

Module 1:

Making Sense of Number Sense

TAB

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Teachers will learn to deepen students’ understanding of number sense by implementing effective math instructional strategies within a numeracy-rich environment.

Objectives

- Gain an understanding of number sense in the early childhood classroom
- Learn how to create a numeracy rich environment
- Learn strategies and activities to teach subitizing and number representations

Standards

Know number names and the count sequence.

Pre-K Standards	Kindergarten Standards
<p>PK.CC.3. Understand the relationships between numerals, names of numbers, and quantities up to 10 (includes subitizing—the ability to look at a quantity and say the quantity [one through four] quickly, just by looking).</p>	<p>K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at one).</p>

TEAM Alignment

- Teacher Content Knowledge
- Activities and Materials
- Problem Solving

Discussion

- What do you think is important in an early childhood program? List your ideas below.

NCTM Position

Young learners' future understanding of mathematics requires an early foundation based on a high-quality, challenging, and accessible mathematics education. Young children in every setting should experience mathematics through effective, research-based curricula and teaching practices. Such practices in turn require that teachers have the support of policies, organizational structures, and resources that enable them to succeed in this challenging and important work.

Retrieved from <http://www.nctm.org/Standards-and-Positions/Position-Statements/Mathematics-in-Early-Childhood-Learning/>

- **How does the NCTM position match or differ from your thinking?**

Creating a Numeracy-Rich Environment

Read the following quote and think about your classroom. In what ways do you promote mathematical learning for your students?

“A numeracy rich environment promotes mathematical learning by students. Borrowing from the research regarding literacy education, where immersion in a literacy-rich environment is considered essential to promote learning, it is important that students are immersed in a world of mathematics. As students see numbers and math-related materials throughout the classroom and participate in real-world, meaningful problem-solving opportunities, they begin to see the connection mathematics has to their own lives. Mathematics is no longer solely problems in a textbook, but it becomes something to ponder.”

Sammons, L. & Janis K. Drab Fackler, *Guided Math, A Framework for Mathematics Instruction*.

List your ideas here:

Vocabulary Activity

In order to create a numeracy-rich environment, we must first understand the key vocabulary in early childhood numeracy. These terms will be discussed over the course of this training:

- subitizing
- number representations
- five/10 frame
- decomposing numbers
- number talk
- think aloud
- part-part whole
- number sense

Teaching Strategies to Build Number Sense

Read the article below, *Understanding Number Sense*. After reading, divide into six groups. Each group will make a list of activities and strategies they use to teach one of the components.

Understanding Number Sense—Its Importance and Research-Based Teaching That Improve It

Retrieved from <http://mathsolutions.com/making-sense-of-math/number-sense/understanding-number-sense/>.

What Is Number Sense?

Number sense essentially refers to a student's "fluidity and flexibility with numbers," (Gersten & Chard, 2001). He/She has sense of what numbers mean, understands their relationship to one another, is able to perform mental math, understands symbolic representations, and can use those numbers in real world situations. In her book, "About Teaching Mathematics", Marilyn Burns describes students with a strong number sense in the following way: "[They] can think and reason flexibly with numbers, use numbers to solve problems, spot unreasonable answers, understand how numbers can be taken apart and put together in different ways, see connections among operations, figure mentally, and make reasonable estimates."

The National Council of Teachers in 1989 identified the following five components that characterize number sense:

- Number meaning
- Number relationships
- Number magnitude
- Operations involving numbers and referents for number
- Referents for numbers and quantities

Why Is Number Sense Important?

Number sense is important because it encourages students to think flexibly and promotes confidence with numbers—they “make friends with numbers” as Carlyle and Mercado charmingly refer to it in their book "Teaching Preschool and Kindergarten Math". Students come to understand that numbers are meaningful and outcomes are sensible and expected (Burns, 2007). Conversely, students who lack a strong number sense have trouble developing the foundation needed for even simple arithmetic much less more complex math.

In a recent study of 180 seventh graders conducted by the University of Missouri, researchers found that, “those who lagged behind their peers in a test of core math skills needed to function as adults were the same kids who had the least number sense or fluency way back when they started first grade.” (Neergaard, 2013) This is particularly sobering when one considers that one in five U.S. adults lacks the math competency of a middle school student—leaving them unqualified for most jobs.

