Module 1:
Teaching Mathematics for Understanding

[TAB PAGE]

Module 1: Teaching Mathematics for Understanding

Objective

- To be able to define what teaching for understanding means in the context of mathematics instruction

Tennessee Standards

The focus of this two day training will be on the Numbers in Base Ten (NBT) and Operations and Algebraic Thinking (OA) standards in both Grade 1 and 2.

Module 1 introduces key understandings to build both teacher content knowledge and pedagogy.

TEAM Alignment

- Presenting Instructional Content
- Questioning
- Teacher Content Knowledge
- Thinking
- Problem Solving

Module Activities

- Participants will read, reflect, and discuss what it means to teach for mathematical understanding
- Participants will sequence student mathematical work to build their math horizon knowledge
- Participants will consider classroom features and routines that promote mathematical understanding
- Participants will connect oral language development to building mathematical understanding by engaging in a vocabulary activity and viewing a number talk strategy

What is mathematical understanding?

Before we begin, take a moment to reflect on what mathematical understanding means to you.

An understanding can never be "covered" if it is to be understood.

-Wiggins and McTighe, 2005, p.229

Understanding is being able to think and act flexibly with a topic or concept. It goes beyond knowing; it is more than a collection of information, facts and data. It is more than being able to follow steps in a procedure. One hallmark of mathematical understanding is a student's ability to justify why a given mathematical claim or answer is true or why a mathematical rule makes sense.

-Council of Chief State School Officers, 2010

Discussion

- How do these two quotes fit with your philosophy of teaching?

- As "understanding" is the foundation of the next two days, what are some specific goals you hope to achieve during this training?

Building Mathematical Knowledge

Read the following quote and highlight the words/phrases that stand out to you.

"Children engaged in the process of problem solving build mathematical knowledge and understanding by grappling with and solving genuine problems, as opposed to completing routine exercises. They use reasoning and proof to make sense of mathematical tasks and concepts and to develop, justify, and evaluate mathematical arguments and solutions. Children create and use representations to reason through problems. They also engage in communication as they explain their ideas and reasoning verbally, in writing, and through representations. Children develop and use connections between mathematical ideas as they learn new mathematical concepts and procedures. They also build connections between mathematics and other disciplines by applying mathematics to real-world situations. By engaging in these processes, children learn mathematics by doing mathematics."

