Tennessee Math Standards

Introduction

The Process

The Tennessee State Math Standards were reviewed and developed by Tennessee teachers for Tennessee schools. The rigorous process used to arrive at the standards in this document began with a public review of the then-current standards. After receiving 130,000+ reviews and 20,000+ comments, a committee composed of Tennessee educators spanning elementary through higher education reviewed each standard. The committee scrutinized and debated each standard using public feedback and the collective expertise of the group. The committee kept some standards as written, changed or added imbedded examples, clarified the wording of some standards, moved some standards to different grades, and wrote new standards that needed to be included for coherence and rigor. From here the standards went before the appointed Standards Review Committee to make further recommendations before being presented to the Tennessee Board of Education for final adoption.

The result is Tennessee Math Standards for Tennessee Students by Tennesseans.

Mathematically Prepared

Tennessee students have various mathematical needs that their K-12 education should address.

All students should be able to recall and use their math education when the need arises. That is, a student should know certain math facts and concepts such as the multiplication table, how to add, subtract, multiply, and divide basic numbers, how to work with simple fractions and percentages, etc. There is a level of procedural fluency that a student's K-12 math education should provide him or her along with conceptual understanding so that this can be recalled and used throughout his or her life. Students also need to be able to reason mathematically. This includes problem solving skills in work and non-work related settings and the ability to critically evaluate the reasoning of others.

A student's K-12 math education should also prepare him or her to be free to pursue post-secondary education opportunities. Students should be able to pursue whatever career choice, and its post-secondary education requirements, that they desire. To this end, the K-12 math standards lay the foundation that allows any student to continue further in college, technical school, or with any other post-secondary educational needs.

A college and career ready math class is one that addresses all of the needs listed above. The standards' role is to define what our students should know, understand, and be able to do mathematically so as to fulfill these needs. To that end, the standards address conceptual understanding, procedural fluency, and application.

Conceptual Understanding, Procedural Fluency, and Application

In order for our students to be mathematically proficient, the standards focus on a balanced development of conceptual understanding, procedural fluency, and application. Through this balance, students gain understanding and critical thinking skills that are necessary to be truly college and career ready.

Conceptual understanding refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.

Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly. One cannot stop with memorization of facts and procedures alone. It is about recognizing when one strategy or procedure is more appropriate to apply than another. Students need opportunities to justify both informal strategies and commonly used procedures through distributed practice. Procedural fluency includes computational fluency with the four arithmetic operations. In the early grades, students are expected to develop fluency with whole numbers in addition, subtraction, multiplication, and division. Therefore, computational fluency expectations are addressed throughout the standards. Procedural fluency extends students' computational fluency and applies in all strands of mathematics. It builds from initial exploration and discussion of number concepts to using informal strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014).

Application provides a valuable context for learning and the opportunity to practice skills in a relevant and a meaningful way. As early as Kindergarten, students are solving simple "word problems" with meaningful contexts. In fact, it is in solving word problems that students are building a repertoire of procedures for computation. They learn to select an efficient strategy and determine whether the solution(s) makes sense. Problem solving provides an important context in which students learn about numbers and other mathematical topics by reasoning and developing critical thinking skills (Adding It Up, 2001).

Progressions

The standards for each grade are not written to be nor are they to be considered as an island in and of themselves. There is a flow, or progression, from one grade to the next, all the way through to the high school standards. There are four main progressions that are composed of mathematical domains/conceptual categories (see the Structure section below and color chart on the following page).

The progressions are grouped as follows:

Grade	Domain/Conceptual Category
K	Counting and Cardinality
K-5	Number and Operations in Base Ten
3-5	Number and Operations – Fractions
6-7	Ratios and Proportional Relationships
6-8	The Number System
9-12	Number and Quantity
K-5	Operations and Algebraic Thinking
6-8	Expressions and Equations
8	Functions
9-12	Algebra and Functions
K-12	Geometry
K-5	Measurement and Data
6-12	Statistics and Probability

State Standards – Mathematics Learning Progressions

<table>
<tr><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations - Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="6">Geometry</td><td colspan="3">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
</table>

Each of the progressions begins in Kindergarten, with a constant movement toward the high school standards as a student advances through the grades. This is very important to guarantee a steady, age appropriate progression which allows the student and teacher alike to see the overall coherence of and connections among the mathematical topics. It also ensures that gaps are not created in the mathematical education of our students.

Structure of the Standards

Most of the structure of the previous state standards has been maintained. This structure is logical and informative as well as easy to follow. An added benefit is that most Tennessee teachers are already familiar with it.

The structure includes:

- **Content Standards -** Statements of what a student should know, understand, and be able to do.

- **Clusters -** Groups of related standards. Cluster headings may be considered as the big idea(s) that the group of standards they represent are addressing. They are therefore useful as a quick summary of the progression of ideas that the standards in a domain are covering and can help teachers to determine the focus of the standards they are teaching.

- **Domains -** A large category of mathematics that the clusters and their respective content standards delineate and address. For example, *Number and Operations – Fractions* is a domain under which there are a number of clusters (the big ideas that will be addressed) along with their respective content standards, which give the specifics of what the student should know, understand, and be able to do when working with fractions.

- **Conceptual Categories –** The content standards, clusters, and domains in the 9^{th}-12^{th} grades are further organized under conceptual categories. These are very broad categories of mathematical thought and lend themselves to the organization of high school course work. For example, Algebra is a conceptual category in the high school standards under which are domains such as Seeing Structure in Expressions, Creating Equations, Arithmetic with Polynomials and Rational Expressions, etc.

Standards and Curriculum

It should be noted that the standards are what students should know, understand, and be able to do; but, they do not dictate how a teacher is to teach them. In other words, the standards do not dictate curriculum. For example, students are to understand and be able to add, subtract, multiply, and divide fractions according to the standards. Although within the standards algorithms are mentioned and examples are given for clarification, how to approach these concepts and the order in which the standards are taught within a grade or course are all decisions determined by the local district, school, and teachers.

Example from the Standards' Document for K – 8

Taken from 3rd Grade Standards:

Measurement and Data (MD)	
Cluster Headings	**Content Standards**
A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve contextual problems involving addition and subtraction of time intervals in minutes. *For example, students may use a number line to determine the difference between the start time and the end time of lunch.* 3.MD.A.2 Measure the mass of objects and liquid volume using standard units of grams (g), kilograms (kg), milliliters (ml), and liters (l). Estimate the mass of objects and liquid volume using benchmarks. *For example, a large paper clip is about one gram, so a box of about 100 large clips is about 100 grams. Therefore, ten boxes would be about 1 kilogram.*

The domain is indicated at the top of the table of standards. The left column of the table contains the cluster headings. A light green coloring of the cluster heading (and codes of each of the standards within that cluster) indicates the major work of the grade. Supporting standards have no coloring. In this way, printing on a non-color printer, the standards belonging to the major work of the grade will be lightly shaded and stand distinct from the supporting standards. This color coding scheme will be followed throughout all standards K – 12. Next to the clusters are the content standards that indicate specifically what a student is to know, understand, and do with respect to that cluster. The numbering scheme for K-8 is intuitive and consistent throughout the grades. The numbering scheme for the high school standards will be somewhat different.

Example coding for grades K-8 standards:
3.MD.A.1
3 is the grade level.
Measurement and Data (**MD**) is the domain.
A is the cluster (ordered by A, B, C, etc. for first cluster, second cluster, etc.).
1 is the standard number (the standards are numbered consecutively throughout each domain regardless of cluster).

Example from the Standards' Document for 9 – 12

Taken from Integrated Math 1 Standards:

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**M1.A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.	*For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* *Tasks are limited to linear and exponential expressions, including related numerical expressions.*

The high school standards follow a slightly different coding structure. They start with the course indicator (M1 – Integrated Math 1, A1 – Algebra 1, G – Geometry, etc.), then the conceptual category (in the example below – Algebra) and then the domain (just above the table of standards it represents – Seeing Structure in Expressions). There are various domains under each conceptual category. The table of standards contains the cluster headings (see explanation above), content standards, and the scope and clarifications column, which gives further clarification of the standard and the extent of its coverage in the course. A ★ with a standard indicates a modeling standard (see MP4 on p.11). The color coding is light green for the major work of the grade and no color for the supporting standards.

Example coding for grades 9-12 standards:
M1.A.SSE.A.1
Integrated Math 1 (**M1**) is the course.
Algebra (**A**) is the conceptual category.
Seeing Structure in Expressions (**SSE**) is the domain.
A is the cluster (ordered by A, B, C, etc. for first cluster, second cluster, etc.).
1 is the standard number (the standards are numbered consecutively throughout each domain regardless of cluster).

Tennessee State Math Standards

The Standards for Mathematical Practice

Being successful in mathematics requires that development of approaches, practices, and habits of mind be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop within their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics.

Processes and proficiencies are two words that address the purpose and intent of the practice standards. Process is used to indicate a particular course of action intended to achieve a result, and this ties to the process standards from NCTM that pertain to problem solving, reasoning and proof, communication, representation, and connections. Proficiencies pertain to being skilled in the command of fundamentals derived from practice and familiarity. Mathematically, this addresses concepts such as adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive dispositions toward the work at hand. The practice standards are written to address the needs of the student with respect to being successful in mathematics.

These standards are most readily developed in the solving of high-level mathematical tasks. High-level tasks demand a greater level of cognitive effort to solve than routine practice problems do. Such tasks require one to make sense of the problem and work at solving it. Often a student must reason abstractly and quantitatively as he or she constructs an approach. The student must be able to argue his or her point as well as critique the reasoning of others with respect to the task. These tasks are rich enough to support various entry points for finding solutions. To develop the processes and proficiencies addressed in the practice standards, students must be engaged in rich, high-level mathematical tasks that support the approaches, practices, and habits of mind which are called for within these standards.

The following are the eight standards for mathematical practice:

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

A full description of each of these standards follows.

MP1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2: Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MP3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

MP7: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8: Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Literacy Skills for Mathematical Proficiency

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others and analyze and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies.
2. Understand and use correct mathematical vocabulary.
3. Discuss and articulate mathematical ideas.
4. Write mathematical arguments.

Reading

Reading in mathematics is different from reading literature. Mathematics contains expository text along with precise definitions, theorems, examples, graphs, tables, charts, diagrams, and exercises. Students are expected to recognize multiple representations of information, use mathematics in context, and draw conclusions from the information presented. In the early grades, non-readers and struggling readers benefit from the use of multiple representations and contexts to develop mathematical connections, processes, and procedures. As students' literacy skills progress, their skills in mathematics develop so that by high school, students are using multiple reading strategies, analyzing context-based problems to develop understanding and comprehension, interpreting and using multiple representations, and fully engaging with mathematics textbooks and other mathematics-based materials. These skills support Mathematical Practices 1 and 2.

Vocabulary

Understanding and using mathematical vocabulary correctly is essential to mathematical proficiency. Mathematically proficient students use precise mathematical vocabulary to express ideas. In all grades, separating mathematical vocabulary from everyday use of words is important for developing an understanding of mathematical concepts. For example, a "table" in everyday use means a piece of furniture, while in mathematics, a "table" is a way of organizing and presenting information. Mathematically proficient students are able to parse a mathematical term, definition, or theorem, provide examples and counterexamples, and use precise mathematical vocabulary in reading, speaking, and writing arguments and explanations. These skills support Mathematical Practice 6.

Speaking and Listening

Mathematically proficient students can listen critically, discuss, and articulate their mathematical ideas clearly to others. As students' mathematical abilities mature, they move from communicating through reiterating others' ideas to paraphrasing, summarizing, and drawing their own conclusions. A mathematically proficient student uses appropriate mathematics vocabulary in verbal discussions, listens to mathematical arguments, and dissects an argument to recognize flaws or determine validity. These skills support Mathematical Practice 3.

Writing

Mathematically proficient students write mathematical arguments to support and refute conclusions and cite evidence for these conclusions. Throughout all grades, students write reflectively to compare and contrast problem-solving approaches, evaluate mathematical processes, and analyze their thinking and decision-making processes to improve their mathematical strategies. These skills support Mathematical Practices 2, 3, and 4.

Mathematics | Grade K

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout Kindergarten.

Counting and Cardinality
Students use numbers, including written numerals and counting, to develop concepts about quantity. Students use numbers to solve contextual problems and represent quantities, such as counting objects in a set, counting out a given number of objects, and comparing sets or numerals. Students use effective strategies for counting and answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects and learning about counting sequences.

Operations and Algebraic Thinking
Students develop an understanding of addition and subtraction and determine when to add or subtract in a given context. Students should solve a variety of problem types in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations). Students choose from multiple representations (including using objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations) when solving addition and subtraction problems within 10. Students decompose quantities within 10 in various ways, and fluently add and subtract using mental strategies. By the end of Kindergarten, students should fluently add and subtract within 10.

Number and Operations in Base Ten
Students understand that numbers from 11 to 19 represent ten ones and some more ones by using objects or drawings, and record each composition or decomposition by a drawing and/or write an equation to represent this relationship.

Measurement and Data
Students describe and sort objects in many different ways. This includes length, weight, and coins. They classify objects in categories and compare measurable attributes. Students begin to learn to graph and analyze collections of objects. Students learn to identify the penny, nickel, dime, and quarter and know the value of each.

Geometry
Students describe their physical world using geometric ideas, vocabulary, and positional words. Regardless of orientation, students name two-dimensional and three-dimensional shapes, compare shapes, and combine shapes to create new shapes. Students identify patterns they discover in numbers, counting, and shapes.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Counting and Cardinality (CC)

Cluster Headings	Content Standards
A. Know number names and the counting sequence.	**K.CC.A.1** Count to 100 by ones, fives, and tens. Count backward from 10. **K.CC.A.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1). **K.CC.A.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20.
B. Count to tell the number of objects.	**K.CC.B.4** Understand the relationship between numbers and quantities; connect counting to cardinality. **a.** When counting objects, say the number names in the standard order, using one-to-one correspondence. **b.** Recognize that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **c.** Recognize that each successive number name refers to a quantity that is one greater. **K.CC.B.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, a circle, or as many as 10 things in a scattered configuration. Given a number from 1-20, count out that many objects.
C. Compare numbers.	**K.CC.C.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. **K.CC.C.7** Compare two given numbers up to 10, when written as numerals, using the terms *greater than, less than*, or *equal to.*

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. (See Table 1 - Addition and Subtraction Situations)	**K.OA.A.1** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. **K.OA.A.2** Add and subtract within 10 to solve contextual problems using objects or drawings to represent the problem.

Cluster Headings	Content Standards
A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. (See Table 1 - Addition and Subtraction Situations)	**K.OA.A.3** Decompose numbers less than or equal to 10 into addend pairs in more than one way (e.g., 5 = 2 + 3 and 5 = 4 + 1) by using objects or drawings. Record each decomposition using a drawing or writing an equation. **K.OA.A.4** Find the number that makes 10, when added to any given number, from 1 to 9 using objects or drawings. Record the answer using a drawing or writing an equation. **K.OA.A.5** Fluently add and subtract within 10 using mental strategies.

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Work with numbers 11–19 to gain foundations for place value.	**K.NBT.A.1** Compose and decompose numbers from 11 to 19 into ten ones and some more ones by using objects or drawings. Record the composition or decomposition using a drawing or by writing an equation.

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Describe and compare measurable attributes.	**K.MD.A.1** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **K.MD.A.2** Directly compare two objects with a measurable attribute in common, to see which object has more of/less of the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
B. Work with money.	**K.MD.B.3** Identify the penny, nickel, dime, and quarter and recognize the value of each.
C. Classify objects and count the number of objects in each category.	**K.MD.C.4** Sort a collection of objects into a given category, with 10 or less in each category. Compare the categories by group size.

Geometry (G)

Cluster Headings	Content Standards
A. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	**K.G.A.1** Describe objects in the environment using names of shapes. Describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind*, *between*, and *next to*. **K.G.A.2** Correctly name shapes regardless of their orientations or overall size. **K.G.A.3** Identify shapes as two-dimensional or three-dimensional.
B. Analyze, compare, create, and compose shapes.	**K.G.B.4** Describe similarities and differences between two- and three-dimensional shapes, in different sizes and orientations. **K.G.B.5** Model shapes in the world by building and drawing shapes. **K.G.B.6** Compose larger shapes using simple shapes and identify smaller shapes within a larger shape.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ? **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5 **One-Step Problem** **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ? **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3 **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3 **One-Step Problem** **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ? **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ? **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2 **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1st)**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? 5 – 3 = ? ? + 3 = 5 **One-Step Problem** **(2nd)**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ? **(1st)**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ? **One-Step Problem** **(2nd)**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1st)**

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Mathematics | Grade 1

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 1st grade.

Operations and Algebraic Thinking
Students extend previous understanding of addition and subtraction to solve contextual problems within 20, add three addends, and recognize subtraction as an unknown addend problem. Students solve a variety of problem types, with unknowns in all positions, in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations). Students should apply properties of operations as strategies to add and subtract when needed (See Table 3 - Properties of Operations). By the end of 1st grade, students should know from memory sums of 10 and fluently add and subtract within 20.

