. in the diagram.] **and 17.5 in.** [Point at 17.5 in. in the diagram.] **will be the leg measures.**
We will substitute in and see if the resulting statement is true. Follow along with me [Write and read aloud as you go.]

$$a^2 + b^2 = c^2$$

Substitute in 6 and 17.5 for the leg measures and 18.5 for the hypotenuse measure.

$$6^2 + 17.5^2 = 18.5^2$$

Now simplify both sides and determine if the resulting statement is true or false. As before, you can use your calculator or calculator app to help with the arithmetic.

$$36 + 306.25 = 342.25$$
$$342.25 = 342.25 \text{ (true)}$$

So, yes, these measures do form a right triangle since the resulting statement is true!

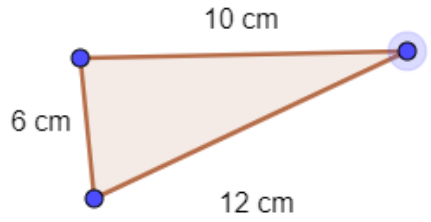
Are these measures also Pythagorean Triples? [Pause]
If you said no, you are right! Although they do form a right triangle, the measures are NOT integers.

We Do:
Let's work on another set. I'll read this problem aloud, and you can sketch a model and label the sides. Listen to the problem. [Show and read aloud.]

A mosaic consists of triangular tiles. Most of the tiles have side lengths of 6 cm, 10 cm, and 12 cm. Are these tiles in the shape of right triangles?

I'll read it one more time while you finish your sketch. [Show and read aloud.]
A mosaic consists of triangular tiles. Most of the tiles have side lengths of 6 cm, 10 cm, and 12 cm. Are these tiles in the shape of right triangles?

Does your sketch look something like this? [Show the diagram.]



What do we need to do first in order to test these measures to see if they form a right triangle? [Pause]
Right! Determine which lengths would be the leg measures and which one would be the hypotenuse measure. You take a second and decide that. [Pause]
Did you choose 12 cm for the hypotenuse? [Pause]
Great! It is the longest side measure, and the other two will be the legs.
Now, we've done several of these already. Take a minute to see if you can set this one up and test it on your own. [Pause]
Did you start it out this way? [Show or write and read aloud.]

$$a^2 + b^2 = c^2$$

Substitute in 6 and 10 for the leg measures and 12 for the hypotenuse measure.

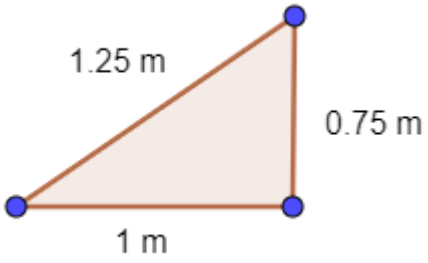
$$6^2 + 10^2 = 12^2$$

Now simplify both sides and determine if the resulting statement is true or false. As before, you can use your calculator or calculator app to help with the arithmetic, but you might not need it on this one. I'll give you a second to finish. [Pause]
Do your final statements look like this? [Show or write and read aloud.]

$$36 + 100 = 144$$
$$136 = 144 \text{ (false)}$$

So, what is the final answer to the question? [Pause]
Exactly! The measures of 6, 10, and 12 DO NOT form a right triangle.

[You Do]
You are definitely getting ready to tackle these on your own. Here's one more for you to look at. Here's a diagram. [Show the diagram.]



I'll give you time to work through this one by yourself to determine if these side lengths form a right triangle. [Pause allowing time to work the problem.]
Well? What did you decide? [Pause]
Yes! These measures do form a right triangle because this statement is true if you used 1.25 m as the hypotenuse. [Show or write and read aloud.]

$$a^2 + b^2 = c^2$$
$$0.75^2 + 1^2 = 1.25^2$$
$$0.5625 + 1 = 1.5625$$
$$1.5625 = 1.5625 \text{ (true)}$$

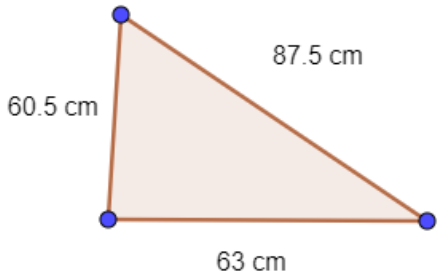
One more quick bonus! Are these measures a Pythagorean Triple? [Pause]
If you said no, then you are correct! They do form a right triangle, but the measurements are NOT integers.