Retrieved from Teaching Student Centered Mathematics

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

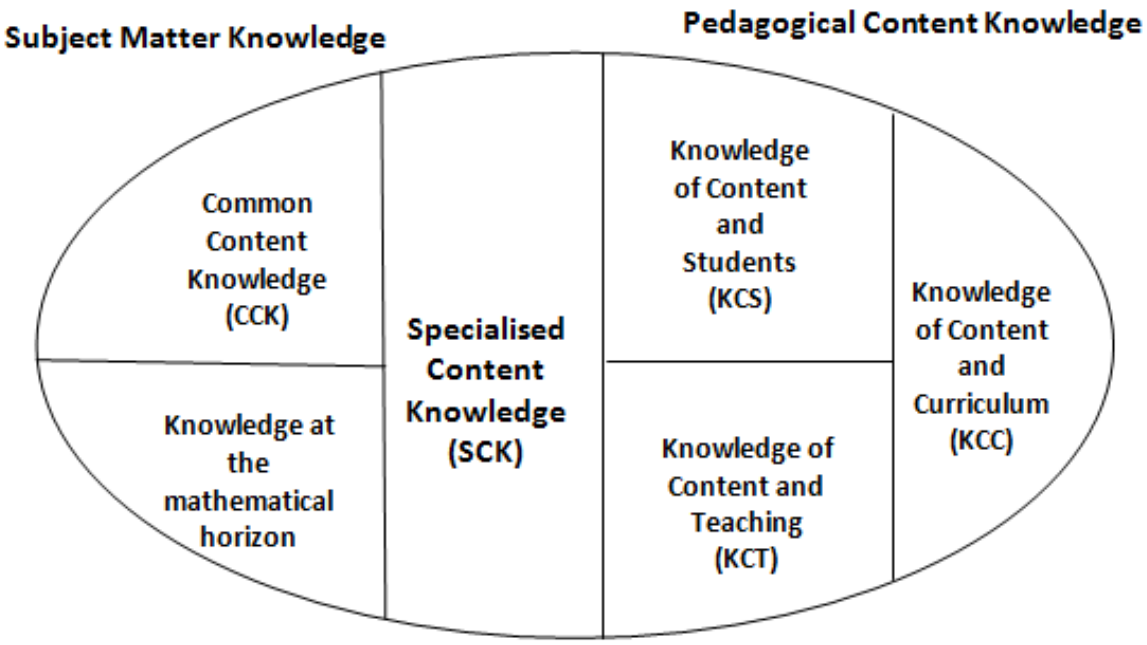
Discussion

- Look at the words or phrases you highlighted. Discuss with your table what you highlighted and why.

- How do these phrases connect to our notion of "understanding"?

MKT: Mathematical Knowledge for Teaching

Mathematical Knowledge for Teaching (MKT) is a kind of complex mathematical understanding, skill, and fluency used in the work of helping others build mathematical knowledge. Beyond being well versed in the content of the curriculum, teachers need significant mathematical skill, perspective, and judgment. Mathematical Knowledge for Teaching is divided into two domains: subject matter knowledge and pedagogical content knowledge. We want to develop each area to improve our teaching effectiveness and ensure students are learning mathematics for understanding. A key component for many early elementary teachers is the Knowledge at the Mathematical Horizon.



Knowledge at the mathematical horizon is an awareness of how mathematical topics are related over the span of mathematics curriculum. First grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later. It also includes the vision useful in seeing connections to much later mathematical ideas. Having this sort of knowledge of the mathematical horizon can help in making decisions about how, for example, to talk about the number line.

- Thames & Ball, 2010

Math Horizon Participant Activity

- Use the cards provided to think about the mathematical horizon on which our students develop number sense.
- Based on what you know about the expectations for learning from kindergarten to third grade standards, order the cards from introductory understanding of the concept to the most developed understanding and be prepared to justify your decisions.
- When everyone at your table has come to an agreement on the order of the cards, tape your cards to the chart paper.

Gallery Walk

- Circulate and analyze the sequence of tasks/activities
- Visit two or three groups' sequence of activities. Consider the order of the tasks/activities as identified by other groups.
- What is the same and what is different among the different sequences on display?
- Leave feedback for each group by recording a "wondering" and "noticing" on sticky notes.

Discussion

- Why is the knowledge of the mathematical horizon important as an educator?
- How could this knowledge guide you in the decisions you make for your classroom?
- How could deepening your knowledge of the mathematical horizon help you teach mathematics for understanding?

Common addition and subtraction situations.

Note: Beginning in kindergarten, students should be exposed to all addition and subtraction problem types. The different shading of the boxes in the table below indicates when mastery of that problem type is expected.

- The darkest shading indicates the problem types to be mastered at the end of kindergarten.
- The lighter shading indicates the problem types to be mastered at the end of first grade.
- The unshaded boxes indicate the problem types to be mastered at the end of second grade.

	Results Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two of them. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put together/take apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many can she put in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 4 + 1, 5 = 1 + 4 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	("How many more?" Version) Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? 2 + ? = 5, 5 – 2 = ?	(Version with "More") Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "More") Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5
	("How many fewer?" Version) Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "Fewer") Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "Fewer") Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Classroom Planning Tool

The template below can be used as a planning tool to help balance the types of situational problems that students have the opportunity to solve during units of instruction.

Action		Result Unknown	Change Unknown	Start Unknown
	Join			
	Separate			

Nonaction		Whole Unknown	Part Unknown
	Part-Part-Whole		

Nonaction			Difference Unknown	Greater Unknown	Lesser Unknown
	Compare	"How many more?"			
		"How many fewer?"			

Making Sense of Mathematics for Teaching Grades K–2 ©2016 Solution Tree Press • solution-tree.com
Visit go.SolutionTree.com/mathematics to download this page.