Students demonstrate their understanding of the equal sign (=) by determining if addition/subtraction equations are true or false and writing equations to represent a given situation.

Numbers and Operations in Base Ten
Students read, write, and represent a given number of objects numerically and extend the counting sequence to 120. They demonstrate the ability to count from any number up to 120 and count backward from 20. Students understand that two-digit numbers represent groups of tens and ones and each two-digit number can be composed and decomposed in a variety of ways. Using place value understanding, students compare two-digit numbers based on the number of tens and ones represented in the given numbers using symbols for comparison.

Students build number sense and use increasingly sophisticated strategies based on place value and properties of operations to add and subtract.

Measurement and Data
This is the first time students develop an understanding of the meaning and processes of measurement including iteration of non-standard equal-sized units. Students compare two objects using a third object as a benchmark and also order objects by length. Students are introduced to writing and telling time to the nearest hour and half-hour. Students build on their previous work in kindergarten and count the value of like coins using the ¢ symbol. Students interpret data to answer questions such as how many more or less.

Geometry
Students build on previous knowledge to explore attributes of shapes and to build, draw, and identify two-dimensional shapes. Two and three-dimensional shapes are used to create composite shapes. This is the first time students partition circles and rectangles to create halves and fourths/quarters.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Represent and solve problems involving addition and subtraction.	**1.OA.A.1** Add and subtract within 20 to solve contextual problems, with unknowns in all positions, involving situations of *add to*, *take from*, *put together/take apart*, and *compare*. Use objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Table 1 - Addition and Subtraction Situations) **1.OA.A.2** Add three whole numbers whose sum is within 20 to solve contextual problems using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
B. Understand and apply properties of operations and the relationship between addition and subtraction. (See Table 3 - Properties of Operations)	**1.OA.B.3** Apply properties of operations (additive identity, commutative, and associative) as strategies to add and subtract. (Students need not use formal terms for these properties.) **1.OA.B.4** Understand subtraction as an unknown-addend problem. *For example, to solve 10 – 8 = ___, a student can use 8 + ___ = 10.*
C. Add and subtract within 20.	**1.OA.C.5** Add and subtract within 20 using strategies such as counting on, counting back, making 10, using fact families and related known facts, and composing/ decomposing numbers with an emphasis on making ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9 or adding 6 + 7 by creating the known equivalent 6 + 4 + 3 = 10 + 3 = 13). **1.OA.C.6** Fluently add and subtract within 20 using mental strategies. By the end of 1st grade, know from memory all sums up to 10.
D. Work with addition and subtraction equations.	**1.OA.D.7** Understand the meaning of the equal sign (e.g., 6 = 6; 5 + 2 = 4 + 3; 7 = 8 - 1). Determine if equations involving addition and subtraction are true or false. **1.OA.D.8** Determine the unknown whole number in an addition or subtraction equation, with the unknown in any position (e.g., 8 + ? = 11, 5 = ? - 3, 6 + 6 = ?).

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Extend the counting sequence.	**1.NBT.A.1** Count to 120, starting at any number. Read and write numerals to 120 and represent a number of objects with a written numeral. Count backward from 20.
B. Understand place value.	**1.NBT.B.2** Know that the digits of a two-digit number represent groups of tens and ones (e.g., 39 can be represented as 39 ones, 2 tens and 19 ones, or 3 tens and 9 ones). **1.NBT.B.3** Compare two two-digit numbers based on the meanings of the digits in each place and use the symbols >, =, and < to show the relationship.

Cluster Headings	Content Standards
C. Use place value understanding and properties of operations to add and subtract.	**1.NBT.C.4** Add a two-digit number to a one-digit number and a two-digit number to a multiple of ten (within 100). Use concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to explain the reasoning used. **1.NBT.C.5** Mentally find 10 more or 10 less than a given two-digit number without having to count by ones and explain the reasoning used. **1.NBT.C.6** Subtract multiples of 10 from multiples of 10 in the range 10-90 using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Measure lengths indirectly and by iterating length units.	**1.MD.A.1** Order three objects by length. Compare the lengths of two objects indirectly by using a third object. *For example, to compare indirectly the heights of Bill and Susan: if Bill is taller than mother and mother is taller than Susan, then Bill is taller than Susan.* **1.MD.A.2** Measure the length of an object using non-standard units and express this length as a whole number of units.
B. Work with time and money.	**1.MD.B.3** Tell and write time in hours and half-hours using analog and digital clocks. **1.MD.B.4** Count the value of a set of like coins less than one dollar using the ¢ symbol only.
C. Represent and interpret data.	**1.MD.C.5** Organize, represent, and interpret data with up to three categories. Ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry (G)

Cluster Headings	Content Standards
A. Reason about shapes and their attributes.	**1.G.A.1** Distinguish between attributes that define a shape (e.g., number of sides and vertices) versus attributes that do not define the shape (e.g., color, orientation, overall size); build and draw two-dimensional shapes to possess defining attributes. **1.G.A.2** Create a composite shape and use the composite shape to make new shapes by using two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, rectangular prisms, cones, and cylinders). **1.G.A.3** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as *two of*, or *four of* the shares. Understand for these examples that partitioning into more equal shares creates smaller shares.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[2]
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1st)**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** **(2nd)**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ **(1st)**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ **One-Step Problem** **(2nd)**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1st)**

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.
2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Mathematics | Grade 2

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 2nd grade.

Operations & Algebraic Thinking

Students solve one- and two-step addition and subtraction contextual problems within 100 with an unknown in any position. Students should solve a variety of problem types in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations). Students also represent these problems with objects, drawings, and/or equations. Students build upon previously taught strategies to mentally add and subtract within 30. Students know from memory all sums of two one-digit numbers and related subtraction facts.

Numbers & Operations in Base Ten

Students extend their understanding of the base-ten place value system to 1,000. This includes counting by ones, fives, tens, and hundreds. Students write numbers using standard form, word form, and expanded form. They deepen their understanding of different ways a number can be composed and decomposed. Students extend their understanding of place value, properties of operations, and the relationship between addition and subtraction to add and subtract within 1,000 and fluently add and subtract within 100 (See Table 3 - Properties of Operations). They add up to four two-digit numbers. They should also be able to explain why these strategies work. Students mentally add and subtract 10 or 100 from a given number 100-900.

Measurement & Data

In previous grades, students measured with non-standard units. Students in 2nd grade measure with standard units (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length. Students use addition and subtraction to solve contextual problems involving lengths in the same units and represent lengths on a number line.

Geometry

Students describe and analyze shapes by examining their sides and angles. Students recognize and draw shapes based on given attributes, such as draw a shape with 3 vertices. Students also are able to partition circles and rectangles into two, three, and four equal shares and rectangles into rows and columns, laying the foundation for fractions and area.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Represent and solve problems involving addition and subtraction. (See Table 1 - Addition and Subtraction Situations)	**2.OA.A.1** Add and subtract within 100 to solve one- and two-step contextual problems, with unknowns in all positions, involving situations of *add to*, *take from*, *put together/take apart*, and *compare*. Use objects, drawings, and equations with a symbol for the unknown number to represent the problem.
B. Add and subtract within 30.	**2.OA.B.2** Fluently add and subtract within 30 using mental strategies. By the end of 2nd grade, know from memory all sums of two one-digit numbers and related subtraction facts.
C. Work with equal groups of objects to gain foundations for multiplication.	**2.OA.C.3** Determine whether a group of objects (up to 20) has an odd or even number of members by pairing objects or counting them by 2s. Write an equation to express an even number as a sum of two equal addends. **2.OA.C.4** Use repeated addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Understand place value.	**2.NBT.A.1** Know that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 can be represented in multiple ways as 7 hundreds, 0 tens, and 6 ones; 706 ones; or 70 tens and 6 ones). **2.NBT.A.2** Count within 1000. Skip-count within 1000 by 5s, 10s, and 100s, starting from any number in its skip counting sequence. **2.NBT.A.3** Read and write numbers to 1000 using standard form, word form, and expanded form. **2.NBT.A.4** Compare two three-digit numbers based on the meanings of the digits in each place and use the symbols >, =, and < to show the relationship.
B. Use place value understanding and properties of operations to add and subtract. (See Table 3 - Properties of Operations)	**2.NBT.B.5** Fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. **2.NBT.B.6** Add up to four two-digit numbers using properties of operations and strategies based on place value.

Cluster Headings	Content Standards
B. Use place value understanding and properties of operations to add and subtract. (See Table 3 - Properties of Operations)	**2.NBT.B.7** Add and subtract within 1000 using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to explain the reasoning used. **2.NBT.B.8** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100– 900. **2.NBT.B.9** Explain why addition and subtraction strategies work using properties of operations and place value. (Explanations may include words, drawing, or objects.)

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Measure and estimate lengths in standard units.	**2.MD.A.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.MD.A.2** Measure the length of an object using two different units of measure and describe how the two measurements relate to the size of the unit chosen. **2.MD.A.3** Estimate lengths using units of inches, feet, yards, centimeters, and meters. **2.MD.A.4** Measure to determine how much longer one object is than another and express the difference in terms of a standard unit of length.
B. Relate addition and subtraction to length.	**2.MD.B.5** Add and subtract within 100 to solve contextual problems involving lengths that are given in the same units by using drawings and equations with a symbol for the unknown to represent the problem. **2.MD.B.6** Represent whole numbers as lengths from 0 on a number line and know that the points corresponding to the numbers on the number line are equally spaced. Use a number line to represent whole number sums and differences of lengths within 100.
C. Work with time and money.	**2.MD.C.7** Tell and write time in quarter hours and to the nearest five minutes (in a.m. and p.m.) using analog and digital clocks. **2.MD.C.8** Solve contextual problems involving dollar bills, quarters, dimes, nickels, and pennies using ¢ and $ symbols appropriately.

Cluster Headings	Content Standards
D. Represent and interpret data.	**2.MD.D.9** Generate measurement data by measuring lengths of several objects to the nearest whole unit. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **2.MD.D.10** Draw a pictograph and a bar graph (with intervals of one) to represent a data set with up to four categories. Solve addition and subtraction problems related to the data in a graph.

Geometry (G)

Cluster Headings	Content Standards
A. Reason about shapes and their attributes.	**2.G.A.1** Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Draw two-dimensional shapes having specified attributes (as determined directly or visually, not by measuring), such as a given number of angles or a given number of sides of equal length. **2.G.A.2** Partition a rectangle into rows and columns of same-sized squares and find the total number of squares. **2.G.A.3** Partition circles and rectangles into two, three, and four equal shares, describe the shares using the words *halves, thirds, fourths, half of, a third of, and a fourth of*, and describe the whole as *two halves, three thirds, four fourths*. Recognize that equal shares of identical wholes need not have the same shape.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[2]
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1st)**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** **(2nd)**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ **(1st)**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ **One-Step Problem** **(2nd)**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1st)**

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.
2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Mathematics | Grade 3

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 3rd grade.

Operations and Algebraic Thinking
Students build on their understanding of addition and subtraction to develop an understanding of the meanings of multiplication and division of whole numbers. Students use increasingly sophisticated strategies based on properties of operations to fluently solve multiplication and division problems within 100 (See Table 3 - Properties of Operations). Students interpret multiplication as finding an unknown product in situations involving equal-sized groups, arrays, area and measurement models, and division as finding an unknown factor in situations involving the unknown number of groups or the unknown group size. Students use these interpretations to represent and solve contextual problems with unknowns in all positions. By the end of 3rd grade, students should know from memory all products of single-digit numbers and the related division facts.

Students use all four operations to solve two-step word problems and use place value, mental computation, and estimation strategies to assess the reasonableness of solutions. They build number sense by investigating numerical representations, such as addition or multiplication tables for the purpose of identifying arithmetic patterns. Students should solve a variety of problem types in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations).

Number and Operations in Base Ten
Students begin to develop an understanding of rounding whole numbers to the nearest ten or hundred. Students fluently add and subtract within 1000 using strategies and algorithms. Students multiply one-digit whole numbers by multiples of 10.

Number and Operations in Fractions
This domain builds on the previous skill of partitioning shapes in geometry. This is the first time students are introduced to unit fractions. Students understand that fractions are composed of unit fractions and they use visual fraction models to represent parts of a whole. Students build on their understanding of number lines to represent fractions as locations and lengths on a number line. Students use fractions to represent numbers equal to, less than, and greater than 1 and are able to generate simple equivalent fractions by using drawings and/or reasoning about fractions. Students understand that the size of a fractional part is relative to the size of the whole.

Measurement and Data
In 2nd grade, students tell time in five minute increments, measure lengths, and create bar graphs, pictographs, and line plots with whole number units. In 3rd grade, students tell and write time to the nearest minute and solve contextual problems involving addition and subtraction. They use appropriate tools to measure and estimate liquid volume and mass. Students draw scaled pictographs and bar graphs and answer two-step questions about these graphs. Students generate measurement data and represent the data on line plots marked with whole number, half, or quarter units. Students recognize area as an attribute of two-dimensional shapes and measure the area of a shape using the standard unit (a square) by finding the total number of same-sized units required to cover the shape without gaps or overlaps. Students connect area to multiplication and use multiplication to justify the area of a rectangle by decomposing rectangles into rectangular arrays of squares.

Geometry
Students understand that shapes in given categories have shared attributes and they identify polygons. Students continue their understanding of shapes and fractions by partitioning shapes into parts with equal areas and identify the parts with unit fractions.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Represent and solve problems involving multiplication and division.	**3.OA.A.1** Interpret the factors and products in whole number multiplication equations (e.g., 4 x 7 is 4 groups of 7 objects with a total of 28 objects or 4 strings measuring 7 inches each with a total of 28 inches.) **3.OA.A.2** Interpret the dividend, divisor, and quotient in whole number division equations (e.g., 28 ÷ 7 can be interpreted as 28 objects divided into 7 equal groups with 4 objects in each group or 28 objects divided so there are 7 objects in each of the 4 equal groups). **3.OA.A.3** Multiply and divide within 100 to solve contextual problems, with unknowns in all positions, in situations involving equal groups, arrays, and measurement quantities using strategies based on place value, the properties of operations, and the relationship between multiplication and division (e.g., contexts including computations such as 3 x ? = 24, 6 x 16 = ?, ? ÷ 8 = 3, or 96 ÷ 6 = ?) (See Table 2 - Multiplication and Division Situations). **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers within 100. *For example, determine the unknown number that makes the equation true in each of the equations: 8 x ? = 48, 5 = ? ÷ 3, 6 x 6 =?*
B. Understand properties of multiplication and the relationship between multiplication and division. (See Table 3 - Properties of Operations)	**3.OA.B.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (Commutative property of multiplication). 3 x 5 x 2 can be solved by (3 x 5) x 2 or 3 x (5 x 2) (Associative property of multiplication). One way to find 8 x 7 is by using 8 x (5 + 2) = (8 x 5) + (8 x 2). By knowing that 8 x 5 = 40 and 8 x 2 = 16, then 8 x 7 = 40 + 16 = 56 (Distributive property of multiplication over addition).* **3.OA.B.6** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
C. Multiply and divide within 100.	**3.OA.C.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of 3^{rd} grade, know from memory all products of two one-digit numbers and related division facts.

Cluster Headings	Content Standards
D. Solve problems involving the four operations and identify and explain patterns in arithmetic.	**3.OA.D.8** Solve two-step contextual problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations). **3.OA.D.9** Identify arithmetic patterns (including patterns in the addition and multiplication tables) and explain them using properties of operations. *For example, analyze patterns in the multiplication table and observe that 4 times a number is always even (because 4 x 6 = (2 x 2) x 6 = 2 x (2 x 6), which uses the associative property of multiplication)* (See Table 3 - Properties of Operations).

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Use place value understanding and properties of operations to perform multi-digit arithmetic.	**3.NBT.A.1** Round whole numbers to the nearest 10 or 100 using understanding of place value. **3.NBT.A.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.NBT.A.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 *(e.g., 9 x 80, 5 x 60)* using strategies based on place value and properties of operations.

Number and Operations - Fractions (NF)

Limit denominators of fractions to 2, 3, 4, 6, and 8.