Additional Problems (if needed):
Here's another example. Let's determine if these side lengths of a triangle are right triangles. [Write or show and read aloud.]

12 ft., 35 ft., 37 ft.

As we have done previously, we will use the Converse of the Pythagorean Theorem to determine if these measures form a right triangle. What do we need to do first? [Pause]
Right! Determine which length would be the hypotenuse if this is a right triangle. Which one do you think it should be? [Pause]
Did you say 37 ft? That's correct! It is the longest measurement of the set. So, by the Converse of the

Pythagorean Theorem, the sum of the squares of 12 and 35 should be the same as the square of 37 if this is a right triangle. **Let's test it out. Follow along with me.** [Show or write and read aloud.] $a^2 + b^2 = c^2$ **Substitute in 12 and 35 for the leg measures and 37 for the hypotenuse measure.** $12^2 + 35^2 = 37^2$ **Now simplify both sides and determine if the resulting statement is true or false. As before, you can use your calculator or calculator app to help with the arithmetic, but you might not need it on this one. I'll give you a second to finish.** [Pause] **Do your final statements look like this?** [Show or write and read aloud.] $144 + 1225 = 1369$ $1369 = 1369$ **(true)** **So, what is the final answer to the question?** [Pause] **Exactly! The measures of 12, 35, and 37 DO form a right triangle.** **Here's one more example. Listen as I read the problem, and make a sketch on your own paper.** [Show and read aloud.] **Savannah is an artist making a triangular base for a piece of artwork. The triangle she made measures 63 cm, 60.5 cm, and 87.5 cm. Is this triangle a right triangle?** **I read it again while you finish up a sketch.** [Read aloud a second time.] **Savannah is an artist making a triangular base for a piece of artwork. The triangle she made measures 63 cm, 60.5 cm, and 87.5 cm. Is this triangle a right triangle?** **Does your triangle sketch look something like this?** [Show the diagram.]	



Yours might look a little different, but as long as you have three sides labeled as 63, 60.5, and 87.5, we are good to go!

Now, let's use the Converse of the Pythagorean Theorem to help us answer the question about whether or not this is a right triangle.

First, decide which measurement is the hypotenuse if this is a right triangle. [Pause]
What did you decide? [Pause]
I picked 87.5 cm because it is the longest side, and I hope you did too! Now, the other two measures would be the legs.
So, let's test. Follow along with me. [Show or write and read aloud.]

$$a^2 + b^2 = c^2$$

Substitute in 63 and 60.5 for the leg measures and 87.5 for the hypotenuse measure.

$$63^2 + 60.5^2 = 87.5^2$$

Now simplify both sides and determine if the resulting statement is true or false. As before, you can use your calculator or calculator app to help with the arithmetic, but you might not need it on this one. I'll give you a second to finish. [Pause]
Do your final statements look like this? [Show or write and read aloud.]

$$3969 + 3660.25 = 7656.25$$
$$7629.25 = 7656.25 \text{ (false)}$$

So, what is the final answer to the question? [Pause]
Exactly! The measures of 63, 60.5, and 87.5 DO NOT form a right triangle since the sum of the squares of the legs DO NOT equal square of the hypotenuse or longest side.

Independent Practice (1 min.)
How are you feeling about using the Converse of the Pythagorean Theorem to determine if the side measures of a triangle will form a right triangle? [Pause] **Terrific! After the**

lesson, you will have some problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education. [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **Students, I enjoyed exploring the Converse of the Pythagorean Theorem and identifying some Pythagorean Triples today with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	