Classroom Planning Tool
Task Analysis Guide (TAG)

Another excellent planning tool is the Task Analysis Guide (TAG) that was introduced in previous summer trainings. The TAG can help us make sure that there is an appropriate balance of high-level instructional tasks.

Lower-Level Demands	*Higher-Level Demands*
Memorization Tasks • Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	**Procedures With Connections Tasks** • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	**Doing Mathematics Tasks** • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Mathematics Teaching in the Middle School. Also in: Stein, Smith, Henningsen, & Silver (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 16. New York: Teachers College Press.

Classrooms that Promote Mathematical Understanding

What are characteristics of a classroom that promotes mathematical understanding?

"The essence of developing rational understanding is to keep the children's ideas at the forefront of classroom activities by emphasizing the process standards, mathematical proficiencies and the Standards for Mathematical Practice. This requires that the teacher create a classroom culture in which children can learn from one another."

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

"The purpose of discourse is not for children to state their answers and get validation from the teacher, but to engage all learners and keep the cognitive demand high."

-Breyfogle & Williams, 2009

Participant Activity: Take a moment and read the two quotes above. Discuss these quotes with your table. As a group decide on a list of features that are a "must do" in a classroom that promotes mathematical understanding. Chart the group's "must" classroom characteristics and be prepared to share with the whole group.

Below is a list of features of a mathematics classroom that promote understanding:

- *Children's ideas are key*- mathematical ideas expressed by children are important and have the potential to contribute to everyone's learning. Learning mathematics is about coming to understand the ideas of the mathematical community.

- *Opportunities for children to talk about mathematics are common*- Learning is enhanced when children are engaged with others who are working on the same ideas. Encouraging student to student dialogue can help children think of themselves as capable of making sense of mathematics. Children are also more likely to question each other's ideas than the teacher's ideas.

- *Multiple approaches are encouraged*- Children must recognize that there is often a variety of methods that will lead to a solution. Respect for the ideas shared by others is critical if real discussion takes place.

- *Mistakes are good opportunities for learning*- Children must come to realize that errors provide opportunities for growth as they are uncovered and explained. Trust must be established with an understanding that it is okay to make mistakes. Without this trust, many ideas will never be shared.

- *Math makes sense*- Children must come to understand that mathematics makes sense. Teachers should resist always evaluating children's answers. In fact, when teachers routinely respond with "Yes, that's correct," or "No, that's wrong," children will stop trying to make sense of ideas in the classroom and discussion and learning will be curtailed.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Reflection

- What are the differences and similarities between the above list and the list your group created?

- As you think about your own classroom, is there anything that you would like to refine in your own teaching practices?

"Mistakes are not only opportunities for learning, as students consider the mistakes but also time when our brain grows."

- Boaler, 2016.

NOTE: It is important for students to see counterexamples, so purposefully point them out and have discussions about them. These discussions allow children to explore misunderstandings.

Video: My Favorite No

https://www.teachingchannel.org/videos/class-warm-up-routine

- As you watch the video, what are some strategies this teacher uses that makes this strategy effective?

- What connections can you make to developing oral language skills and the Strands of Mathematical Proficiency?

Small Group Activity

With your small group, discuss some of the favorite no's you may address with your students on these various problems.

- Write Two Hundred Eighty-Three in standard form
- 60+8 = _____ tens and _____ ones
- 5 tens and 3 ones = 13 ones and _____ tens
- 17 + 18
- 6 - ☐ = 3
- 58 + 29
- Write Three Hundred Four in standard form

Oral Language Needs for Developing Mathematical Understanding

"Oral language development is one of the most important, yet basic, foundational skills of children. If children come to school with well-developed oral language, it must be expanded. If children come to school with underdeveloped oral language, it must be developed."
"It is time we reflect on and explore how oral language can affect not only reading proficiency but math proficiency as well."

Pace, M. H., & Ortiz, E. (2015). Oral Language Needs: Making Math Meaningful. *Teaching Children Mathematics*, 494-500.