Cluster Headings	Content Standards
A. Develop understanding of fractions as numbers.	**3.NF.A.1** Understand a fraction, $\frac{1}{b}$, as the quantity formed by 1 part when a whole is partitioned into *b* equal parts (unit fraction); understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$. *For example, $\frac{3}{4}$ represents a quantity formed by 3 parts of size $\frac{1}{4}$.*

Cluster Headings	Content Standards
A. Develop understanding of fractions as numbers.	**3.NF.A.2** Understand a fraction as a number on the number line. Represent fractions on a number line. **a.** Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint locates the number $\frac{1}{b}$ on the number line. *For example, on a number line from 0 to 1, students can partition it into 4 equal parts and recognize that each part represents a length of $\frac{1}{4}$ and the first part has an endpoint at $\frac{1}{4}$ on the number line.* **b.** Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off *a* lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. *For example, $\frac{5}{3}$ is the distance from 0 when there are 5 iterations of $\frac{1}{3}$.*
	3.NF.A.3 Explain equivalence of fractions and compare fractions by reasoning about their size. **a.** Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. **b.** Recognize and generate simple equivalent fractions (e.g., $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$) and explain why the fractions are equivalent using a visual fraction model. **c.** Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *For example, express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point on a number line diagram.* **d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	**3.MD.A.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve contextual problems involving addition and subtraction of time intervals in minutes. *For example, students may use a number line to determine the difference between the start time and the end time of lunch.* **3.MD.A.2** Measure the mass of objects and liquid volume using standard units of grams (g), kilograms (kg), milliliters (ml), and liters *(l)*. Estimate the mass of objects and liquid volume using benchmarks. *For example, a large paper clip is about one gram, so a box of about 100 large clips is about 100 grams.*

Cluster Headings	Content Standards
B. Represent and interpret data.	**3.MD.B.3** Draw a scaled pictograph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs. **3.MD.B.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units: whole numbers, halves, or quarters.
C. Geometric measurement: understand and apply concepts of area and relate area to multiplication and to addition.	**3.MD.C.5** Recognize that plane figures have an area and understand concepts of area measurement. **a.** Understand that a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area. **b.** Understand that a plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.C.6** Measure areas by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units). **3.MD.C.7** Relate area of rectangles to the operations of multiplication and addition. **a.** Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. **b.** Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. **c.** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a x b* and *a x c.* Use area models to represent the distributive property in mathematical reasoning. *For example, in a rectangle with dimensions 4 by 6, students can decompose the rectangle into 4 x 3 and 4 x 3 to find the total area of 4 x 6.* (See Table 3 - Properties of Operations) **d.** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.
D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	**3.MD.D.8** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry (G)

Cluster Headings	Content Standards
A. Reason about shapes and their attributes.	**3.G.A.1** Understand that shapes in different categories may share attributes and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories. **3.G.A.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area and describe the area of each part as 1/4 of the area of the shape.* **3.G.A.3** Determine if a figure is a polygon.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1st)**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** **(2nd)**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ **(1st)**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ **One-Step Problem** **(2nd)**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1st)**

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.
2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] **Area**[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Mathematics | Grade 4

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 4th grade.

Operations and Algebraic Thinking
Students build on their knowledge of multiplication and begin to interpret and represent multiplication as a comparison. They multiply and divide to solve contextual problems involving multiplicative situations, distinguishing their solutions from additive comparison situations. Students solve multi-step whole number contextual problems using the four operations representing the unknown as a variable within equations (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations). They apply appropriate methods to estimate and check for reasonableness. This is the first time students find and interpret remainders in context. Students find factors and multiples, and they identify prime and composite numbers. Students generate number or shape patterns following a given rule.

Number and Operations in Base Ten
Students generalize place value understanding to read and write numbers to 1,000,000, using standard form, word form, and expanded form. They compare the relative size of the numbers and round numbers to the nearest hundred thousand, which builds on 3rd grade rounding concepts. By the end of 4th grade, students should fluently add and subtract multi-digit whole numbers to 1,000,000. Students use strategies based on place value and the properties of operations to multiply a whole number up to four-digits by a one-digit number, and multiply two two-digit numbers. They use these strategies and the relationship between multiplication and division to find whole number quotients and remainders up to four-digit dividends and one-digit divisors (See Table 3 - Properties of Operations).

Number and Operations-Fractions
Students continue to develop an understanding of fraction equivalence by reasoning about the size of the fractions, using a benchmark fraction to compare the fractions, or finding a common denominator. Students extend previous understanding of unit fractions to compose and decompose fractions in different ways. They use the meaning of fractions and the meaning of multiplication as repeated addition to multiply a whole number by a fraction. Students solve contextual problems involving addition and subtraction of fractions with like denominators and multiplication of a whole number by a fraction (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions). Students learn decimal notation for the first time to represent fractions with denominators of 10 and 100. They express these fractions and their equivalents as decimals and are able to read, write, compare, and locate these decimals on a number line.

Measurement and Data
Students know the relative sizes of measurement units within one system of units. They use the four operations to solve contextual problems involving measurement. Students build on their previous understanding of area and perimeter to generate and apply formulas for finding the area and perimeter of rectangles. Students also build on their understanding of line plots and solve problems involving fractions using operations appropriate for the grade. For the first time, students learn concepts of angle measurement.

Geometry
Students extend their previous understanding to analyze and classify shapes based on line and angle types. Students also use knowledge of line and angle types to identify right triangles. Students recognize and draw lines of symmetry for the first time.

Standards for Mathematical Practice

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Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Use the four operations with whole numbers to solve problems. (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations)	**4.OA.A.1** Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.A.2** Multiply or divide to solve contextual problems involving multiplicative comparison, and distinguish multiplicative comparison from additive comparison. *For example, school A has 300 students and school B has 600 students: to say that school B has two times as many students is an example of multiplicative comparison; to say that school B has 300 more students is an example of additive comparison.* **4.OA.A.3** Solve multi-step contextual problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
B. Gain familiarity with factors and multiples.	**4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
C. Generate and analyze patterns.	**4.OA.C.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Generalize place value understanding for multi-digit whole numbers.	**4.NBT.A.1** Recognize that in a multi-digit whole number (less than or equal to 1,000,000), a digit in one place represents 10 times as much as it represents in the place to its right. *For example, recognize that 7 in 700 is 10 times bigger than the 7 in 70 because 700 ÷ 70 = 10 and 70 x 10 = 700.* **4.NBT.A.2** Read and write multi-digit whole numbers (less than or equal to 1,000,000) using standard form, word form, and expanded form (e.g. the expanded form of 4256 is written as 4 x 1000 + 2 x 100 + 5 x 10 + 6 x 1). Compare two multi-digit numbers based on meanings of the digits in each place and use the symbols >, =, and < to show the relationship. **4.NBT.A.3** Round multi-digit whole numbers to any place (up to and including the hundred-thousand place) using understanding of place value.

Cluster Headings	Content Standards
B. Use place value understanding and properties of operations to perform multi-digit arithmetic. (See Table 3 - Properties of Operations)	**4.NBT.B.4** Fluently add and subtract within 1,000,000 using appropriate strategies and algorithms. **4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations - Fractions (NF)

Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Cluster Headings	Content Standards
A. Extend understanding of fraction equivalence and comparison.	**4.NF.A.1** Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{a \times n}{b \times n}$ or $\frac{a \div n}{b \div n}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. *For example,* $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$. **4.NF.A.2** Compare two fractions with different numerators and different denominators by creating common denominators or common numerators or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

Cluster Headings	Content Standards
B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations for whole number situations that can be applied for fractions.)	**4.NF.B.3** Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$. *For example*, $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$), recording each decomposition by an equation. Justify decompositions by using a visual fraction model. **c.** Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction. **d.** Solve contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators
	4.NF.B.4 Apply and extend previous understandings of multiplication as repeated addition to multiply a whole number by a fraction. **a.** Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as the product* $5 \times \frac{1}{4}$, *recording the conclusion by the equation* $\frac{5}{4} = 5 \text{ x } \frac{1}{4}$. **b.** Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and use this understanding to multiply a whole number by a fraction. *For example, use a visual fraction model to express* $3 \times \frac{2}{5}$ *as* $6 \times \frac{1}{5}$, *recognizing this product as* $\frac{6}{5}$. *(In general,* $n \text{ x } \frac{a}{b} = \frac{(n \text{ x } a)}{b} = (n \text{ x } a) \text{ x } \frac{1}{b}$.*)* **c.** Solve contextual problems involving multiplication of a whole number by a fraction (e.g., by using visual fraction models and equations to represent the problem). *For example, if each person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be 4 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
C. Understand decimal notation for fractions and compare decimal fractions.	**4.NF.C.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express,* $\frac{3}{10}$ *as* $\frac{30}{100}$ *and add* $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$. **4.NF.C.6** Read and write decimal notation for fractions with denominators 10 or 100. Locate these decimals on a number line. **4.NF.C.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Estimate and solve problems involving measurement.	**4.MD.A.1** Measure and estimate to determine relative sizes of measurement units within a single system of measurement involving length, liquid volume, and mass/weight of objects using customary and metric units. **4.MD.A.2** Solve one- or two-step real-world problems involving whole number measurements with all four operations within a single system of measurement including problems involving simple fractions. **4.MD.A.3** Know and apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
B. Represent and interpret data.	**4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
C. Geometric measurement: understand concepts of angle and measure angles.	**4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. **a.** Understand that an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. **b.** Understand that an angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees and represents a fractional portion of the circle. **4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems *(e.g., by using an equation with a symbol for the unknown angle measure)*.

Geometry (G)

Cluster Headings	Content Standards
A. Draw and identify lines and angles and classify shapes by properties of their lines and angles.	**4.G.A.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse, straight, reflex), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.A.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles. **4.G.A.3** Recognize and draw lines of symmetry for two-dimensional figures.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1st)**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** **(2nd)**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ **(1st)**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ **One-Step Problem** **(2nd)**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1st)**

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.
2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2 Common multiplication and division situations[1]

	Unknown Product **3 × 6 = ?**	**Group Size Unknown (“How many in each group?” Division)** **3 × ? = 18, and 18 ÷ 3 = ?**	**Number of Groups Unknown (“How many groups?” Division)** **? × 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] **Area**[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Mathematics | Grade 5

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 5^{th} grade.

Operations and Algebraic Thinking
Students build on their understanding of patterns to generate two numerical patterns using given rules and identify relationships between the patterns. For the first time, students form ordered pairs and graph them on a coordinate plane. In addition, students write and evaluate numerical expressions using parentheses and/or brackets.

Number and Operations in Base Ten
Students generalize their understanding of place value to include decimals by reading, writing, comparing, and rounding numbers. They recognize that in a multi-digit number, the value of each digit has a relationship to the value of the same digit in another position. Students explain patterns in products when multiplying a number by a power of 10. Whole-number exponents are used to denote powers of 10 for the first time. By the end of 5^{th} grade, students should fluently multiply multi-digit whole numbers (up to 4 digits by 3 digits).

Students build on their understanding of why division procedures work based on place value and the properties of operations to find whole number quotients and remainders (See Table 3 - Properties of Operations). They apply their understanding of models for decimals, decimal notation, and properties of operations to add, subtract, multiply, and divide decimals to hundredths. (Limit division problems so that either the dividend or the divisor is a whole number.) They develop fluency in these computations and make reasonable estimates of their results. Students finalize their understanding of multi-digit addition, subtraction, multiplication, and division with whole numbers.

Number and Operations in Fractions
Students apply their understanding of equivalent fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions and make reasonable estimates of them. For the first time, students develop an understanding of fractions as division problems. They use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Limit to dividing unit fractions by whole numbers or whole numbers by unit fractions.) Students reason about the size of products compared to the size of the factors. Students should solve a variety of problem types in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions).

Measurement and Data
Students build on their understanding of area and recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-sized units of volume required to fill the space without gaps or overlaps. Students decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of cubes. Students build on their understanding of measurements to convert from larger units to smaller units within a single system of measurement and solve multistep problems involving these conversions. Students solve problems with data from line plots involving fractions using operations appropriate for the grade.

Geometry
Students plot points on the coordinate plane to solve real-world and mathematical problems. Students classify two-dimensional figures into categories based on their properties.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Write and interpret numerical expressions.	**5.OA.A.1** Use parentheses and/or brackets in numerical expressions and evaluate expressions having these symbols using the conventional order (Order of Operations). **5.OA.A.2** Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.*
B. Analyze patterns and relationships.	**5.OA.B.3** Generate two numerical patterns using two given rules. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences.* **a.** Identify relationships between corresponding terms in two numerical patterns. *For example, observe that the terms in one sequence are twice the corresponding terms in the other sequence.* **b.** Form ordered pairs consisting of corresponding terms from two numerical patterns and graph the ordered pairs on a coordinate plane.

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Understand the place value system.	**5.NBT.A.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.A.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.A.3** Read and write decimals to thousandths using standard form, word form, and expanded form (e.g., the expanded form of 347.392 is written as 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000)). Compare two decimals to thousandths based on meanings of the digits in each place and use the symbols >, =, and < to show the relationship. **5.NBT.A.4** Round decimals to the nearest hundredth, tenth, or whole number using understanding of place value.

Cluster Headings	Content Standards
B. Perform operations with multi-digit whole numbers and with decimals to hundredths. (See Table 3 - Properties of Operations)	**5.NBT.B.5** Fluently multiply multi-digit whole numbers (up to three-digit by four-digit factors) using appropriate strategies and algorithms. **5.NBT.B.6** Find whole-number quotients and remainders of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; assess the reasonableness of answers using estimation strategies. (Limit division problems so that either the dividend or the divisor is a whole number.)

Number and Operations - Fractions (NF)

Cluster Headings	Content Standards
A. Use equivalent fractions as a strategy to add and subtract fractions. (See Table 1 - Addition and Subtraction Situations for whole number situations that can be applied to fractions)	**5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. *(In general* $\frac{a}{b}+\frac{c}{d}=\frac{(ad+bc)}{bd}$*.)* **5.NF.A.2** Solve contextual problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$*, by observing that* $\frac{3}{7}<\frac{1}{2}$.
B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (See Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions)	**5.NF.B.3** Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b}=a\div b$). *For example,* $\frac{3}{4}=3\div 4$ *so when 3 wholes are shared equally among 4 people, each person has a share of size* $\frac{3}{4}$. Solve contextual problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem. *For example, if 8 people want to share 49 sheets of construction paper equally, how many sheets will each person receive? Between what two whole numbers does your answer lie?*

Cluster Headings	Content Standards
B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (See Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions)	**5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number or a fraction by a fraction. **a.** Interpret the product $\frac{a}{b}$ x *q* as *a* x (*q* ÷ *b*) (partition the quantity *q* into *b* equal parts and then multiply by *a*). Interpret the product $\frac{a}{b}$ x *q* as (*a* x *q*) ÷ *b* (multiply *a* times the quantity *q* and then partition the product into *b* equal parts). *For example, use a visual fraction model or write a story context to show that* $\frac{2}{3}$ *x 6 can be interpreted as 2 x (6 ÷ 3) or (2 x 6) ÷ 3. Do the same with* $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{\mathrm{a}}{\mathrm{b}} \times \frac{\mathrm{c}}{\mathrm{d}} = \frac{\mathrm{ac}}{\mathrm{bd}}$.) **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.
	5.NF.B.5 Interpret multiplication as scaling (resizing). **a.** Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, know if the product will be greater than, less than, or equal to the factors*. **b.** Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product less than the given number; and relate the principle of fraction equivalence $\frac{a}{b} = \frac{(a \ x \ n)}{(b \ x \ n)}$ to the effect of multiplying $\frac{a}{b}$ by 1.
	5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.
	5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. **a.** Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example*, *use visual models and the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.* **b.** Interpret division of a whole number by a unit fraction and compute such quotients. *For example*, *use visual models and the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.* **c.** Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3 cup servings are in 2 cups of raisins?*

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Convert like measurement units within a given measurement system from a larger unit to a smaller unit.	**5.MD.A.1** Convert customary and metric measurement units within a single system by expressing measurements of a larger unit in terms of a smaller unit. Use these conversions to solve multi-step real-world problems involving distances, intervals of time, liquid volumes, masses of objects, and money (including problems involving simple fractions or decimals). *For example, 3.6 liters and 4.1 liters can be combined as 7.7 liters or 7700 milliliters*
B. Represent and interpret data.	**5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
C. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	**5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a.** Understand that a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume. **b.** Understand that a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
	5.MD.C.4 Measure volume by counting unit cubes, using cubic centimeters, cubic inches, cubic feet, and improvised units.
	5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume of right rectangular prisms. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent whole-number products of three factors as volumes *(e.g., to represent the associative property of multiplication).* **b.** Know and apply the formulas $V = l \times w \times h$ and $V = B \times h$ (where B represents the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Geometry (G)

Cluster Headings	Content Standards
A. Graph points on the coordinate plane to solve real-world and mathematical problems.	**5.G.A.1** Graph ordered pairs and label points using the first quadrant of the coordinate plane. Understand in the ordered pair that the first number indicates the horizontal distance traveled along the x-axis from the origin and the second number indicates the vertical distance traveled along the y-axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). **5.G.A.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.
B. Classify two-dimensional figures into categories based on their properties.	**5.G.B.3** Classify two-dimensional figures in a hierarchy based on properties. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1st)**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** **(2nd)**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ **(1st)**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ **One-Step Problem** **(2nd)**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1st)**

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.
2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Mathematics | Grade 6

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 6^{th} grade.

Ratios and Proportional Relationships
6^{th} grade begins the formal study of ratios and proportions. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates. Proportional relationships are added and studied in the 7^{th} grade.

The Number System
Students use fractions, multiplication, and division along with an understanding of the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students also extend their previous understandings of numbers and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

Expressions and Equations
Students begin to use properties of arithmetic operations systematically to work with numerical expressions that contain whole-number exponents. Students come to understand more fully the use of variables and variable expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students explore how algebraic expressions can represent written situations and generalize relationships from specific cases.