Teaching Strategies to Build Students’ Number Sense

We know from a wide body of research that number sense develops gradually and over time resulting from an exploration of numbers, visualizing numbers in a variety of contexts, and relating to numbers in different ways. About Teaching Mathematics. A K-8 Resource, 3rd Edition, Marilyn Burns (2007) highlights the following key, research-based teaching strategies to build numbers sense:

Model different methods for computing.

When a teacher publicly records a number of different approaches to solving a problem—solicited from the class or by introducing her own—it exposes students to strategies that they may not have considered. As Marilyn Burns explains, “When children think that there is one right way to compute, they focus on learning and applying it, rather than thinking about what makes sense for the numbers at hand.”

Ask students regularly to calculate mentally.

Mental math encourages students to build on their knowledge about numbers and numerical relationships. When they cannot rely on memorized procedures or hold large quantities in their heads, students are forced to think more flexibly and efficiently, and to consider alternate problem solving strategies. (Parrish, 2010)

Have class discussions about strategies for computing.

Classroom discussions about strategies help students to crystalize their own thinking while providing them the opportunity to critically evaluate their classmates’ approaches. In guiding the discussion, be sure to track ideas on the board to help students make connections between mathematical thinking and symbolic representation (Conklin &

Sheffield, 2012). As noted in *Classroom Discussions: Using Math Talk to Help Students Learn*, the goal is “not to increase the amount of talk but the amount of high quality talk.”

Make estimation an integral part of computing.

Most of the math that we do every day—deciding when to leave for school, how much paint to buy, what type of tip to leave in a restaurant, which line to get in at the grocery store relies not only on mental math but estimations. However traditional textbook rounding exercises don’t provide the necessary context for students to understand estimating or build number sense. To do that, estimation must be embedded in problem situations.

Question students about how they reason numerically.

Asking students about their reasoning—both when they make mistakes AND when they arrive at the correct answer—communicates to them that you value their ideas, that math is about reasoning, and, most importantly, that math should make sense to them. Exploring reasoning is also extremely important for the teacher as a formative assessment tool. It helps her understand each student’s strengths and weaknesses, content knowledge, reasoning strategies and misconceptions.

Pose numerical problems that have more than one possible answer.

Problems with multiple answers provide plenty of opportunities for students to reason numerically. It’s a chance to explore numbers and reasoning perhaps more creatively than if there was “one right answer.”

“Just as our understanding of phonemic awareness has revolutionized the teaching of beginning reading, the influence of number sense on early math development and more complex mathematical thinking carries implications for instruction” (Gersten & Chard, 2001).

Sources

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Discussion

As a group, discuss and list different ways or activities you use to teach this strategy in your classroom.

Group 1: Model different methods for computing

Computing in pre-K and Kindergarten can encompass counting, comparing numbers, addition or subtraction, and problem solving. What are some ways that you model these skills for your students?

Group 2: Ask students regularly to calculate mentally

Asking students to use mental math helps them to formulate strategies. How do you ask your students to use mental math?

Group 3: Have class discussions about strategies for computing

Number Talks and Think Alouds are two strategies that can be used to help students discuss their mathematical thinking. What are some other ways you help students vocalize their thinking?

Group 4: Make estimation an integral part of computing.

Using an estimation jar and varying the objects can help students learn estimation skills. What are some other ways you use estimation in your classroom?

Group 5: Question students about how they reason numerically.

During calendar time, ask students to figure out what the date is for that day. Ask them how they came up with that number. One might say they counted to that number, another might say they knew what number came next on the calendar. Both students are showing their reasoning. How do you find out the reasoning of your students?

Group 6: Pose numerical problems that have more than one possible answer

During Morning Meeting, write the number 10 on the board. Ask students how they can make 10. Some might draw 10 pictures, some might write $5+5$. They can show many possible answers. What do you do in your classroom to foster this type of thinking?

How can we use Subitizing and Number Representations to foster Number Sense?

What is Subitizing?

Subitizing is the ability to understand “how many” without counting.