Activity: Vocabulary Strategy

In the article quoted above, the authors describe a mathematics vocabulary strategy that helps young students make meaning of mathematical vocabulary through pictures, words, and oral communication.

(Pace & Ortiz, 2015)

Read the excerpt below that describes how the author implemented the vocabulary strategy in her classroom:

"I created an introductory mathematics lesson focusing on the word joining. After many modeling examples, students were ready to create the vocabulary chart by creating their own pictures to explain what the word joining meant. They were asked to take an index card back to their seat and draw a picture that would explain to the class what joining meant. I explained to them that they must be able to tell the class about their picture and why it represented *joining*."

"After ten minutes of watching my little mathematicians draw furiously, we started our group sharing. One by one, hands flew into the air, eager to share their work. Students were able to define the word *join* by creating a meaningful picture. At this point in the lesson, I could tell no observable difference between the varying academic levels of my students."

"In my classroom, students were making sense of the term *joining* in their own way, making their understanding meaningful and observable trhough their pictures and dialogue."

(Pace & Ortiz, 2015)

Discussion

- How does this vocabulary strategy strengthen student mathematical understanding?

- What activities or strategies do you do in your classroom to help your students make sense of mathematical vocabulary and develop oral language?

Vocabulary Chart Activity

- Split into grade level groups
- Count off by 4's. Gather each number together.
- Each group will utilize the strategy from the article and plan out various ways in which they would present it to their students. Be specific: what types of experiences do you want them to have.
- Draw your own visual for the word you are given and display on chart paper.
- Suggested vocabulary
    - 1st grade- Greater than, less than, Expression, Equal
    - 2nd grade- Equation, Compose, Decompose