Geometry
Students build on their work with area from earlier grades by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can more easily determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in the 7^{th} grade by drawing polygons in the coordinate plane.

Statistics and Probability
6^{th} grade students begin to formally develop their ability to think statistically. They understand that a set of data (collected to answer a question) will have a distribution, which can be described by its center, spread, and shape. Students calculate the median, mean, and mode and relate these to the overall shape of the distribution. They recognize that the median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. They understand that the mode refers to the most frequently occurring number found in a set of numbers and is found by collecting and organizing the data in order to count the frequency of each result. Students display, summarize and describe numerical data sets, considering the context in which the data were collected. Students use number lines, dot plots, box plots, and pie charts to display numerical data.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Ratios and Proportional Relationships (RP)

Cluster Headings	Content Standards
A. Understand ratio concepts and use ratio reasoning to solve problems.	**6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, the ratio of wings to beaks in a bird house at the zoo was 2:1, because for every 2 wings there was 1 beak. Another example could be for every vote candidate A received, candidate C received nearly three votes*
	6.RP.A.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*. Use rate language in the context of a ratio relationship. *For example, this recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar. Also, we paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.* (Expectations for unit rates in 6th grade are limited to non-complex fractions).
	6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). **a.** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if a runner ran 10 miles in 90 minutes, running at that speed, how long will it take him to run 6 miles? How fast is he running in miles per hour?* **c.** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert customary and metric measurement units (within the same system); manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System (NS)

Cluster Headings	Content Standards
A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	**6.NS.A.1** Interpret and compute quotients of fractions, and solve contextual problems involving division of fractions by fractions (e.g., using visual fraction models and equations to represent the problem is suggested). *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 times 8/9 is 2/3 ((a/b) ÷ (c/d) = ad/bc.)* *Further example: How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Cluster Headings	Content Standards
B. Compute fluently with multi-digit numbers and find common factors and multiples.	**6.NS.B.2** Fluently divide multi-digit numbers using a standard algorithm. **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
C. Apply and extend previous understandings of numbers to the system of rational numbers.	**6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself. *For example, – (–3) = 3, and that 0 is its own opposite.* **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7 Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.* **c.** Understand the absolute value of a rational number as its distance from 0 on the number line and distinguish comparisons of absolute value from statements about order in a real-world context. *For example, an account balance of -24 dollars represents a greater debt than an account balance -14 dollars because -24 is located to the left of -14 on the number line*

Cluster Headings	Content Standards
C. Apply and extend previous understandings of numbers to the system of rational numbers.	**6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations (EE)

Cluster Headings	Content Standards
A. Apply and extend previous understandings of arithmetic to algebraic expressions.	**6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents.
	6.EE.A.2 Write, read, and evaluate expressions in which variables stand for numbers. **a.** Write expressions that record operations with numbers and with variables. *For example, express the calculation "Subtract y from 5" as 5 - y.* **b.** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* **c.** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
	6.EE.A.3 Apply the properties of operations (including, but not limited to, commutative, associative, and distributive properties) to generate equivalent expressions. The distributive property is prominent here. *For example, apply the distributive property to the expression* 3 (2 + x) *to produce the equivalent expression* 6 + 3x*; apply the distributive property to the expression* 24x + 18y *to produce the equivalent expression* 6 (4x + 3y)*; apply properties of operations to* y + y + y *to produce the equivalent expression* 3y.
	6.EE.A.4 Identify when expressions are equivalent (i.e., when the expressions name the same number regardless of which value is substituted into them). *For example, the expression* 5b + 3b *is equivalent to* (5 +3) b*, which is equivalent to* 8b.
B. Reason about and solve one-variable equations and inequalities.	**6.EE.B.5** Understand solving an equation or inequality is carried out by determining if any of the values from a given set make the equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Cluster Headings	Content Standards
B. Reason about and solve one-variable equations and inequalities.	**6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving one-step equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.B.8** Interpret and write an inequality of the form $x > c$ or $x < c$ which represents a condition or constraint in a real-world or mathematical problem. Recognize that inequalities have infinitely many solutions; represent solutions of inequalities on number line diagrams.
C. Represent and analyze quantitative relationships between dependent and independent variables.	**6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another. *For example, Susan is putting money in her savings account by depositing a set amount each week (50). Represent her savings account balance with respect to the number of weekly deposits (*s = 50w*, illustrating the relationship between balance amount* s *and number of weeks* w*).* **a.** Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. **b.** Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Geometry (G)

Cluster Headings	Content Standards
A. Solve real-world and mathematical problems involving area, surface area, and volume.	**6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; know and apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Know and apply the formulas $V = \ell wh$ and $V = Bh$ *where B is the area of the base* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Cluster Headings	Content Standards
A. Solve real-world and mathematical problems involving area, surface area, and volume.	**6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side that joins two vertices (vertical or horizontal segments only). Know and apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability (SP)

Cluster Headings	Content Standards
A. Develop understanding of statistical variability.	**6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean, median, mode), spread (range), and overall shape. **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
B. Summarize and describe distributions.	**6.SP.B.4** Display a single set of numerical data using dot plots (line plots), box plots, pie charts and stem plots.
	6.SP.B.5 Summarize numerical data sets in relation to their context. **a.** Report the number of observations. **b.** Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Give quantitative measures of center (median and/or mean) and variability (range) as well as describing any overall pattern with reference to the context in which the data were gathered. **d.** Relate the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Mathematics | Grade 7

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 7th grade.

Ratios and Proportional Relationships
Students extend their understanding of ratios from 6th grade and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

The Number System
Students develop a unified understanding of numbers, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percent as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. These properties are further explored with respect to negative numbers. This exploration is carried out in problems from everyday contexts so that the student can gain a deeper understanding and appreciation for the mathematical concepts being studied.

Expressions and Equations
By applying the properties of operations as strategies, students explore working with expressions, equations, and inequalities. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve multi-step real-world problems. They use variables to represent quantities and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Geometry
Students continue their work with area from 6th grade, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability
Students continue their work from 6th grade in order to build a strong foundation for statistics and probability needed for high school. Students understand that statistics can be used to gain information about a population through sampling. They work with drawing inferences about a population based on a sample and use measures of center and of variability to draw informal comparative inferences about two populations. Students investigate the chance processes and develop, use, and evaluate probability models. Students summarize numerical data sets with respect to their context using quantitative measures and describe any overall patterns or deviations from the overall pattern.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Ratios and Proportional Relationships (RP)

Cluster Headings	Content Standards
A. Analyze proportional relationships and use them to solve real-world and mathematical problems.	**7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*
	7.RP.A.2 Recognize and represent proportional relationships between quantities. **a**. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). **b**. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c**. Represent proportional relationships by equations. *For example, if total cost* t *is proportional to the number* n *of items purchased at a constant price* p*, the relationship between the total cost and the number of items can be expressed as* t = pn. **d.** Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.
	7.RP.A.3 Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System (NS)

Cluster Headings	Content Standards
A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	**7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a.** Describe situations in which opposite quantities combine to make 0. **b.** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **c.** Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **d.** Apply properties of operations as strategies to add and subtract rational numbers.

Cluster Headings	Content Standards
A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	**7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. **c.** Apply properties of operations as strategies to multiply and divide rational numbers. **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
	7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Expressions and Equations (EE)

Cluster Headings	Content Standards
A. Use properties of operations to generate equivalent expressions.	**7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.A.2** Understand that rewriting an expression in different forms in a contextual problem can provide multiple ways of interpreting the problem and how the quantities in it are related. *For example, shoes are on sale at a 25% discount. How is the discounted price* P *related to the original cost* C *of the shoes?* C - .25C = P. *In other words,* P *is 75% of the original cost for* C - .25C *can be written as* .75C.
B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations and inequalities.	**7.EE.B.3** Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers presented in any form (whole numbers, fractions, and decimals). **a.** Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate. **b.** Assess the reasonableness of answers using mental computation and estimation strategies.

Cluster Headings	Content Standards
B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations and inequalities.	**7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a.** Solve contextual problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **b.** Solve contextual problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality on a number line and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* (Note that inequalities using >, <, ≤, ≥ are included in this standard).

Geometry (G)

Cluster Headings	Content Standards
A. Draw, construct, and describe geometrical figures and describe the relationships between them.	**7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **7.G.A.2** Draw geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	**7.G.B.3** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **7.G.B.4** Know and use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.B.5** Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability (SP)

Cluster Headings	Content Standards
A. Use random sampling to draw inferences about a population.	**7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
B. Draw informal comparative inferences about two populations.	**7.SP.B.3** **I**nformally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team; on a dot plot or box plot, the separation between the two distributions of heights is noticeable.* **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a 7th grade science book are generally longer than the words in a chapter of a 4th grade science book.*
C. Investigate chance processes and develop, use, and evaluate probability models.	**7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.C.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

Cluster Headings	Content Standards
C. Investigate chance processes and develop, use, and evaluate probability models.	**7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **a.** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
D. Summarize and describe numerical data sets.	**7.SP.D.8** Summarize numerical data sets in relation to their context. **a.** Give quantitative measures of center (median and/or mean) and variability (range and/or interquartile range), as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **b.** Know and relate the choice of measures of center (median and/or mean)and variability (range and/or interquartile range) to the shape of the data distribution and the context in which the data were gathered.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		Supporting Content

Mathematics | Grade 8

The descriptions below provide an overview of the concepts and skills that students explore throughout the 8th grade.

The Number System

This is the culminating area for the number system from 6th and 7th grade. Students now know there are numbers that are not rational, called irrational numbers. Students approximate irrational numbers by rational numbers locating them on a number line and students estimate the value of irrational expressions.

Expressions and Equations

Students work with radicals and integer exponents. Students understand the connections between proportional relationships, lines, and linear equations. Students advance their knowledge developed in 7th grade about equations to analyze and solve linear equations and pairs of simultaneous linear equations. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

Functions

This begins the formal study of functions, a mathematical concept that for the student will continue throughout high school. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

Geometry

Students use ideas about distance and angles and how they behave under translations, rotations, reflections, and dilations, to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse and can explain why the Pythagorean Theorem holds. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Statistics and Probability

Students extend their knowledge from 7th grade by working with scatter plots for bivariate data and understand linear associations and the use of linear models to solve problems interpreting the slope and intercept. Students continue work with probability by finding probability of compound events and represent the data using organized lists, tables, and tree diagrams.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

The Number System (NS)

Cluster Headings	Content Standards
A. Know that there are numbers that are not rational, and approximate them by rational numbers.	**8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually or terminates, and convert a decimal expansion which repeats eventually or terminates into a rational number. **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers locating them approximately on a number line diagram. Estimate the value of irrational expressions such as π^2. *For example, by truncating the decimal expansion of* $\sqrt{2}$, *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations (EE)

Cluster Headings	Content Standards
A. Work with radicals and integer exponents.	**8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **8.EE.A.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9, *and determine that the world population is more than 20 times larger.* **8.EE.A.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
B. Understand the connections between proportional relationships, lines, and linear equations.	**8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; know and derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Cluster Headings	Content Standards
C. Analyze and solve linear equations and systems of two linear equations.	**8.EE.C.7** Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
	8.EE.C.8 Analyze and solve systems of two linear equations. **a.** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **b.** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.* **c.** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions (F)

Cluster Headings	Content Standards
A. Define, evaluate, and compare functions.	**8.F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in 8^{th} grade.) **8.F.A.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and another linear function represented by an algebraic expression, determine which function has the greater rate of change.* **8.F.A.3** Know and interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Cluster Headings	Content Standards
B. Use functions to model relationships between quantities.	**8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. **8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry (G)

Cluster Headings	Content Standards
A. Understand and describe the effects of transformations on two-dimensional figures and use informal arguments to establish facts about angles.	**8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations: **a.** Lines are taken to lines, and line segments to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines.
	8.G.A.2 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
	8.G.A.3 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
B. Understand and apply the Pythagorean Theorem.	**8.G.B.4** Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.5** Know and apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **8.G.B.6** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	**8.G.C.7** Know and understand the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

Statistics and Probability (SP)

Cluster Headings	Content Standards
A. Investigate patterns of association in bivariate data.	**8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line. **8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
B. Investigate chance processes and develop, use, and evaluate probability models	**8.SP.B.4** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

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	Major Content		**Supporting Content**

Algebra I | A1

Algebra I emphasizes linear and quadratic expressions, equations, and functions. This course also introduces students to polynomial and exponential functions with domains in the integers. Students explore the structures of and interpret functions and other mathematical models. Students build upon previous knowledge of equations and inequalities to reason, solve, and represent equations and inequalities numerically and graphically.

The major work of Algebra I is from the following domains and clusters:

- **Seeing Structure in Expressions**
  - Interpret the structure of expressions.
  - Write expressions in equivalent forms to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Perform arithmetic operations on polynomials.
- **Creating Equations**
  - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
  - Understand solving equations as a process of reasoning and explain the reasoning.
  - Solve equations and inequalities in one variable.
  - Represent and solve equations and inequalities graphically.
- **Interpreting Functions**
  - Understand the concept of a function and use function notation.
  - Interpret functions that arise in applications in terms of the context.
- **Interpreting Categorical and Quantitative Data**
  - Interpret linear models.

Supporting work is from the following domains and clusters:

- **Quantities**
  - Reason quantitatively and use units to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Understand the relationship between zeros and factors of polynomials.
- **Reasoning with Equations and Inequalities**
  - Solve systems of equations.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Building Functions**
  - Build a function that models a relationship between two quantities.
  - Build new functions from existing functions.
- **Linear, Quadratic, and Exponential Models**
  - Construct and compare linear, quadratic, and exponential models and solve problems.
  - Interpret expressions for functions in terms of the situation they model.
- **Interpreting Categorical and Quantitative Data**
  - Summarize, represent, and interpret data on a single count or measurement variable.
  - Summarize, represent, and interpret data on two categorical and quantitative variables.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

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Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

Quantities* (N.Q)

Cluster Headings	Content Standards	Scope & Clarifications
A. Reason quantitatively and use units to solve problems.	**A1.N.Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.N.Q.A.2 Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.	*Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.N.Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**A1.A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.	*For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**A1.A.SSE.A.2** Use the structure of an expression to identify ways to rewrite it.	*For example, recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 + 47)(53 – 47)$. See an opportunity to rewrite* *$a^2 + 9a + 14$ as $(a + 7)(a + 2)$.* *Tasks are limited to numerical expressions and polynomial expressions in one variable.*
B. Write expressions in equivalent forms to solve problems.	**A1.A.SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Factor a quadratic expression to reveal the zeros of the function it defines. **b.** Complete the square in a quadratic expression in the form $Ax^2 + Bx + C$ to reveal the maximum or minimum value of the function it defines. **c.** Use the properties of exponents to rewrite exponential expressions.	*For A1.A.SSE.B.3c:* *For example, the growth of bacteria can be modeled by either $f(t) = 3^{(t+2)}$ or $g(t) = 9(3^t)$ because the expression $3^{(t+2)}$ can be rewritten as $(3^t)(3^2) = 9(3^t)$.* *i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.* *ii) Tasks are limited to exponential expressions with integer exponents.*

Arithmetic with Polynomials and Rational Expressions (A.APR)

Cluster Headings	Content Standards	Scope & Clarifications
A. Perform arithmetic operations on polynomials.	**A1.A.APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Understand the relationship between zeros and factors of polynomials.	**A1.A.APR.B.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	*Graphing is limited to linear and quadratic polynomials.*

Creating Equations★ (A.CED)

Cluster Headings	Content Standards	Scope & Clarifications
A. Create equations that describe numbers or relationships.	**A1.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.	*Tasks are limited to linear, quadratic, or exponential equations with integer exponents.*
	A1.A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations with two variables on coordinate axes with labels and scales.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.A.CED.A.3 Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	*For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	*i) Tasks are limited to linear, quadratic, and absolute value equations with integer exponents.* *ii) Tasks have a real-world context.*