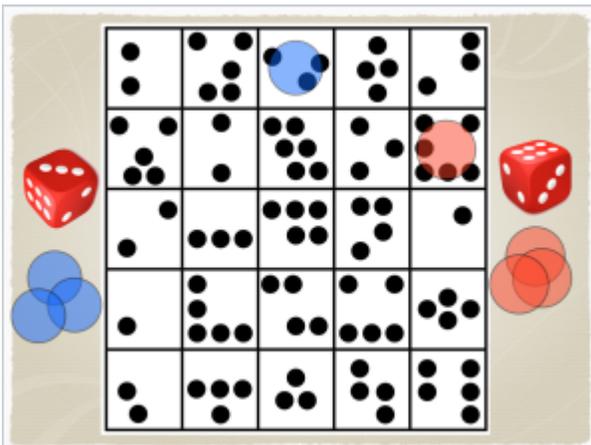
The ability to subitize is an important part of developing a strong mathematical foundation. There are two components of subitizing; conceptual and perceptual. Perceptual subitizing is the instant visual recognition of a pattern such as the dots on a die. Conceptual subitizing is recognizing smaller groups and adding them together, such as two dots plus two dots equals four dots. Subitizing can help children learn to:

- Develop estimation skills
- Count on from a known patterned set
- Combine numbers from sets
- Develop mathematical fluency

Retrieved from <http://www.pre-kpages.com/subitizing/>

Classroom Activities that Help to Develop Subitizing

- **Find-It Game:** You can download the game board from this site:
<https://mindfull.wordpress.com/2015/01/24/find-it-a-subitizing-bingo-game/>.



- **On/Off Game:** Use a seasonal picture and matching manipulatives to play this game. In the picture you see an igloo and fish crackers. Decide on the number you want to work with and put that many manipulatives in a small cup. Students pour out the manipulatives on the work mat. They tell how many are “on” the mat, and how many are “off” the mat. **This is a fun partner game.**
- **Five frames/10 frames:** Use five frames first with dot stickers for subitizing practice. After students master the five frames, you can move into using 10 frames, and then double 10 frames for the teen numbers.
- **Finger games:** Say a number and have students hold up that many fingers. It is amazing how many students cannot do this. You hold up fingers for a few seconds and let them tell you how many you are holding up. You can also hold up some fingers for a few seconds then have them match you. This is a great activity to do while waiting in line for something.
- **Dominoes:** Students can sort dominoes by number.
- **Subitizing concentration game:** Print the game cards from the link below. Students match the cards.
http://teachmath.openschoolnetwork.ca/wpcontent/uploads//2015/12/concentration1_k.pdf
- **Dot cards/plates:** Use plastic plates and colored dot stickers to make different groups and combinations. Hold up a plate for a few seconds for students to “see” how many.
- **Doubles:** Teacher flashes a dot plate. Students respond with double the number of dots. The number range should reflect the grade level.

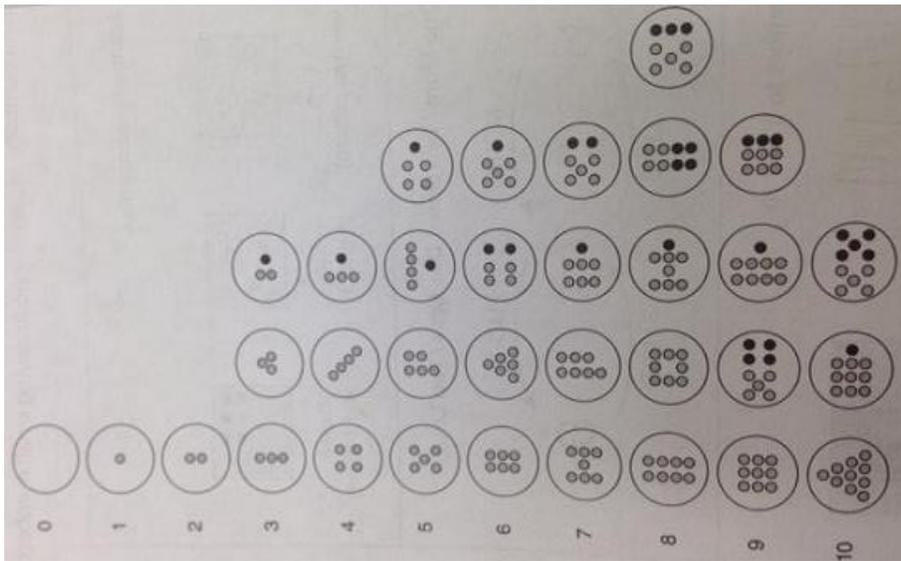


Think about the above activities. Talk with the people at your table and decide which of these activities show conceptual subitizing and which ones show perceptual subitizing. Do any of the activities show both types of subitizing? How do these activities foster Number Sense in the classroom?