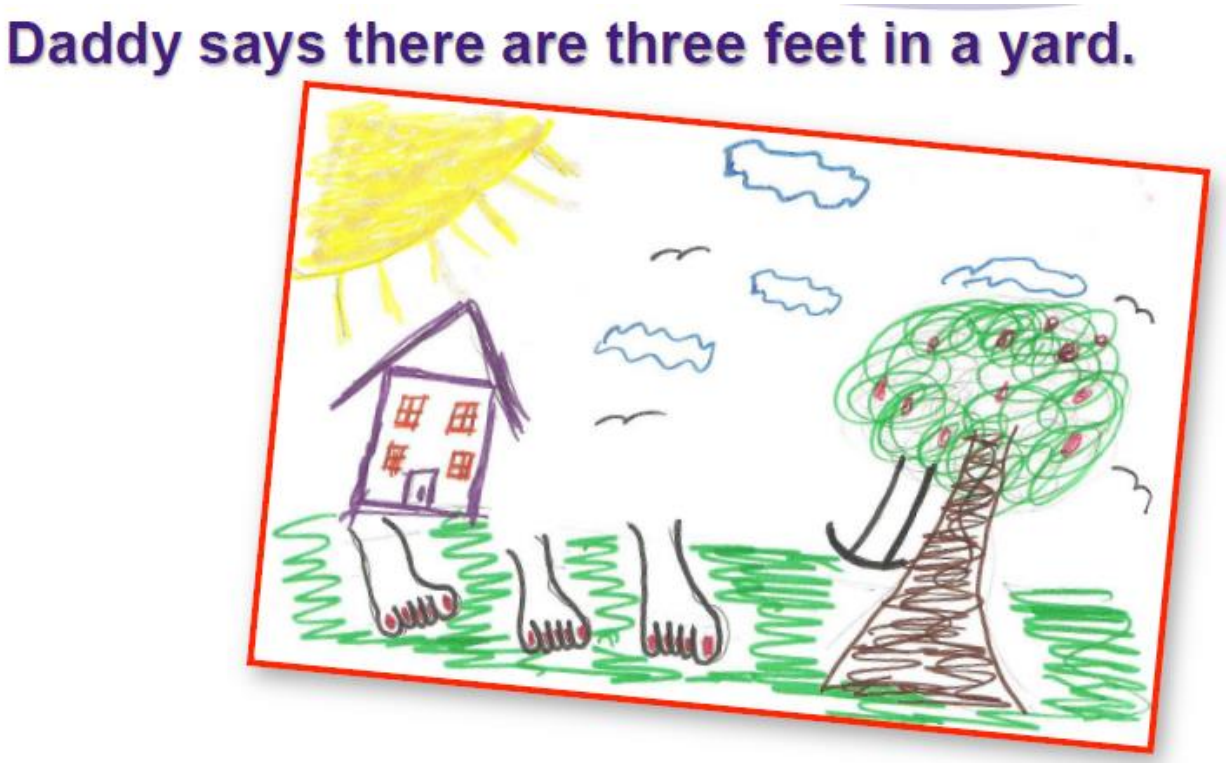
Gallery Walk

- What do you notice about other participants' depictions of the vocabulary words?

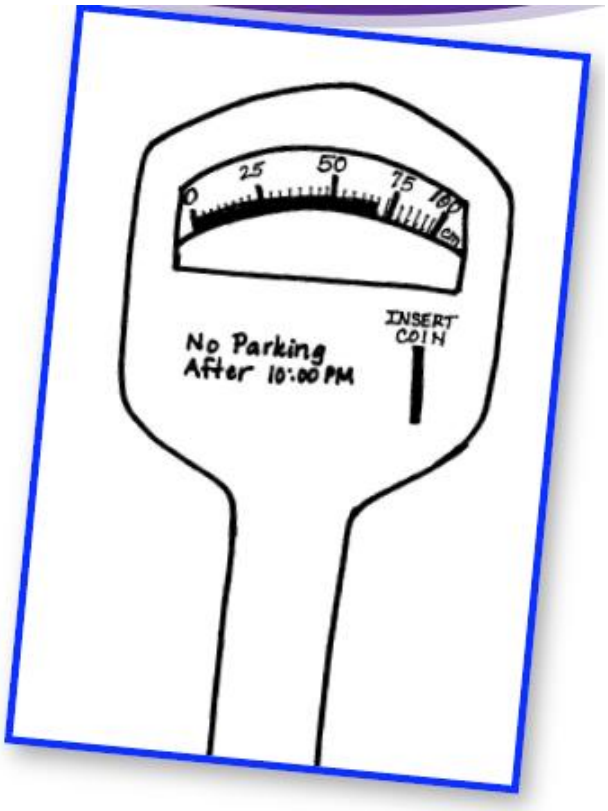
- How is this type of activity beneficial for a classroom promoting mathematical understanding?

Vocabulary in Mathematics

Without specific mathematics vocabulary instruction, students may become confused as some words are shared by a content area and everyday English, but have different meanings.



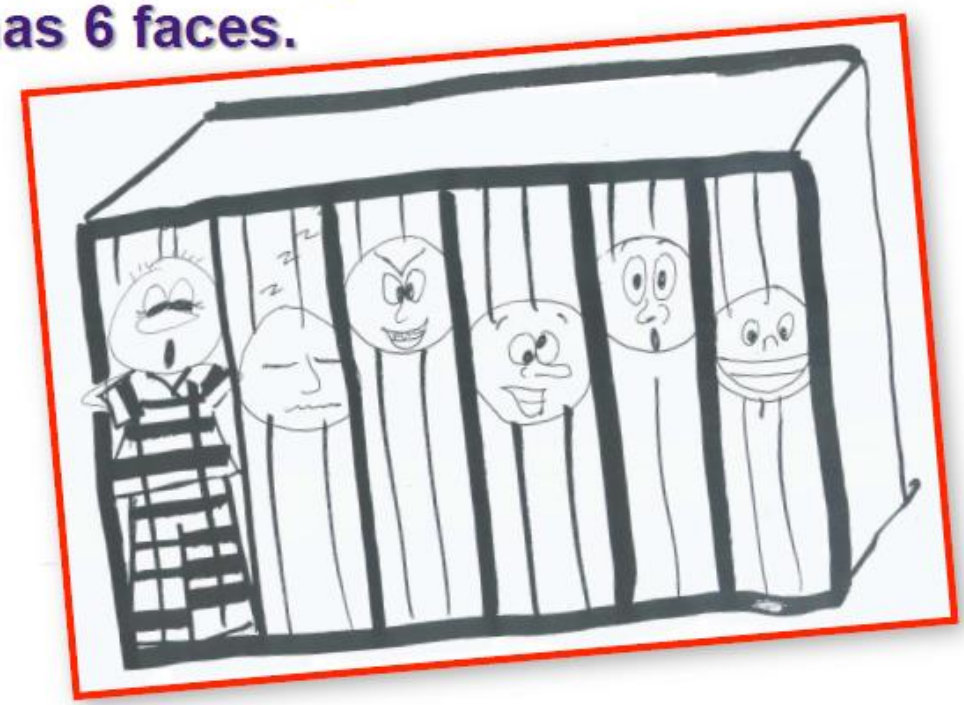
The teacher said there are 100 centimeters in a meter.