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand solving equations as a process of reasoning and explain the reasoning.	**A1.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	*Tasks are limited to linear, quadratic, and absolute value equations with integer exponents.*
B. Solve equations and inequalities in one variable.	**A1.A.REI.B.2** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Solve equations and inequalities in one variable.	**A1.A.REI.B.3** Solve quadratic equations and inequalities in one variable. **a.** Use the method of completing the square to rewrite any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions.	*For A1.A.REI.B.3b:* *Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions.* *Note: solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials. This is formally assessed in Algebra II.*
C. Solve systems of equations.	**A1.A.REI.C.4** Write and solve a system of linear equations in context.	*Solve systems both algebraically and graphically.* *Systems are limited to at most two equations in two variables.*
D. Represent and solve equations and inequalities graphically.	**A1.A.REI.D.5** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.A.REI.D.6 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the approximate solutions using technology. ★	*Include cases where f(x) and/or g(x) are linear, quadratic, absolute value, and exponential functions. For example,* $f(x) = 3x + 5$ *and* $g(x) = x^2 + 1$. *Exponential functions are limited to domains in the integers.*
	A1.A.REI.D.7 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand the concept of function and use function notation.	**A1.F.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Interpret functions that arise in applications in terms of the context.	**A1.F.IF.B.3** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. ⋆	*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* *i) Tasks have a real-world context.* *ii) Tasks are limited to linear functions, quadratic functions, absolute value functions, and exponential functions with domains in the integers.*
	A1.F.IF.B.4 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ⋆	*For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Interpret functions that arise in applications in terms of the context.	**A1.F.IF.B.5** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	*i) Tasks have a real-world context.* *ii) Tasks are limited to linear functions, quadratic functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.*
C. Analyze functions using different representations.	**A1.F.IF.C.6** Graph functions expressed symbolically and show key features of the graph, by hand and using technology. **a**. Graph linear and quadratic functions and show intercepts, maxima, and minima. **b**. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	*Tasks in A1.F.IF.C.6b are limited to piecewise, step and absolute value functions.*
	A1.F.IF.C.7 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a**. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.F.IF.C.8 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	*i) Tasks have a real-world context.* *ii) Tasks are limited to linear functions, quadratic functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.*

Building Functions (F.BF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Build a function that models a relationship between two quantities.	**A1.F.BF.A.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.	*i) Tasks have a real-world context.* *ii) Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.*
B. Build new functions from existing functions.	**A1.F.BF.B.2** Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	*i) Identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), and f(x+k) for specific values of k (both positive and negative) is limited to linear, quadratic, and absolute value functions.* *ii) f(kx) will not be included in Algebra 1. It is addressed in Algebra 2.* *iii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, absolute value, and exponential functions with domains in the integers.* *iv) Tasks do not involve recognizing even and odd functions.*

Linear, Quadratic, and Exponential Models★ (F.LE)

<table>
<tr><th>Cluster Headings</th><th>Content Standards</th><th>Scope & Clarifications</th></tr>
<tr><td rowspan="3">A. Construct and compare linear, quadratic, and exponential models and solve problems.</td><td>A1.F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
a. Recognize that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant factor per unit interval relative to another.</td><td>There are no assessment limits for this standard. The entire standard is assessed in this course.</td></tr>
<tr><td>A1.F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.</td><td>Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).</td></tr>
<tr><td>A1.F.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</td><td>There are no assessment limits for this standard. The entire standard is assessed in this course.</td></tr>
<tr><td>B. Interpret expressions for functions in terms of the situation they model.</td><td>A1.F.LE.B.4 Interpret the parameters in a linear or exponential function in terms of a context.</td><td>For example, the total cost of an electrician who charges 35 dollars for a house call and 50 dollars per hour would be expressed as the function $y = 50x + 35$. If the rate were raised to 65 dollars per hour, describe how the function would change.

i) Tasks have a real-world context.

ii) Exponential functions are limited to those with domains in the integers.</td></tr>
</table>

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards	Scope & Clarifications
A. Summarize, represent, and interpret data on a single count or measurement variable.	**A1.S.ID.A.1** Represent single or multiple data sets with dot plots, histograms, stem plots (stem and leaf), and box plots.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**A1.S.ID.B.4** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. **b.** Fit a linear function for a scatter plot that suggests a linear association.	*Emphasize linear models, quadratic models, and exponential models with domains in the integers.* *For A1.S.ID.B.4a:* *i) Tasks have a real-world context.* *ii) Exponential functions are limited to those with domains in the integers.*
C. Interpret linear models.	**A1.S.ID.C.5** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.S.ID.C.6 Use technology to compute and interpret the correlation coefficient of a linear fit.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A1.S.ID.C.7 Distinguish between correlation and causation.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major content of the course is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Geometry | G

Geometry emphasizes similarity, right triangle trigonometry, congruence, and modeling geometry concepts in real life situations. Students build upon previous knowledge of similarity, congruence, and triangles to prove theorems and reason mathematically. This course also introduces students to geometric constructions and circles. Students show a progression of mastery and understanding of the use and application of surface area and volume.

The major work of Geometry is from the following domains and clusters:

- **Congruence**
  - Understand congruence in terms of rigid motions.
  - Prove geometric theorems.
- **Similarity, Right Triangles, and Trigonometry**
  - Understand similarity in terms of similarity transformations.
  - Prove theorems involving similarity.
  - Define trigonometric ratios and solve problems involving triangles.
- **Expressing Geometric Properties with Equations**
  - Use coordinates to prove simple geometric theorems algebraically.
- **Modeling with Geometry**
  - Apply geometric concepts in modeling situations.

Supporting work is from the following domains and clusters:

- **Congruence**
  - Experiment with transformations in the plane.
  - Make geometric constructions.
- **Circles**
  - Understand and apply theorems about circles.
  - Find areas of sectors of circles.
- **Expressing Geometric Properties with Equations**
  - Translate between the geometric description and the equation for a circle.
- **Geometric Measurement and Dimension**
  - Explain volume and surface area formulas and use them to solve problems.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Geometry

Congruence (G.CO)

Cluster Headings	Content Standards	Scope & Clarifications
A. Experiment with transformations in the plane.	**G.CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, plane, distance along a line, and distance around a circular arc.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.CO.A.2 Represent transformations in the plane in multiple ways, including technology. Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry the shape onto itself.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.CO.A.5 Given a geometric figure and a rigid motion, draw the image of the figure in multiple ways, including technology. Specify a sequence of rigid motions that will carry a given figure onto another.	*Rigid motions include rotations, reflections, and translations.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Understand congruence in terms of rigid motions.	**G.CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to determine informally if they are congruent.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Understand congruence in terms of rigid motions.	**G.CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, AAS, and SSS) follow from the definition of congruence in terms of rigid motions.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
C. Prove geometric theorems.	**G.CO.C.9** Prove theorems about lines and angles.	*Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs.* *Theorems include but are not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
	G.CO.C.10 Prove theorems about triangles.	*Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs.* *Theorems include but are not limited to: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

Cluster Headings	Content Standards	Scope & Clarifications
C. Prove geometric theorems.	**G.CO.C.11** Prove theorems about parallelograms.	*Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs.* *Theorems include but are not limited to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*
D. Make geometric constructions.	**G.CO.D.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	*Constructions include but are not limited to: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; constructing a line parallel to a given line through a point not on the line, and constructing the following objects inscribed in a circle: an equilateral triangle, square, and a regular hexagon.*

Similarity, Right Triangles, and Trigonometry (G.SRT)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand similarity in terms of similarity transformations.	**G.SRT.A.1** Verify informally the properties of dilations given by a center and a scale factor.	*Properties include but are not limited to: a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center of the dilation unchanged; the dilation of a line segment is longer or shorter in the ratio given by the scale factor.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand similarity in terms of similarity transformations	**G.SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Prove theorems involving similarity.	**G.SRT.B.4** Prove theorems about similar triangles.	*Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs.* *Theorems include but are not limited to: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
	G.SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
C. Define trigonometric ratios and solve problems involving triangles.	**G.SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
C. Define trigonometric ratios and solve problems involving triangles.	**G.SRT.C.8** Solve triangles. ★ **a.** Know and use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **b.** Know and use the Law of Sines and Law of Cosines to solve problems in real life situations. Recognize when it is appropriate to use each.	*Ambiguous cases will not be included in assessment.*

Circles (G.C)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand and apply theorems about circles.	**G.C.A.1** Recognize that all circles are similar.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.C.A.2 Identify and describe relationships among inscribed angles, radii, and chords.	*Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle, and properties of angles for a quadrilateral inscribed in a circle.*
	G.C.A.3 Construct the incenter and circumcenter of a triangle and use their properties to solve problems in context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Find areas of sectors of circles.	**G.C.B.4** Know the formula and find the area of a sector of a circle in a real-world context.	*For example, use proportional relationships and angles measured in degrees or radians.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Expressing Geometric Properties with Equations (G.GPE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Translate between the geometric description and the equation for a circle.	**G.GPE.A.1** Know and write the equation of a circle of given center and radius using the Pythagorean Theorem.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Use coordinates to prove simple geometric theorems algebraically.	**G.GPE.B.2** Use coordinates to prove simple geometric theorems algebraically.	*For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,* $\sqrt{3}$*) lies on the circle centered at the origin and containing the point (0, 2).* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.GPE.B.3 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.	*For example, find the equation of a line parallel or perpendicular to a given line that passes through a given point.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.GPE.B.4 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.GPE.B.5 Know and use coordinates to compute perimeters of polygons and areas of triangles and rectangles.★	*For example, use the distance formula.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Geometric Measurement and Dimension (G.GMD)

Cluster Headings	Content Standards	Scope & Clarifications
A. Explain volume and surface area formulas and use them to solve problems.	**G.GMD.A.1** Give an informal argument for the formulas for the circumference of a circle and the volume and surface area of a cylinder, cone, prism, and pyramid.	*Informal arguments may include but are not limited to using the dissection argument, applying Cavalieri's principle, and constructing informal limit arguments.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.GMD.A.2 Know and use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems.★	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Modeling with Geometry (G.MG)

Cluster Headings	Content Standards	Scope & Clarifications
A. Apply geometric concepts in modeling situations.	**G.MG.A.1** Use geometric shapes, their measures, and their properties to describe objects.★	*For example, modeling a tree trunk or a human torso as a cylinder.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	G.MG.A.2 Apply geometric methods to solve real-world problems.★	*Geometric methods may include but are not limited to using geometric shapes, the probability of a shaded region, density, and design problems.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major content of the course is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		Supporting Content

Algebra II | A2

Algebra II emphasizes polynomial, rational and exponential expressions, equations, and functions. This course also introduces students to the complex number system, basic trigonometric functions, and foundational statistics skills such as interpretation of data and making statistical inferences. Students build upon previous knowledge of equations and inequalities to reason, solve, and represent equations and inequalities numerically and graphically.

The major work of Algebra II is from the following domains and clusters:

- **The Real Number System**
    - Extend the properties of exponents to rational exponents.
- **Seeing Structure in Expressions**
    - Interpret the structure of expressions.
    - Use expressions in equivalent forms to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
    - Understand the relationship between zeros and factors of polynomials.
- **Reasoning with Equations and Inequalities**
    - Understand solving equations as a process of reasoning and explain the reasoning.
    - Represent and solve equations graphically.
- **Interpreting Functions**
    - Interpret functions that arise in applications in terms of the context.
- **Building Functions**
    - Build a function that models a relationship between two quantities.
- **Making Inferences and Justifying Conclusions**
    - Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Supporting work is from the following domains and clusters:

- **Quantities**
    - Reason quantitatively and use units to solve problems.
- **The Complex Number System**
    - Perform arithmetic operations with complex numbers.
    - Use complex numbers in quadratic equations.
- **Arithmetic with Polynomials and Rational Expressions**
    - Use polynomial identities to solve problems.
    - Rewrite rational expressions.
- **Creating Equations**
    - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
    - Solve equations and inequalities in one variable.
    - Solve systems of equations.
- **Interpreting Functions**
    - Analyze functions using different representations.
- **Building Functions**
    - Build new functions from existing functions.
- **Linear, Quadratic, and Exponential Models**
    - Construct and compare linear, quadratic, and exponential models and solve problems.
    - Interpret expressions for functions in terms of the situation they model.

- **Trigonometric Functions**
  - Extend the domain of trigonometric functions using the unit circle.
  - Prove and apply trigonometric identities.
- **Interpreting Categorical and Quantitative Data**
  - Summarize, represent, and interpret data on a single count or measurement variable.
  - Summarize, represent, and interpret data on two categorical and quantitative variables.
- **Conditional Probability and the Rules of Probability**
  - Understand independence and conditional probability and use them to interpret data.
  - Use the rules of probability to compute probabilities of compound events in a uniform probability model.

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Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

The Real Number System (N.RN)

Cluster Headings	Content Standards	Scope & Clarifications
A. Extend the properties of exponents to rational exponents.	**A2.N.RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	*For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Quantities★ (N.Q)

Cluster Headings	Content Standards	Scope & Clarifications
A. Reason quantitatively and use units to solve problems.	**A2.N.Q.A.1** Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.	*Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

The Complex Number System (N.CN)

Cluster Headings	Content Standards	Scope & Clarifications
A. Perform arithmetic operations with complex numbers.	**A2.N.CN.A.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Perform arithmetic operations with complex numbers.	**A2.N.CN.A.2** Know and use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Use complex numbers in quadratic equations.	**A2.N.CN.B.3** Solve quadratic equations with real coefficients that have complex solutions.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.	*For example, see $2x^4 + 3x^2 - 5$ as its factors $(x^2 - 1)$ and $(2x^2 + 5)$; see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$; see $(x^2 + 4)/(x^2 + 3)$ as $((x^2+ 3) + 1)/(x^2 + 3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.* *Tasks are limited to polynomial, rational, or exponential expressions.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Use expressions in equivalent forms to solve problems.	**A2.A.SSE.B.2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Use the properties of exponents to rewrite expressions for exponential functions.	*For example the expression 1.15^t can be rewritten as $((1.15)^{1/12})^{12t} \approx 1.012^{12t}$ to reveal that the approximate equivalent monthly interest rate is 1.2% if the annual rate is 15%.* *i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.* *ii) Tasks are limited to exponential expressions with rational or real exponents.*
	A2.A.SSE.B.3 Recognize a finite geometric series (when the common ratio is not 1), and use the sum formula to solve problems in context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Arithmetic with Polynomials and Rational Expressions (A.APR)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand the relationship between zeros and factors of polynomials.	**A2.A.APR.A.1** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.A.APR.A.2 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	*Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Use polynomial identities to solve problems.	**A2.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.	*For example, compare (31)(29) = (30 + 1) (30 – 1) = $30^2 - 1^2$ with (x + y) (x – y) = $x^2 - y^2$.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
C. Rewrite rational expressions.	**A2.A.APR.C.4** Rewrite rational expressions in different forms.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Creating Equations★ (A.CED)

Cluster Headings	Content Standards	Scope & Clarifications
A. Create equations that describe numbers or relationships.	**A2.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.	*Include equations arising from linear and quadratic functions, and rational and exponential functions.* *Tasks have a real-world context.*
	A2.A.CED.A.2 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	*i) Tasks are limited to square root, cube root, polynomial, rational, and logarithmic functions.* *ii) Tasks have a real-world context.*

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand solving equations as a process of reasoning and explain the reasoning.	**A2.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	*Tasks are limited to square root, cube root, polynomial, rational, and logarithmic functions.*
	A2.A.REI.A.2 Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Solve equations and inequalities in one variable.	**A2.A.REI.B.3** Solve quadratic equations and inequalities in one variable. **a**. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	*In the case of equations that have roots with nonzero imaginary parts, students write the solutions as a ± bi for real numbers a and b.*
C. Solve systems of equations.	**A2.A.REI.C.4** Write and solve a system of linear equations in context.	*When solving algebraically, tasks are limited to systems of at most three equations and three variables. With graphic solutions systems are limited to only two variables.*
	A2.A.REI.C.5 Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
D. Represent and solve equations graphically.	**A2.A.REI.D.6** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the approximate solutions using technology. ★	*Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* *Tasks may involve any of the function types mentioned in the standard.*

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret functions that arise in applications in terms of the context.	**A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. ★	*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* *i) Tasks have a real-world context.* *ii) Tasks may involve square root, cube root, polynomial, exponential, and logarithmic functions.*
	A2.F.IF.A.2 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	*i) Tasks have a real-world context.* *ii) Tasks may involve polynomial, exponential, and logarithmic functions.*
B. Analyze functions using different representations.	**A2.F.IF.B.3** Graph functions expressed symbolically and show key features of the graph, by hand and using technology.★ **a.** Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions. **b.** Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. **c.** Graph exponential and logarithmic functions, showing intercepts and end behavior.	*A2.F.IF.B.3a: Tasks are limited to square root and cube root functions. The other functions are assessed in Algebra 1.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Analyze functions using different representations.	**A2.F.IF.B.4** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Know and use the properties of exponents to interpret expressions for exponential functions.	*For example, identify percent rate of change in functions such as $y = 2^x$, $y = (1/2)^x$, $y = 2^{-x}$, $y = (1/2)^{-x}$.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.F.IF.B.5 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	*Tasks may involve polynomial, exponential, and logarithmic functions.*