Make and Take

Using the plates and dot stickers, make a set of dot plates you can use in your classroom. Think about your students' math knowledge at the beginning of the year, and make the plates so you can help build on that knowledge. We will be using these plates later in the training.

Dot plate examples



- Retrieved from

<http://teachmath.openschoolnetwork.ca/wpcontent/uploads/documents/dotplatepatternsVDW.pdf>

Video

<https://www.teachingchannel.org/videos/visualizing-number-combinations>

- Why is it helpful for students to discuss the mental images they formed?
- Ms. Latimer takes multiple student answers to the same question. What effect does this have?
- How could this lesson be connected with other skills?

Literature Connection:

Ten Black Dots by Donald Crews

This book uses colorful illustrations and rhymes to show common objects whose parts can be seen as various numbers of black dots. You can show the pages for three to four seconds, and then ask the children how many black dots they see. After learning the pattern of the book, have students make a prediction of how many dots will be on the next page, and what they are being used for.

The Very Hungry Caterpillar by Eric Carle

This book is great for students to see the “one more” pattern of numbers. On each day of the week, the caterpillar eats one more fruit than the day before. You can ask students to make predictions, or extend the learning by asking how much the caterpillar ate on Monday and Wednesday together.



Talk with the people at your table. List ideas you would like to use to support subitizing in your classroom below. Be ready to share.

What are Number Representations?

Number representations are ways to show numbers and their quantities. To help students develop an understanding of mathematical concepts, they need to understand that numbers can be represented in many different ways.

Classroom Activities for Number Representations:

- **Match-me games:** students can match different representations of numbers—quantities to numbers, tally marks to ten frames, 10 frame to words, etc.
- **Number of the day:** choose a number of the day and make multiple representations of it.
- **Numbers about Me:** Use a student’s birthdate, age, etc. to make multiple representations.
- **Number posters:** Make different representations of numbers and let the students sort them onto the correct number poster (tally marks, 10 frame, numeral, number word, pictures).



Talk with the people at your table List ideas you would like to use to support number representations in your classroom below. Be ready to share.

Make and Take: Make a Number Poster

Using the template at the end of your manual, choose a number and make a number poster. Show different ways that the number can be represented.

Number Sense Resources

- Number and Number Sense Module:
http://www.doe.virginia.gov/instruction/mathematics/elementary/number_sense_module/nns_gradek.pdf

Literature Connection:

How Many Snails? by Paul Giganti, Jr.

A series of simple questions directs young readers to determine the differences between seemingly similar objects, encouraging them to develop powers of observation, discrimination, and visual analysis. There's plenty of opportunity to practice counting, too (but that's just the beginning!).

What Comes in 2s, 3s, and 4s? by Suzanne Aker

This picture book presents the set number concept through pictures of everyday objects.

Vocabulary Connection for Subitizing and Number Representations

- **Number:** how many of something you have
- **Group:** a collection of items
- **Count:** saying numbers as you touch objects in a group
- **Tally Marks:** lines used to help count
- **Five Frame:** a box used to show five things
- **Ten Frame:** a box used to show 10 things

Discussion

- What other math vocabulary terms do students need to know?

Listening and Speaking Connection

Possible Teacher Questions	Possible Student Responses
How do you know there are four dots on the plate?	I saw 2 on the top and 2 on the bottom.
How did you know there were five dots in the 10 frame?	The top row was full.
How did you make the number 10?	I drew two circles. I filled in the whole 10 frame. I drew 5 orange dots and 5 red dots.

- What other questions could you ask?

Writing Connection: Math Journals

- Using the dot plates you made, show an image, then have students copy it in their math journals.
- After reading the book *Ten Black Dots*, give students some black dot stickers and have them make something and write about it in their journals.
- Tell students to divide their journal page into four sections and draw dots for a given number in different ways: array, line, circle, scattered.
- Tell students to divide their journal paper into four sections and give them a number to represent four different ways: pictures, equation, word, tally marks, etc.



Talk with the people at your table. What is one idea from this morning that you want to use in your classroom? Why?

Closing Activity

Inside/Outside Circle: Participants will divide into two groups. One group will form the inside circle, the other group will form the outside circle. The circle will rotate and participants will share an idea that they have learned, or something they can expand on in their own classrooms.



Key Idea #1

Number sense is a “good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.”

- Hilde Howden