Spiral Calculation Guide

Roadway Design Division

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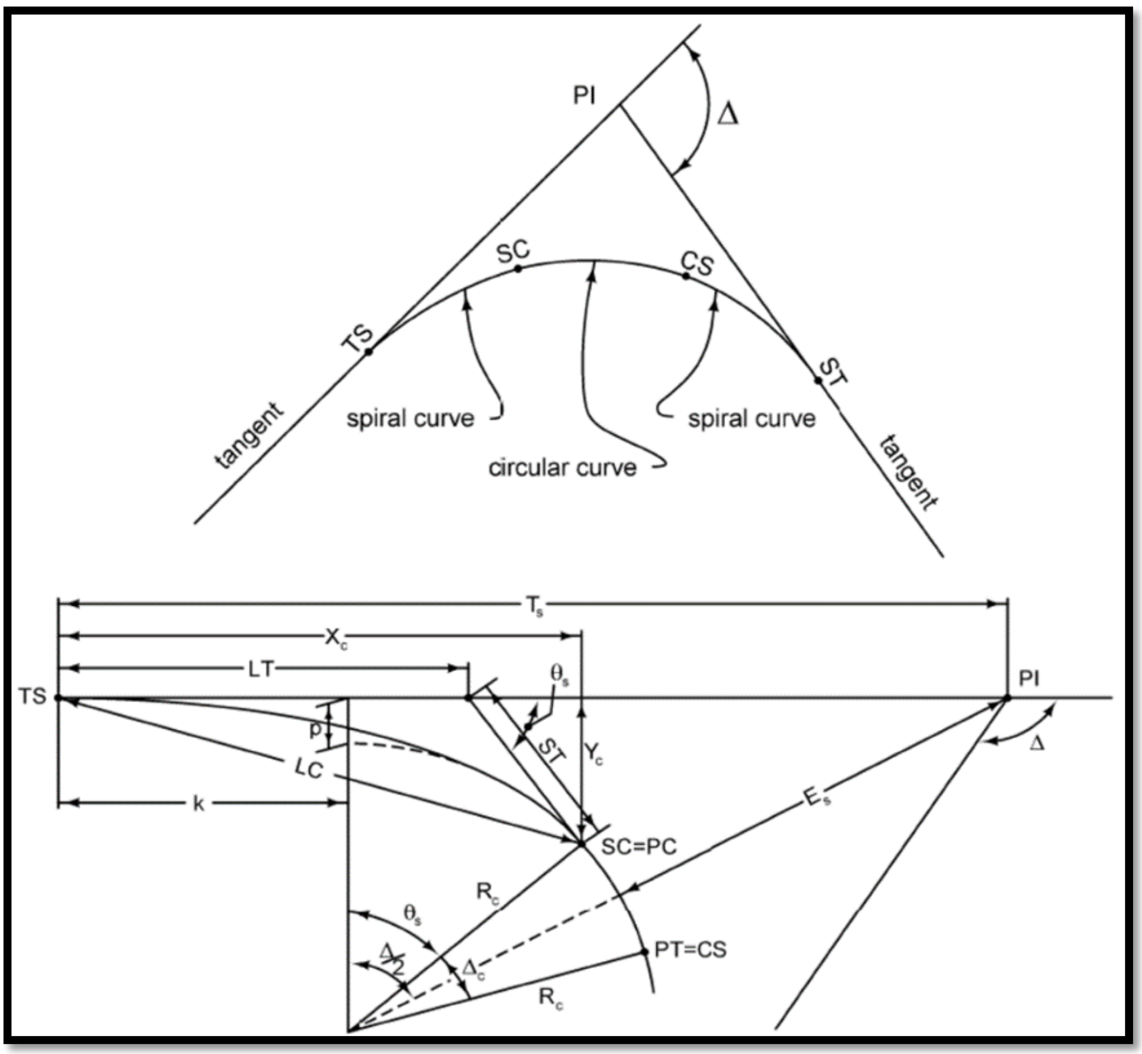
A spiral curve can be used to provide a gradual transition between tangent sections and circular curves. While a circular curve has a radius that is constant, a spiral curve has a radius that varies along its length. The radius decreases from infinity at the tangent to the radius of the circular curve it is intended to meet. This allows vehicles to transition into and out of a curve more easily while staying within the travel lane. The superelevation tables in the RD11-LR and RD11-SE series define when spiral curves shall be placed on projects. Spirals are recommended for curves 50 MPH or greater and superelevation of three percent or greater. This guide covers the steps of finding the placement of a spiral transition for a high-speed horizontal curve.