My teacher says the Earth is like a giant sphere.

My teacher said we needed to find the volume of the cereal box.

Mom says a rectangular prism has 6 faces.



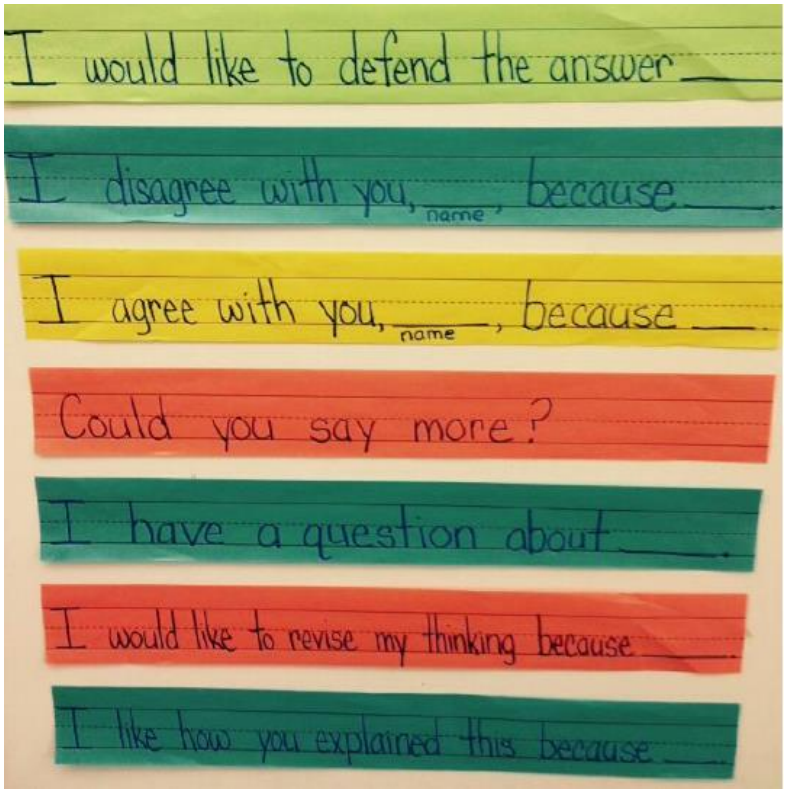
Retrieved from https://www.mentoringminds.com/documents/VocabularyinMathematics.pdf

Supporting Oral Language Development with Number Talks

Video: Language Supports for Number Talks

https://www.teachingchannel.org/videos/sentence-frames-ousd

As you view the video, record your key takeaways below:



Discussion:

- How can sentence frames empower students during number talks?

- How are the sentence frames related to Accountable Talk?

- Create some sentence frames with your table group.

Module 1 Closing Reflection

- How has this module changed the way you think about mathematics instruction?

- What will you do differently as a result?

- What are you still wondering about?

Module 1 Collaborative Notes

Ideas for collaboration: vocabulary strategies or routines; math talk or accountable talk strategies; resources on standards for mathematical practice; resources for addition and subtraction problem types.