Building Functions (F.BF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Build a function that models a relationship between two quantities.	**A2.F.BF.A.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context. **b.** Combine standard function types using arithmetic operations.	*For example, given cost and revenue functions, create a profit function.* *For A2.F.BF.A.1a:* *i) Tasks have a real-world context.* *ii) Tasks may involve linear functions, quadratic functions, and exponential functions.*
	A2.F.BF.A.2 Write arithmetic and geometric sequences with an explicit formula and use them to model situations.★	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Build new functions from existing functions.	**A2.F.BF.B.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	*i) Tasks may involve polynomial, exponential, and logarithmic functions.* *ii) Tasks may involve recognizing even and odd functions.*
	A2.F.BF.B.4 Find inverse functions. **a**. Find the inverse of a function when the given function is one-to-one.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Linear, Quadratic, and Exponential Models★ (F.LE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Construct and compare linear, quadratic, and exponential models and solve problems.	**A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.F.LE.A.2 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Interpret expressions for functions in terms of the situation they model.	**A2.F.LE.B.3** Interpret the parameters in a linear or exponential function in terms of a context.	*For example, the equation $y = 5000 (1.06)^x$ models the rising population of a city with 5000 residents when the annual growth rate is 6 percent. What will be the effect on the equation if the city's growth rate was 7 percent instead of 6 percent?* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Trigonometric Functions (F.TF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Extend the domain of trigonometric functions using the unit circle.	**A2.F.TF.A.1** Understand and use radian measure of an angle. **a.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **b.** Use the unit circle to find *sin* θ, *cos* θ, and *tan* θ when θ is a commonly recognized angle between 0 and 2π.	*Commonly recognized angles include all multiples $n\pi/6$ and $n\pi/4$, where n is an integer.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Prove and apply trigonometric identities.	**A2.F.TF.B.3** Know and use trigonometric identities to to find values of trig functions. **a.** Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of *sin* θ, *cos* θ, and *tan* θ to evaluate the trigonometric functions. **b.** Given the quadrant of the angle, use the identity sin^2 θ + cos^2 θ = 1 to find *sin* θ given *cos* θ, or vice versa.	*Commonly recognized angles include all multiples nπ /6 and nπ /4, where n is an integer.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards	Scope & Clarifications
A. Summarize, represent, and interpret data on a single count or measurement variable.	**A2.S.ID.A.1** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using the Empirical Rule.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**A2.S.ID.B.2** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data.	*Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* *i) Tasks have a real-world context.* *ii) Tasks are limited to exponential functions with domains not in the integers.*

Making Inferences and Justifying Conclusions (S.IC)

Cluster Headings	Content Standards	Scope & Clarifications
A. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	**A2.S.IC.A.1** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	*For example, in a given situation, is it more appropriate to use a sample survey, an experiment, or an observational study? Explain how randomization affects the bias in a study.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.S.IC.A.2 Use data from a sample survey to estimate a population mean or proportion; use a given margin of error to solve a problem in context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Conditional Probability and the Rules of Probability (S.CP)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand independence and conditional probability and use them to interpret data.	**A2.S.CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.S.CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.S.CP.A.3 Know and understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand independence and conditional probability and use them to interpret data.	**A2.S.CP.A.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	*For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**A2.S.CP.B.5** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* and interpret the answer in terms of the model.	*For example, a teacher gave two exams. 75 percent passed the first quiz and 25 percent passed both. What percent who passed the first quiz also passed the second quiz?* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.S.CP.B.6 Know and apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.	*For example, in a math class of 32 students, 14 are boys and 18 are girls. On a unit test 6 boys and 5 girls made an A. If a student is chosen at random from a class, what is the probability of choosing a girl or an A student?* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major content of the course is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Integrated Math I | M1

Integrated Math I emphasizes linear and exponential expressions, equations, and functions. This course also focuses on geometric congruence and interpreting linear models from quantitative data. Students continue their learning and understanding of categorical and quantitative data. Students are also introduced to reasoning with equations by solving systems of equations in two variables.

The major work of Integrated Math I is from the following domains and clusters:

- **Seeing Structure in Expressions**
  - Interpret the structure of expressions.
  - Write expressions in equivalent forms to solve problems.
- **Creating Equations**
  - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
  - Solve equations and inequalities in one variable.
  - Represent and solve equations and inequalities graphically.
- **Interpreting Functions**
  - Understand the concept of a function and use function notation.
  - Interpret functions that arise in applications in terms of the context.
- **Building Functions**
  - Build a function that models a relationship between two quantities.
- **Congruence**
  - Understand congruence in terms of rigid motions.
  - Prove geometric theorems.
- **Interpreting Categorical and Quantitative Data**
  - Interpret linear models.

Supporting work is from the following domains and clusters:

- **Quantities**
  - Reason quantitatively and use units to solve problems.
- **Reasoning with Equations and Inequalities**
  - Solve systems of equations.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Linear and Exponential Models**
  - Construct and compare linear and exponential models and solve problems.
  - Interpret expressions for functions in terms of the situation they model.
- **Congruence**
  - Experiment with transformations in the plane.
- **Interpreting Categorical and Quantitative Data**
  - Summarize, represent, and interpret data on a single count or measurement variable.
  - Summarize, represent, and interpret data on two categorical and quantitative variables.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

Quantities* (N.Q)

Cluster Headings	Content Standards	Scope & Clarifications
A. Reason quantitatively and use units to solve problems.	**M1.N.Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.N.Q.A.2 Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.	*Clarification: Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.* *Tasks are limited to linear or exponential equations with integer exponents.*
	M1.N.Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**M1.A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.	*For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* *Tasks are limited to linear and exponential expressions, including related numerical expressions.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Write expressions in equivalent forms to solve problems.	**M1.A.SSE.B.2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Use the properties of exponents to rewrite exponential expressions.	*For M1.A.SSE.B.2a:* *For example, the growth of bacteria can be modeled by either $f(t) = 3^{(t+2)}$ or $g(t) = 9(3^t)$ because the expression $3^{(t+2)}$ can be rewritten as $(3^t)(3^2) = 9(3^t)$.* *Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.*

Creating Equations★ (A.CED)

Cluster Headings	Content Standards	Scope & Clarifications
A. Create equations that describe numbers or relationships	**M1.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.	*i) Tasks are limited to linear or exponential equations with integer exponents.* *ii) Tasks have a real-world context.* *iii) In the linear case, tasks have more of the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).*
	M1.A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations with two variables on coordinate axes with labels and scales.	*i) Tasks are limited to linear equations* *ii) Tasks have a real-world context.* *iii) Tasks have the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).*

Cluster Headings	Content Standards	Scope & Clarifications
A. Create equations that describe numbers or relationships	**M1.A.CED.A.3** Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	*For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	*i) Tasks are limited to linear equations.* *ii) Tasks have a real-world context.*

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards	Scope & Clarifications
A. Solve equations and inequalities in one variable.	**M1.A.REI.A.1** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Solve systems of equations.	**M1.A.REI.B.2** Write and solve a system of linear equations in context.	*Solve systems both algebraically and graphically.* *Systems are limited to at most two equations in two variables.*
C. Represent and solve equations and inequalities graphically.	**M1.A.REI.C.3** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
C. Represent and solve equations and inequalities graphically.	**M1.A.REI.C.4** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f(x)* and *y* = *g(x)* intersect are the solutions of the equation *f(x)* = *g(x)*; find the approximate solutions using technology. ★	*Include cases where f(x) and/or g(x) are linear, absolute value, and exponential functions. For example: f(x) = 3x + 5.* *i) Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions.* *ii) Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial.* *iii) Tasks are limited to linear and absolute value functions.*
	M1.A.REI.C.5 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand the concept of a function and use function notation.	**M1.F.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand the concept of a function and use function notation.	**M1.F.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Interpret functions that arise in applications in terms of the context.	**M1.F.IF.B.3** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. ★	*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* *i) Tasks have a real-world context.* *ii) Tasks are limited to linear functions, absolute value, and exponential functions with domains in the integers.*
	M1.F.IF.B.4 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★	*For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *i) Tasks have a real-world context.* *ii) Tasks are limited to linear functions, piecewise functions (including step functions and absolute value functions), and exponential functions with domains in the integers.*
	M1.F.IF.B.5 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	*i) Tasks have a real-world context.* *ii) Tasks are limited to linear functions, piecewise functions (including step functions and absolute value functions), and exponential functions with domains in the integers.*

<table>
<tr><th>Cluster Headings</th><th>Content Standards</th><th>Scope & Clarifications</th></tr>
<tr><td rowspan="2">C. Analyze functions using different representations.</td><td>M1.F.IF.C.6 Graph functions expressed symbolically and show key features of the graph, by hand and using technology.
a. Graph linear functions and show it's intercepts.</td><td>Tasks are limited to linear functions.</td></tr>
<tr><td>M1.F.IF.C.7 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</td><td>i) Tasks have a real-world context.
ii) Tasks are limited to linear functions, piecewise functions (including step functions and absolute value functions), and exponential functions with domains in the integers.</td></tr>
</table>

Building Functions (F.BF)

<table>
<tr><th>Cluster Headings</th><th>Content Standards</th><th>Scope & Clarifications</th></tr>
<tr><td rowspan="2">A. Build a function that models a relationship between two quantities.</td><td>M1.F.BF.A.1 Write a function that describes a relationship between two quantities.★
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</td><td>i) Tasks have a real-world context.
ii) Tasks are limited to linear functions and exponential functions with domains in the integers.</td></tr>
<tr><td>M1.F.BF.A.2 Write arithmetic and geometric sequences with an explicit formula and use them to model situations.★</td><td>There are no assessment limits for this standard. The entire standard is assessed in this course.</td></tr>
</table>

Linear and Exponential Models★ (F.LE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Construct and compare linear and exponential models and solve problems.	**M1.F.LE.A.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. **a.** Recognize that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant factor per unit interval relative to another.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.F.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.	*Tasks are limited linear and exponential functions.*
B. Interpret expressions for functions in terms of the situation they model.	**M1.F.LE.B.4** Interpret the parameters in a linear or exponential function in terms of a context.	*For example, the total cost of an electrician who charges 35 dollars for a house call and 50 dollars per hour would be expressed as the function $y = 50x + 35$. If the rate were raised to 65 dollars per hour, describe how the function would change.* *Tasks have a real-world context.*

Geometry

Congruence (G.CO)

Cluster Headings	Content Standards	Scope & Clarifications
A. Experiment with transformations in the plane.	**M1.G.CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, plane, distance along a line, and distance around a circular arc.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.G.CO.A.2 Represent transformations in the plane in multiple ways, including technology. Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry the shape onto itself.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.G.CO.A.5 Given a geometric figure and a rigid motion, draw the image of the figure in multiple ways, including technology. Specify a sequence of rigid motions that will carry a given figure onto another.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Understand congruence in terms of rigid motions.	**M1.G.CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to determine informally if they are congruent.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Understand congruence in terms of rigid motions.	**M1.G.CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.G.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, AAS, and SSS) follow from the definition of congruence in terms of rigid motions.	*There is no additional scope or clarification information for this standard.*
C. Prove geometric theorems.	**M1.G.CO.C.9** Prove theorems about lines and angles.	*Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs.* *Theorems include but are not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
	M1.G.CO.C.10 Prove theorems about triangles.	*Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs.* *Theorems include but are not limited to: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

Cluster Headings	Content Standards	Scope & Clarifications
C. Prove geometric theorems.	**M1.G.CO.C.11** Prove theorems about parallelograms.	*Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs.* *Theorems include but are not limited to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards	Scope & Clarifications
A. Summarize, represent, and interpret data on a single count or measurement variable.	**M1.S.ID.A.1** Represent single or multiple data sets with dot plots, histograms, stem plots (stem and leaf), and box plots.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**M1.S.ID.B.4** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a**. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. **b**. Fit a linear function for a scatter plot that suggests a linear association.	*i) Tasks have real-world context.* *ii) Tasks are limited to linear functions and exponential functions with domains in the integers.*
C. Interpret linear models.	**M1.S.ID.C.5** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.S.ID.C.6 Compute (using technology) and interpret the correlation coefficient of a linear fit.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M1.S.ID.C.7 Distinguish between correlation and causation.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major content of the course is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Integrated Math II | M2

Integrated Math II builds upon concepts taught in Integrated Math I with an emphasis on quadratic and polynomial expressions, equations, and functions. This course also focuses on geometric similarity and interpreting functions from a real life context. Students extend previous knowledge of exponential properties to rational exponents. This course also introduces probability of compound events and the complex number system.

The major work of Integrated Math II is from the following domains and clusters:

- **The Real Number System**
  - Extend the properties of exponents to rational exponents.
- **Seeing Structure in Expressions**
  - Interpret the structure of expressions.
  - Write expressions in equivalent forms to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Perform arithmetic operations on polynomials.
- **Creating Equations**
  - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
  - Understand solving equations as a process of reasoning and explain the reasoning.
  - Solve equations and inequalities in one variable.
- **Interpreting Functions**
  - Interpret functions that arise in applications in terms of the context.
- **Similarity, Right Triangles, and Trigonometry**
  - Understand similarity in terms of similarity transformations.
  - Prove theorems involving similarity.
  - Define trigonometric ratios and solve problems involving triangles.

Supporting work is from the following domains and clusters:

- **Quantities**
  - Reason quantitatively and use units to solve problems.
- **The Complex Number System**
  - Perform arithmetic operations with complex numbers.
  - Use complex numbers in polynomial identities and equations.
- **Reasoning with Equations and Inequalities**
  - Solve systems of equations.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Building Functions**
  - Build a function that models a relationship between two quantities.
  - Build new functions from existing functions.
- **Geometric Measurement and Dimension**
  - Explain volume and surface area formulas and use them to solve problems.
- **Interpreting Categorical and Quantitative Data**
  - Summarize, represent, and interpret data on two categorical and quantitative variables.
- **Conditional Probability and the Rules of Probability**
  - Understand independence and conditional probability and use them to interpret data.
  - Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

The Real Number System (N.RN)

Cluster Headings	Content Standards	Scope & Clarifications
A. Extend the properties of exponents to rational exponents.	**M2.N.RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	*For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M2.N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Quantities* (N.Q)

Cluster Headings	Content Standards	Scope & Clarifications
A. Reason quantitatively and use units to solve problems.	**M2.N.Q.A.1** Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.	*Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.* *Tasks are limited to linear, quadratic, exponential equations with integer exponents, square root, and cube root functions.*

The Complex Number System (N.CN)

Cluster Headings	Content Standards	Scope & Clarifications
A. Perform arithmetic operations with complex numbers.	**M2.N.CN.A.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Perform arithmetic operations with complex numbers.	**M2.N.CN.A.2** Know and use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Use complex numbers in polynomial identities and equations.	**M2.N.CN.B.3** Solve quadratic equations with real coefficients that have complex solutions.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**M2.A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.★ **a.** Interpret complicated expressions by viewing one or more of their parts as a single entity.	*For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* *Tasks are limited to quadratic expressions.*
	M2.A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.	*For example, recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 + 47)(53 - 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$.* *Tasks are limited to numerical expressions and polynomial expressions in one variable.*

B. Write expressions in equivalent forms to solve problems.	**M2.A.SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Factor a quadratic expression to reveal the zeros of the function it defines. **b.** Complete the square in a quadratic expression in the form $Ax^2 + Bx + C$ to reveal the maximum or minimum value of the function it defines.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Arithmetic with Polynomials and Rational Expressions (A.APR)

Cluster Headings	Content Standards	Scope & Clarifications
A. Perform arithmetic operations on polynomials.	**M2.A.APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Creating Equations★ (A–CED)

Cluster Headings	Content Standards	Scope & Clarifications
A. Create equations that describe numbers or relationships.	**M2.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.	*Include equations arising from linear and quadratic functions and rational and exponential functions. Tasks have a real-world context.*
	M2.A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations with two variables on coordinate axes with labels and scales.	*i) Tasks are limited to quadratic, square root, cube root, and piecewise functions.* *ii) Tasks have a real-world context.* *iii) Tasks have the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).*

Cluster Headings	Content Standards	Scope & Clarifications
A. Create equations that describe numbers or relationships.	**M2.A.CED.A.3** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	*i) Tasks are limited to quadratic, square root, and cube root functions.* *ii) Tasks have a real-world context.*

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand solving equations as a process of reasoning and explain the reasoning.	**M2.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	*Tasks are limited to linear, quadratic, exponential equations with integer exponents, square root, and cube root functions.*
B. Solve equations and inequalities in one variable.	**M2.A.REI.B.2** Solve quadratic equations and inequalities in one variable. **a**. Use the method of completing the square to rewrite any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
C. Solve systems of equations.	**M2.A.REI.C.3** Write and solve a system of linear equations in context.	*When solving algebraically, tasks are limited to systems of at most three equations and three variables. With graphic solutions systems are limited to only two variables.*

Cluster Headings	Content Standards	Scope & Clarifications
C. Solve systems of equations.	**M2.A.REI.C.4** Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret functions that arise in applications in terms of the context.	**M2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. ⋆	*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* *i) Tasks have a real-world context.* *ii) Tasks are limited to quadratic, exponential functions with integer exponents, square root, and cube root functions.*
	M2.F.IF.A.2 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ⋆	*For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *Tasks are limited to quadratic, square root, cube root, piecewise, and exponential functions.*
	M2.F.IF.A.3 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ⋆	*i) Tasks have a real-world context.* *ii) Tasks may involve quadratic, square root, cube root, piecewise, and exponential functions.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Analyze functions using different representation.	**M2.F.IF.B.4** Graph functions expressed symbolically and show key features of the graph, by hand and using technology.★ **a.** Graph linear and quadratic functions and show intercepts, maxima, and minima. **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **c**. Graph exponential and logarithmic functions, showing intercepts and end behavior.	*M2.F.IF.B.4a – Tasks are limited to quadratic functions.* *M2.F.IF.B.4c – Tasks are limited to exponential functions.*
	M2.F.IF.B.5 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **b.** Know and use the properties of exponents to interpret expressions for exponential functions.	*For example, identify percent rate of change in functions such as $y = 2^x$, $y = (1/2)^x$, $y = 2^{-x}$, $y = (1/2)^{-x}$.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M2.F.IF.B.6 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	*i) Tasks do not have a real-world context.* *ii) Tasks may involve quadratic, square root, cube root, piecewise, and exponential functions.*