The figure below illustrates the standard components of a spiral curve connecting tangents with a central circular curve. The back and forward tangent sections intersect one another at the PI. The alignment changes from the back tangent to the entrance spiral at the TS point. The entrance spiral meets the circular curve at the SC point. The circular curve meets the exit spiral at the CS point. The alignment changes from the exit spiral to the forward tangent at the ST point. The entrance and exit spiral at each end of the circular curve are geometrically identical.



Below is a list of term and abbrievations of a spiral curve and the definition of each.

PI	Point of intersection of main tangents
TS	Point of change from tangent to spiral curve
SC	Point of change from spiral curve to circular curve
CS	Point of change from circular curve to spiral curve
ST	Point of change from spiral curve to tangent
LC	Long chord
LT	Long tangent
ST	Short tangent
PC	Point of curvature for the adjoining circular curve
PT	Point of tangency for the adjoining circular curve
T_S	Tangent distance from TS to PI or ST to PI
R_C	Radius of the adjoining circular curve
D_C	Degree of curve of the adjoining circular curve, based on a 100-foot arc
ℓ_s	Total length of spiral curve from TS to SC or CS to ST
L	Length of adjoining circular curve
Θ_S **(Theta)**	Spiral angle of arc ℓ_s
Δ **(Delta)**	Total central angle of the circular curve from TS to ST
Δ_C	Central angle of circular curve of length L extending from SC to CS
ρ **(rho)**	Offset from the initial tangent
k	Abscissa of the distance between the shifted PC and TS
Y_C	Tangent offset at the SC
X_C	Tangent distance at the SC

Below is a table of formulas to calculate each component of a spiral curve. Each will be used in an example calculation later in this guide. It is important to remember that there is **NO** unit conversion needed after the calculations. The unit conversion is considered within each calculation.

Eqn. #	Component	Formula	Units
1	Degree of Curvature	$D_C = \frac{5729.57795}{R_C}$	R_C in feet D_C in decimal degrees
2	Spiral Angle	$\theta_s = \frac{\ell_s D_c}{200}$	θ_S and D_C in decimal degrees ℓ_S in feet
3	Central Angle of the circular curve from the SC to the CS	$\Delta_C = \Delta - 2\theta_S$	Δc, Δ and θs measured in decimal degrees
4	Length of the circular Curve from the SC to the CS	$L = 100\left(\frac{\Delta_C}{D_C}\right)$	Δc and D_C in decimal degrees L in feet
5	Tangent Offset from the TS to the SC	$Y_C = \frac{\ell_s{}^2}{6R_C}$	θ_S in decimal degrees ℓ_S and Y_C in feet
6	Tangent Distance from the TS to the SC	$X_C = \ell_s - \frac{Y_C}{2\ell_s}$	θ_S in decimal degrees ℓ_S, Y_C and X_C in feet
7	Offset from Initial Tangent	$\rho = Y_C - R_C[1 - \cos(\theta_s)]$	Y_C, R_C and ρ in feet θ_S in decimal degrees
8	Tangent Distance from TS to Pre-Spiral PC (Shifted PC)	$k = X_C - R_C \sin(\theta_s)$	X_C and k in feet θ_S in decimal degrees
9	Tangent Distance from PI to TS or ST	$T_S = (R_C + p)\tan\left(\frac{\Delta}{2}\right) + k$	T_S, R_C, p and k in feet Δ in decimal degrees
10	TS Station	TS = PI – T_S	TS and PI in station Ts in ft
11	SC Station	SC = TS + ℓ_s	Ts and ℓ_S in feet SC and TS in station
12	CS Station	CS = SC + L	CS and SC in stations L in feet
13	ST Station	ST = CS + ℓ_s	ℓ_S in feet ST and CS in stations

In this portion of the guide, an example will be worked through. It is important to know that this process has a very specific order of steps that must be followed. For this example, the provided information is shown in the table below.

Entry Bearing	S 35.3 E
Exit Bearing	S 20.1 W
Max Superelevation rate	4.0 %
Circular Curve Radius	4000 ft
Spiral Length	275 ft
PI Station	100+00

The spiral length given for this example is 275 ft. Standard Drawing RD11-SE-1 can be used to calculate the spiral length with different parameters. The spiral length is the same as the superelevation transition length that can be calculated on this standard drawing.

The first calculation is to determine the central angle, Δ. The central angle of each curve should be as small as the physical conditions permit, so that the highway will be as directional as practical. One way to think about the central angle is that it is the angle that the vehicles turns throughout the horizontal curve. For this example, the vehicle is traveling from a bearing of S35.3E to a bearing of S20.1W, so the central angle would be as follows:

$$\Delta = 35.3° + 20.1° = 55.4°$$

The central angle calculation is not always as simple as adding up the angles as it depends on the direction of the entry and exit bearings of a curve. More information and examples can be found in Appendix A.

The next calculation is the Degree of Curvature, D_C.

$$D_C = \frac{5729.57795}{R_C} = \frac{5729.57795}{4000} = 1.432° \qquad \text{Eqn. 1}$$

Spiral Angle, θ_s, of the spiral length, ℓ_s

$$\theta_s = \frac{\ell_s D_c}{200} = \frac{275\ x\ 1.432}{200} = 1.970° \qquad \text{Eqn. 2}$$

Central angle of the circular curve from SC to CS:

$$\Delta_C = \Delta - 2\theta_S = 55.4 - (2\ x\ 1.970) = 51.461° \qquad \text{Eqn. 3}$$

Length of the circular curve from SC to CS:

$$L = 100\left(\frac{\Delta_C}{D_C}\right) = 100\left(\frac{51.461}{1.432}\right) = 3593.645\ ft \qquad \text{Eqn. 4}$$

Tangent Offset from TS to SC:

$$Y_C = \frac{\ell_s^2}{6R_C} = \frac{275^2}{6\,x\,4000} = 3.151\,ft$$ Eqn. 5

Tangent Distance from TS to SC:

$$X_C = \ell_s - \frac{Y_C}{2\ell_s} = 275 - \frac{3.151}{2\,x\,275} = 274.982\,ft$$ Eqn. 6

Offset from Initial Angle:

$$\rho = Y_C - R_C[1 - \cos(\theta_s)] = 3.151 - 4000\,[1 - \cos(1.970)] = 0.788\text{ ft}$$ Eqn. 7

Tangent Distance from TS to Pre-Spiral PC:

$$k = X_C - R_C \sin(\theta_s) = 274.982 - 4000\,[\sin(1.970)] = 137.509\,ft$$ Eqn. 8

Tangent Distance from PI to TS or ST

$$T_S = (R_C + \rho)\tan\left(\frac{\Delta}{2}\right) + k = (4000 + 0.788)\tan\left(\frac{55.4}{2}\right) + 137.509 = 2237.970\,ft$$ Eqn. 9

TS Station

$$TS = PI - T_S = 10000 - 2237.970 = 7762.03,\ \textbf{Station 77+62.03}$$ Eqn. 10

SC Station:

$$SC = TS + \ell_s = 7762.03 + 275 = 8037.03,\ \textbf{Station 80+37.03}$$ Eqn. 11

CS Station:

$$CS = SC + L = 8037.03 + 3593.645 = 11129.68,\ \textbf{Station 116+29.68}$$ Eqn. 12

ST Station:

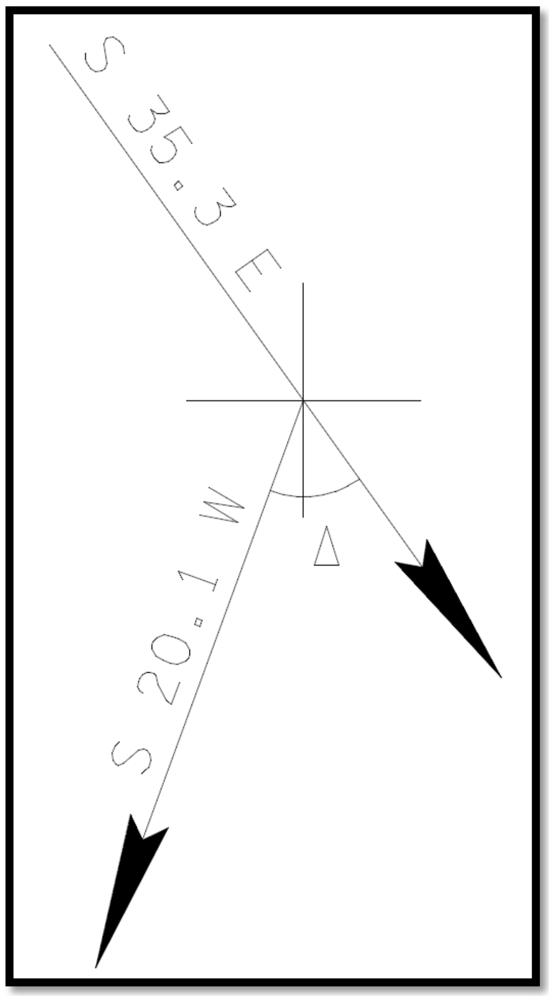
$$ST = CS + \ell_s = 11629.68 + 275 = 11904.68,\ \textbf{Station 119+04.68}$$ Eqn. 13

Appendix A

The calculation of the central angle depends on the entry and exit bears of the horizontal curve. A good way to think about the central angle is that it is the angle in which the headlights of the vehicle make when traveling from the entry bearing to the exit bearing. Below are a couple examples of how the calculations differ based on different bearings.

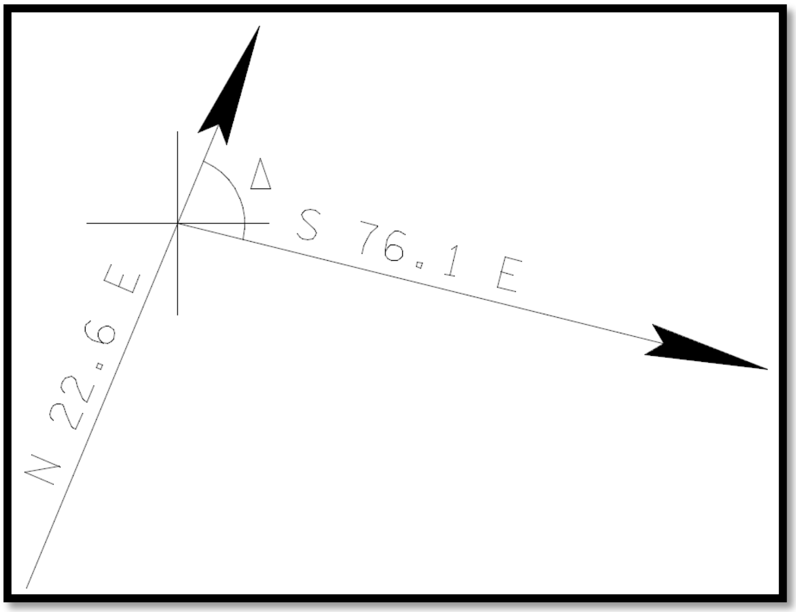
1. From S35.3E to S20.1W

$$\Delta = 36.3 + 20.1 = 55.4$$



2. From N22.6E to S76.1E

$$\Delta = (90 - 22.6) + (90 - 76.1) = 81.3$$



3. From S43.5E to S70.0E

$$\Delta = 70.0 - 43.5 = 26.5$$