Building Functions (F.BF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Build a function that models a relationship between two quantities.	**M2.F.BF.A.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context. **b.** Combine standard function types using arithmetic operations.	*For M2.F.BF.A.1a:* *i) Tasks have a real-world context.* *ii) Tasks may involve linear and quadratic functions.*
B. Build new functions from existing functions.	**M2.F.BF.B.2** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	*i) Identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), and f(x+k) for specific values of k (both positive and negative) is limited to linear, quadratic, and absolute value functions.* *ii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear, quadratic, square root, cube root, and exponential functions.* *iii) Tasks do not involve recognizing even and odd functions.*

Geometry

Similarity, Right Triangles, and Trigonometry (G.SRT)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand similarity in terms of similarity transformations.	**M2.G.SRT.A.1** Verify informally the properties of dilations given by a center and a scale factor.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand similarity in terms of similarity transformations.	**M2.G.SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M2.G.SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Prove theorems involving similarity.	**M2.G.SRT.B.4** Prove theorems about similar triangles.	*Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs.* *Theorems include but are not limited to: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
	M2.G.SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
C. Define trigonometric ratios and solve problems involving triangles.	**M2.G.SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	*There are no assessment limits for this standard.*
	M2.G.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

C. Define trigonometric ratios and solve problems involving triangles.	**M2.G.SRT.C.8** Solve triangles. ★ **a**. Know and use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **b.** Know and use the Law of Sines and the Law of Cosines to solve triangles in applied problems. Recognize when it is appropriate to use each.	*Ambiguous cases will not be included in assessment.*

Geometric Measurement and Dimension (G.GMD)

Cluster Headings	Content Standards	Scope & Clarifications
A. Explain volume and surface area formulas and use them to solve problems.	**M2.G.GMD.A.1** Give an informal argument for the formulas for the circumference of a circle and the volume and surface area of a cylinder, cone, prism, and pyramid.	*Informal arguments may include but are not limited to using the dissection argument, applying Cavalieri's principle, and constructing informal limit arguments.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M2.G.GMD.A.2 Know and use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems.★	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards	Scope & Clarifications
A. Summarize, represent, and interpret data on two categorical and quantitative variables.	**M2.S.ID.A.1** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a**. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.	*Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. Exponential functions are limited to those with domains in the integers.* *Tasks have a real-world context.*

Conditional Probability and the Rules of Probability (S.CP)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand independence and conditional probability and use them to interpret data.	**M2.S.CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M2.S.CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M2.S.CP.A.3 Know and understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M2.S.CP.A.4 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**M2.S.CP.B.5** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* and interpret the answer in terms of the model.	*For example, a teacher gave two exams. 75 percent passed the first exam and 25 percent passed both. What percent who passed the first exam also passed the second exam?* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**M2.S.CP.B.6** Know and apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.	*For example, in a math class of 32 students, 14 are boys and 18 are girls. On a unit test 6 boys and 5 girls made an A. If a student is chosen at random from a class, what is the probability of choosing a girl or an A student?* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major content of the course is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Integrated Math III | M3

Integrated Math III builds upon concepts taught in Integrated Math I and Integrated Math II and emphasizes polynomial and rational expressions, equations, and functions. This course has a focus on geometric modeling and using algebra to prove geometric theorems. This course also introduces students to circles, basic trigonometric functions, and foundational statistics skills such as interpretation of data and making statistical inferences.

The major work of Integrated Math III is from the following domains and clusters:

- **Seeing Structure in Expressions**
  - Interpret the structure of expressions.
  - Write expressions in equivalent forms to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Understand the relationship between zeros and factors of polynomials.
- **Creating Equations**
  - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
  - Understand solving equations as a process of reasoning and explain the reasoning.
  - Represent and solve equations graphically.
- **Interpreting Functions**
  - Interpret functions that arise in applications in terms of the context.
- **Expressing Geometric Properties with Equations**
  - Use coordinates to prove simple geometric theorems algebraically.
- **Modeling with Geometry**
  - Apply geometric concepts in modeling situations.
- **Interpreting Categorical and Quantitative Data**
  - Summarize, represent, and interpret data on a single count or measurement variable.
  - Summarize, represent, and interpret data on two categorical and quantitative variables.
- **Making Inferences and Justifying Conclusions**
  - Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Supporting work is from the following domains and clusters:

- **Quantities**
  - Reason quantitatively and use units to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Use polynomial identities to solve problems.
  - Rewrite rational expressions.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Building Functions**
  - Build new functions from existing functions.
- **Linear, Quadratic, and Exponential Models**
  - Construct and compare linear, quadratic, and exponential models and solve problems.
- **Trigonometric Functions**
  - Extend the domain of trigonometric functions using the unit circle.
  - Prove and apply trigonometric identities.
- **Congruence**
  - Make geometric constructions.

- **Circles**
    - Understand and apply theorems about circles.
    - Find areas of sectors of circles.
- **Expressing Geometric Properties with Equations**
    - Translate between the geometric description and the equation for a circle.

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Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

Quantities* (N.Q)

Cluster Headings	Content Standards	Scope & Clarifications
A. Reason quantitatively and use units to solve problems.	**M3.N.Q.A.1** Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.	*Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**M3.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.	*For example, see $2x^4 + 3x^2 – 5$ as its factors $(x^2 – 1)$ and $(2x^2 + 5)$; see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$; see* *$(x^2 + 4)/(x^2 + 3)$ as $((x^2+ 3) + 1)/(x^2 + 3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.* *Tasks are limited to polynomial, rational, or exponential expressions.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Write expressions in equivalent forms to solve problems.	**M3.A.SSE.B.2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **a.** Use the properties of exponents to rewrite expressions for exponential functions.	*For example, the expression 1.15^t can be rewritten as $((1.15)^{1/12})^{12t} \approx 1.012^{12t}$ to reveal that the approximate equivalent monthly interest rate is 1.2% if the annual rate is 15%.* *i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.* *ii) Tasks are limited to exponential expressions with rational or real exponents.*
	M3.A.SSE.B.3 Recognize a finite geometric series (when the common ratio is not 1), and use the sum formula to solve problems in context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Arithmetic with Polynomials and Rational Expressions (A.APR)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand the relationship between zeros and factors of polynomials.	**M3.A.APR.A.1** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M3.A.APR.A.2 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	*Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2-1)(x^2+1)$.*
B. Use polynomial identities to solve problems.	**M3.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.	*For example, compare $(31)(29)=(30+1)(30-1)=30^2-1^2$ with $(x+y)(x-y)=x^2-y^2$.*

Cluster Headings	Content Standards	Scope & Clarifications
C. Rewrite rational expressions.	**M3.A.APR.C.4** Rewrite rational expressions in different forms.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Creating Equations★ (A.CED)

Cluster Headings	Content Standards	Scope & Clarifications
A. Create equations that describe numbers or relationships.	**M3.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.	*i) Tasks are limited to polynomial, rational, absolute value, exponential, or logarithmic functions.* *ii) Tasks have a real-world context.*
	M3.A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations with two variables on coordinate axes with labels and scales.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M3.A.CED.A.3 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	*i) Tasks have a real-world context.* *ii) Tasks are limited to polynomial, rational, absolute value, exponential, or logarithmic functions.*

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand solving equations as a process of reasoning and explain the reasoning.	**M3.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	*Tasks are limited to simple rational or radical equations.*
	M3.A.REI.A.2 Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Represent and solve equations graphically.	**M3.A.REI.B.3** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the approximate solutions using technology. ⋆	*Tasks may include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, or logarithmic functions.*

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret functions that arise in applications in terms of the context.	**M3.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. ⋆	*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* *i) Tasks have a real-world context.* *ii) Tasks may involve polynomial, exponential, and logarithmic functions.*
	M3.F.IF.A.2 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ⋆	*i) Tasks have a real-world context.* *ii) Tasks may involve polynomial, exponential, and logarithmic functions.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Analyze functions using different representations.	**M3.F.IF.B.3** Graph functions expressed symbolically and show key features of the graph, by hand and using technology.★ **a**. Graph linear and quadratic functions and show intercepts, maxima, and minima. **b**. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **c**. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. **d**. Graph exponential and logarithmic functions, showing intercepts and end behavior.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M3.F.IF.B.4 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	*Tasks may involve polynomial, exponential, and logarithmic functions.*

Building Functions (F.BF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Build new functions from existing functions.	**M3.F.BF.A.1** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	*i) Tasks may involve polynomial, exponential, and logarithmic functions.* *ii) Tasks may involve recognizing even and odd functions.*
	M3.F.BF.A.2 Find inverse functions. **a**. Find the inverse of a function when the given function is one-to-one.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Linear, Quadratic, and Exponential Models★ (F.LE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Construct and compare linear, quadratic, and exponential models and solve problems.	**M3.F.LE.A.1** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	*There are no assessment limits for this standard. The entire standard is assessed in this course*
	M3.F.LE.A.2 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Trigonometric Functions (F.TF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Extend the domain of trigonometric functions using the unit circle.	**M3.F.TF.A.1** Understand and use radian measure of an angle. **a.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **b.** Use the unit circle to find *sin* θ, *cos* θ, and *tan* θ when θ is a commonly recognized angle between 0 and 2π.	*Commonly recognized angles include all multiples of nπ /6 and nπ /4, where n is an integer.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M3.F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Prove and apply trigonometric identities.	**M3.F.TF.B.3** Know and use trigonometric identities to find values of trig functions. **a**. Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of *sin* θ, *cos* θ, and *tan* θ to evaluate the trigonometric functions. **b**. Given the quadrant of the angle, use the identity $\sin^2\theta + \cos^2\theta = 1$ to find *sin* θ given *cos* θ, or vice versa.	*Commonly recognized angles include all multiples of nπ /6 and nπ /4, where n is an integer.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Geometry

Congruence (G.CO)

Cluster Headings	Content Standards	Scope & Clarifications
A. Make geometric constructions.	**M3.G.CO.A.1** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	*Constructions include but are not limited to: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; constructing a line parallel to a given line through a point not on the line, and constructing the following objects inscribed in a circle: an equilateral triangle, square, and a regular hexagon.*

Circles (G.C)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand and apply theorems about circles.	**M3.G.C.A.1** Recognize that all circles are similar.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M3.G.C.A.2 Identify and describe relationships among inscribed angles, radii, and chords.	*Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle, and properties of angles for a quadrilateral inscribed in a circle.*
	M3.G.C.A.3 Construct the incenter and circumcenter of a triangle and use their properties to solve problems in context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Find areas of sectors of circles.	**M3.G.C.B.4** Know the formula and find the area of a sector of a circle in a real-world context.	*For example, use proportional relationships and angles measured in degrees or radians.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Expressing Geometric Properties with Equations (G.GPE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Translate between the geometric description and the equation for a circle.	**M3.G.GPE.A.1** Know and write the equation of a circle of given center and radius using the Pythagorean Theorem.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Use coordinates to prove simple geometric theorems algebraically.	**M3.G.GPE.B.2** Use coordinates to prove simple geometric theorems algebraically.	*For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,* $\sqrt{3}$*) lies on the circle centered at the origin and containing the point (0, 2).* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M3.G.GPE.B.3 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.	*For example, find the equation of a line parallel or perpendicular to a given line that passes through a given point.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M3.G.GPE.B.4 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Use coordinates to prove simple geometric theorems algebraically.	**M3.G.GPE.B.5** Know and use coordinates to compute perimeters of polygons and areas of triangles and rectangles.★	*For example, use the distance formula.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Modeling with Geometry (G.MG)

Cluster Headings	Content Standards	Scope & Clarifications
A. Apply geometric concepts in modeling situations.	**M3.G.MG.A.1** Use geometric shapes, their measures, and their properties to describe objects.★	*There are no assessment limits for this standard. The entire standard is assessed in this course. For example, modeling a tree trunk or a human torso as a cylinder.*
	M3.G.MG.A.2 Apply geometric methods to solve real-world problems.★	*Geometric methods may include but are not limited to using geometric shapes, the probability of a shaded region, density, and design problems.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards	Scope & Clarifications
A. Summarize, represent, and interpret data on a single count or measurement variable.	**M3.S.ID.A.1** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using the Empirical Rule.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**M3.S.ID.B.2** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a**. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. **b.** Fit a linear function for a scatter plot that suggests a linear association.	*Use given functions or choose a function suggested by the context.* *i) Tasks have a real-world context.* *ii) Tasks are limited to linear, quadratic, and exponential functions with domains not in the integers.*

Making Inferences and Justifying Conclusions (S.IC)

Cluster Headings	Content Standards	Scope & Clarifications
A. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	M3.S.IC.A.1 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	*For example, in a given situation, is it more appropriate to use a sample survey, an experiment, or an observational study? Explain how randomization affects the bias in a study.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	M3.S.IC.A.2 Use data from a sample survey to estimate a population mean or proportion; use a given margin of error to solve a problem in context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major content of the course is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		Supporting Content

Fourth-Year Courses
Proposed Standards

Bridge Math | B

Bridge Math is a course intended to build upon concepts taught in previous courses to allow students to gain a deeper knowledge of the real and complex number systems as well as the structure, use, and application of equations, expressions, and functions. Functions emphasized include linear, quadratic and polynomial. Students continue mastery of geometric concepts such as similarity, congruence, right triangles, and circles. Students use categorical and quantitative data to model real life situations and rules of probability to compute probabilities of compound events.

Bridge Math includes the following domains and clusters:

- **The Real Number System**
  - Use properties of rational and irrational numbers.
- **Quantities**
  - Reason quantitatively and use units to solve problems.
- **The Complex Number System**
  - Perform arithmetic operations with complex numbers.
- **Seeing Structure in Expressions**
  - Write expressions in equivalent forms to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Perform arithmetic operations on polynomials.
  - Understand the relationship between zeros and factors of polynomials.
- **Creating Equations**
  - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
  - Understand solving equations as a process of reasoning and explain the reasoning.
  - Solve equations and inequalities in one variable.
  - Solve systems of equations.
  - Represent and solve equations and inequalities graphically.
- **Interpreting Functions**
  - Understand the concept of a function and use function notation.
  - Interpret functions that arise in applications in terms of the context.
  - Analyze functions using different representations
- **Similarity, Right Triangles, and Trigonometry**
  - Understand similarity in terms of similarity transformations.
  - Define trigonometric ratios and solve problems involving right triangles.
- **Circles**
  - Find arc lengths and areas of sectors of circles.
- **Geometric Measurement and Dimension**
  - Visualize relationships between two-dimensional and three-dimensional objects.
- **Modeling with Geometry**
  - Apply geometric concepts in modeling situations.
- **Interpreting Categorical and Quantitative Data**
  - Summarize, represent, and interpret data on a single count or measurement variable.
  - Summarize, represent, and interpret data on two categorical and quantitative variables.
  - Interpret linear models.
- **Conditional probability and the Rules of Probability**
  - Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

The Real Number System (N.RN)

Cluster Headings	Content Standards
A. Use properties of rational and irrational numbers.	**B.N.RN.A.1**. Use rational and irrational numbers in calculations and in real-world context.

Quantities★ (N.Q)

Cluster Headings	Content Standards
A. Reason quantitatively and use units to solve problems.	**B.N.Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **B.N.Q.A.2** Define appropriate quantities for the purpose of descriptive modeling. **B.N.Q.A.3** Solve problems involving squares, square roots of numbers, cubes, and cube roots of numbers.

The Complex Number System (N.CN)

Cluster Headings	Content Standards
A. Perform arithmetic operations with complex numbers.	**B.N.CN.A.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **B.N.CN.A.2** Know and use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards
A. Write expressions in equivalent forms to solve problems.	**B.A.SSE.A.1** Use properties of multiplication and division to solve problems containing scientific notation. **B.A.SSE.A.2** Use algebraic structures to solve problems involving proportional reasoning in real-world context.

Arithmetic with Polynomials and Rational Expressions (A.APR)

Cluster Headings	Content Standards
A. Perform arithmetic operations on polynomials.	**B.A.APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
B. Understand the relationship between zeros and factors of polynomials.	**B.A.APR.B.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Creating Equations★ (A.CED)

Cluster Headings	Content Standards
A. Create equations that describe numbers or relationships.	**B.A.CED.A.1** Create equations and inequalities in one variable and use them to solve real-world problems. **B.A.CED.A.2** Create equations in two or more variables to represent relationships between quantities. **B.A.CED.A.3** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards
A. Understand solving equations as a process of reasoning and explain the reasoning.	**B.A.REI.A.1** Build functions and write expressions, equations, and inequalities for common algebra settings leading to a solution in context (e.g., rate and distance problems and problems that can be solved using proportions).
B. Solve equations and inequalities in one variable.	**B.A.REI.B.2** Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
C. Solve systems of equations.	**B.A.REI.C.3** Solve and explain the solutions to a system of equations using a variety of representations including combinations of linear and non-linear equations.
D. Represent and solve equations and inequalities graphically.	**B.A.REI.D.4** Use algebra and geometry to solve problems involving midpoints and distances. **B.A.REI.D.5** Solve a linear inequality using multiple methods and interpret the solution as it applies to the context.

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards
A. Understand the concept of a function and use function notation.	**B.F.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **B.F.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Cluster Headings	Content Standards
B. Interpret functions that arise in applications in terms of the context.	**B.F.IF.B.3** Recognize functions as mappings of an independent variable into a dependent variable. ★
C. Analyze functions using different representations.	**B.F.IF.C.4** Graph linear, quadratic, absolute value, and piecewise functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated ones. ★ **B.F.IF.C.5** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **B.F.IF.C.6** Use the properties of exponents to interpret expressions for exponential functions.

Geometry

Similarity, Right Triangles and Trigonometry (G.SRT)

Cluster Headings	Content Standards
A. Understand similarity in terms of similarity transformations.	**B.G.SRT.A.1** Apply similar triangles to solve problems, such as finding heights and distances.
B. Define trigonometric ratios and solve problems involving right triangles.	**B.G.SRT.B.2** Apply basic trigonometric ratios to solve right triangle problems. **B.G.SRT.B.3** Apply properties of 30° 60° 90°, 45° 45° 90°, similar, and congruent triangles. **B.G.SRT.B.4** Solve problems involving angles of elevation and angles of depression.

Circles (G.C)

Cluster Headings	Content Standards
A. Find arc lengths and areas of sectors of circles.	**B.G.C.A.1** Apply a variety of strategies to determine the area and circumference of circles after identifying necessary information.

Geometric Measurement and Dimension (G.GMD)

Cluster Headings	Content Standards
A. Visualize relationships between two-dimensional and three-dimensional objects.	**B.G.GMD.A.1** Use relationships involving area, perimeter, and volume of geometric figures to compute another measure. **B.G.GMD.A.2** Use several angle properties to find an unknown angle measure. **B.G.GMD.A.3** Apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures in composite figures (i.e., combinations of basic figures).

Modeling with Geometry (G.MG)

Cluster Headings	Content Standards
A. Apply geometric concepts in modeling situations.	**B.G.MG.A.1** Use appropriate technology to find the mathematical model for a set of non-linear data. **B.G.MG.A.2** Solve problems involving surface area and volume in real-world context.

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards
A. Summarize, represent, and interpret data on a single count or measurement variable.	**B.S.ID.A.1** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**B.S.ID.B.2** Interpret and use data from tables, charts, and graphs.

Cluster Headings	Content Standards
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**B.S.ID.B.3** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
C. Interpret linear models.	**B.S.ID.C.4** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Conditional Probability and the Rules of Probability (S.CP)

Cluster Headings	Content Standards
A. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**B.S.CP.A.1** Understand and use basic counting techniques in contextual settings. **B.S.CP.A.2** Compute a probability when the event and/or sample space are not given or obvious. **B.S.CP.A.3** Recognize the concepts of conditional and joint probability expressed in real-world contexts. **B.S.CP.A.4** Recognize the concept of independence expressed in real-world contexts.

Precalculus | P

Precalculus is designed to prepare students for college level STEM focused courses. Students extend their knowledge of the complex number system to use complex numbers in polynomial identities and equations. Topics for student mastery include vectors and matrix quantities, sequences and series, parametric equations, and conic sections. Students use previous knowledge to continue progressing in their understanding of trigonometric functions and using regression equations to model quantitative data.

Precalculus includes the following domains and clusters:

- **Number Expressions**
  - Represent, interpret, compare, and simplify number expressions.
- **Complex Numbers**
  - Perform complex number arithmetic and understand the representation on the complex plane.
  - Use complex numbers in polynomial identities and equations.
- **Vectors and Matrix Quantities**
  - Represent and model with vector quantities.
  - Understand the graphic representation of vectors and vector arithmetic.
  - Perform operations on matrices and use matrices in applications.
- **Sequences and Series**
  - Understand and use sequences and series.
- **Reasoning with Equations and Inequalities**
  - Solve systems of equations and nonlinear inequalities.
- **Parametric Equations**
  - Describe and use parametric equations.
- **Conic Sections**
  - Understand the properties of conic sections and apply them to model real-world phenomena.
- **Building Functions**
  - Build new functions from existing functions.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Trigonometric Functions**
  - Extend the domain of trigonometric functions using the unit circle.
- **Graphing Trigonometric Functions**
  - Model periodic phenomena with trigonometric functions.
- **Applied Trigonometry**
  - Use trigonometry to solve problems.
- **Trigonometric Identities**
  - Apply trigonometric identities to rewrite expressions and solve equations.
- **Polar Coordinates**
  - Use polar coordinates.
- **Model with Data**
  - Model data using regression equations.

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Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

Number Expressions (N.NE)

Cluster Headings	Content Standards
A. Represent, interpret, compare, and simplify number expressions.	**P.N.NE.A.1** Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them. **P.N.NE.A.2** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.★ **P.N.NE.A.3** Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of π and e. **P.N.NE.A.4** Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not. **P.N.NE.A.5** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Complex numbers (N.CN)

Cluster Headings	Content Standards
A. Perform complex number arithmetic and understand the representation on the complex plane.	**P.N.CN.A.1** Perform arithmetic operations with complex numbers expressing answers in the form $a + bi$. **P.N.CN.A.2** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **P.N.CN.A.3** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **P.N.CN.A.4** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + 3i)^3 = 8$ *because* $(-1 + 3i)$ *has modulus 2 and argument 120°.*

A. Perform complex number arithmetic and understand the representation on the complex plane.	**P.N.CN.A.5** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
B. Use complex numbers in polynomial identities and equations.	**P.N.CN.B.6** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$*.* **P.N.CN.B.7** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (N.VM)

Cluster Headings	Content Standards
A. Represent and model with vector quantities.	**P.N.VM.A.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes *(e.g.,* ***v****,* |***v***|*,* ||***v***||*, v)*. **P.N.VM.A.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **P.N.VM.A.3** Solve problems involving velocity and other quantities that can be represented by vectors.
B. Understand the graphic representation of vectors and vector arithmetic.	**P.N.VM.B.4** Add and subtract vectors. **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **c.** Understand vector subtraction $\boldsymbol{v} – \boldsymbol{w}$ as $\boldsymbol{v} + (–\boldsymbol{w})$, where $–\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	P.N.VM.B.5 Multiply a vector by a scalar. **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. **b.** Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = \|c\|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Cluster Headings	Content Standards
B. Understand the graphic representation of vectors and vector arithmetic.	**P.N.VM.B.6** Calculate and interpret the dot product of two vectors.
C. Perform operations on matrices and use matrices in applications.	**P.N.VM.C.7** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **P.N.VM.C.8** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **P.N.VM.C.9** Add, subtract, and multiply matrices of appropriate dimensions. **P.N.VM.C.10** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **P.N.VM.C.11** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **P.N.VM.C.12** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **P.N.VM.C.13** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Functions

Building Functions (F.BF)

Cluster Headings	Content Standards
A. Build new functions from existing functions.	**P.F.BF.A.1** Understand how the algebraic properties of an equation transform the geometric properties of its graph. *For example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation (i.e., translations, stretches, reflections and changes in periodicity and amplitude).*

Cluster Headings	Content Standards
A. Build new functions from existing functions.	**P.F.BF.A.2** Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.
	P.F.BF.A.3 Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*
	P.F.BF.A.4 Construct the difference quotient for a given function and simplify the resulting expression.
	P.F.BF.A.5 Find inverse functions (including exponential, logarithmic and trigonometric). **a.** Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, *1 / f(x),* and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type. **b.** Verify by composition that one function is the inverse of another. **c.** Read values of an inverse function from a graph or a table, given that the function has an inverse. **d.** Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain.
	P.F.BF.A.6 Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.

Interpreting Functions (F.IF)

Cluster Headings	Content Standards
A. Analyze functions using different representations.	**P.F.IF.A.1** Determine whether a function is even, odd, or neither. **P.F.IF.A.2** Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology).★ **P.F.IF.A.3** Identify or analyze the distinguishing properties of exponential, polynomial, logarithmic, trigonometric, and rational functions from tables, graphs, and equations.

Cluster Headings	Content Standards
A. Analyze functions using different representations.	**P.F.IF.A.4** Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational). **P.F.IF.A.5** Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$. **P.F.IF.A.6** Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur. **P.F.IF.A.7** Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior. **P.F.IF.A.8** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.

Trigonometric Functions (F.TF)

Cluster Headings	Content Standards
A. Extend the domain of trigonometric functions using the unit circle.	**P.F.TF.A.1** Convert from radians to degrees and from degrees to radians. **P.F.TF.A.2** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. **P.F.TF.A.3** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **P.F.TF.A.4** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Graphing Trigonometric Functions (F.GT)

Cluster Headings	Content Standards
A. Model periodic phenomena with trigonometric functions.★	**P.F.GT.A.1** Interpret transformations of trigonometric functions. **P.F.GT.A.2** Determine the difference made by choice of units for angle measurement when graphing a trigonometric function. **P.F.GT.A.3** Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes. **P.F.GT.A.4** Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions. **P.F.GT.A.5** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **P.F.GT.A.6** Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions. **P.F.GT.A.7** Graph the inverse trigonometric functions and identify their key characteristics. **P.F.GT.A.8** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Geometry

Applied Trigonometry (G.AT)

Cluster Headings	Content Standards
A. Use trigonometry to solve problems. ★	**P.G.AT.A.1** Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles. **P.G.AT.A.2** Derive the formula $A = 1/2\ ab\ sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **P.G.AT.A.3** Derive and apply the formulas for the area of sector of a circle. **P.G.AT.A.4** Calculate the arc length of a circle subtended by a central angle.

Cluster Headings	Content Standards
A. Use trigonometry to solve problems. ★	**P.G.AT.A.5** Prove the Laws of Sines and Cosines and use them to solve problems. **P.G.AT.A.6** Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Trigonometric Identities (G.TI)

Cluster Headings	Content Standards
A. Apply trigonometric identities to rewrite expressions and solve equations.★	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. **P.G.TI.A.2** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Polar Coordinates (G.PC)

Cluster Headings	Content Standards
A. Use polar coordinates.	**P.G.PC.A.1** Graph functions in polar coordinates. **P.G.PC.A.2** Convert between rectangular and polar coordinates. **P.G.PC.A.3** Represent situations and solve problems involving polar coordinates.★

Statistics and Probability

Model with Data★ (S.MD)

Cluster Headings	Content Standards
A. Model data using regressions equations.	**P.S.MD.A.1** Create scatter plots, analyze patterns and describe relationships for bivariate data (linear, polynomial, trigonometric or exponential) to model real-world phenomena and to make predictions. **P.S.MD.A.2** Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data. **P.S.MD.A.3** Use a regression equation modeling bivariate data to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.

Statistics | S

Statistics is designed to introduce students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. The major themes in Statistics include: interpreting categorical and quantitative data, conditional probability and other rules of probability, using probability to make decisions, and making inferences and justifying conclusions.

Statistics includes the following domains and clusters:

- **Interpreting Categorical and Quantitative Data**
  - Understand, represent, and use univariate data.
  - Understand, represent, and use bivariate data.
- **Conditional Probability and the Rules of Probability**
  - Understand and apply basic concepts of probability.
  - Use the rules of probability to compare probabilities of compound events in a uniform probability model.
- **Using Probability to Make Decisions**
  - Understand and use discrete probability distributions.
  - Understand the normal probability distribution.
- **Making Inferences and Justifying Conclusions**
  - Know the characteristics of well-defined studies.
  - Design and conduct a statistical experiment to study a problem, then interpret and communicate the outcomes.
  - Make inferences about population parameters based on a random sample from that population.
  - Understand and use confidence intervals.
  - Use distributions to make inferences about a data set.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Exploring Data

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards
A. Understand, represent, and use univariate data.	**S.ID.A.1** Understand the term 'variable' and differentiate between the data types: measurement, categorical, univariate, and bivariate. **S.ID.A.2** Understand histograms, parallel box plots, and scatterplots, and use them to display and compare data. **S.ID.A.3** Summarize distributions of univariate data. **S.ID.A.4** Compute basic statistics and understand the distinction between a statistic and a parameter. **S.ID.A.5** For univariate measurement data, be able to display the distribution and describe its shape; select and calculate summary statistics. **S.ID.A.6** Recognize how linear transformations of univariate data affect shape, center, and spread. **S.ID.A.7** Analyze the effect of changing units on summary measures. **S.ID.A.8** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. **S.ID.A.9** Describe individual performances in terms of percentiles, z-scores, and t-scores.
B. Understand, represent, and use bivariate data.	**S.ID.B.10** Represent and analyze categorical data. **S.ID.B.11** Display and discuss bivariate data where at least one variable is categorical. **S.ID.B.12** For bivariate measurement data, be able to display a scatterplot and describe its shape; use technological tools to determine regression equations and correlation coefficients. **S.ID.B.13** Identify trends in bivariate data; find functions that model the data and that transform the data so that they can be modeled.

Probability

Conditional Probability and the Rules of Probability (S.CP)

Cluster Headings	Content Standards
A. Understand and apply basic concepts of probability.	**S.CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). **S.CP.A.2** Use permutations and combinations to compute probabilities of compound events and solve problems. **S.CP.A.3** Demonstrate an understanding of the Law of Large Numbers (Strong and Weak).
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**S.CP.B.4** Demonstrate an understanding of the addition rule, the multiplication rule, conditional probability, and independence. **S.CP.B.5** Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.

Probability Distributions

Using Probability to Make Decisions (S.MD)

Cluster Headings	Content Standards
A. Understand and use discrete probability distributions.	**S.MD.A.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **S.MD.A.2** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **S.MD.A.3** Design a simulation of random behavior and probability distributions (e.g., drawing by lots, using a random number generator and using the results to make fair decisions.) **S.MD.A.4** Analyze discrete random variables and their probability distributions, including binomial and geometric.

Cluster Headings	Content Standards
A. Understand and use discrete probability distributions.	**S.MD.A.5** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
	S.MD.A.6 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
	S.MD.A.7 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **a.** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **b.** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
	S.MD.A.8 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
B. Understand the normal probability distribution.	**S.MD.B.9** Calculate the mean (expected value) and standard deviation of both a random variable and a linear transformation of a random variable. **S.MD.B.10** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Sampling and Experimentation

Making Inferences and Justifying Conclusions (S.IC)

Cluster Headings	Content Standards
A. Know the characteristics of well-designed studies.	**S.IC.A.1** Understand the differences among various kinds of studies and which types of inferences can be legitimately drawn from each. **S.IC.A.2** Compare census, sample survey, experiment, and observational study. **S.IC.A.3** Describe the role of randomization in surveys and experiments. **S.IC.A.4** Describe the role of experimental control and its effect on confounding. **S.IC.A.5** Identify bias in sampling and determine ways to reduce it to improve results. **S.IC.A.6** Describe the sampling distribution of a statistic and define the standard error of a statistic. **S.IC.A.7** Demonstrate an understanding of the Central Limit Theorem.
B. Design and conduct a statistical experiment to study a problem, then interpret and communicate the outcomes.	**S.IC.B.8** Select a method to collect data and plan and conduct surveys and experiments. **S.IC.B.9** Compare and use sampling methods, including simple random sampling, stratified random sampling, and cluster sampling. **S.IC.B.10** Test hypotheses using appropriate statistics. **S.IC.B.11** Analyze results and make conclusions from observational studies, experiments, and surveys. **S.IC.B.12** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
C. Make inferences about population parameters based on a random sample from that population.	**S.IC.C.13** Develop and evaluate inferences and predictions that are based on data. **S.IC.C.14** Use properties of point estimators, including biased/unbiased, and variability.
D. Understand and use confidence intervals.	**S.IC.D.15** Understand the meaning of confidence level, of confidence intervals, and the properties of confidence intervals.

Cluster Headings	Content Standards
D. Understand and use confidence intervals.	**S.IC.D.16** Construct and interpret a large sample confidence interval for a proportion and for a difference between two proportions. **S.IC.D.17** Construct the confidence interval for a mean and for a difference between two means.
E. Use distributions to make inferences about a data set.	**S.IC.E.18** Apply the properties of a Chi-square distribution in appropriate situations in order to make inferences about a data set. **S.IC.E.19** Apply the properties of the normal distribution in appropriate situations in order to make inferences about a data set. **S.IC.E.20** Interpret the t-distribution and determine the appropriate degrees of freedom.

Applied Mathematical Concepts | AM

Applications and modeling using mathematics are the primary foci of this course.

Sample potential applications for topics are listed below:

Counting, Combinatorics, and Probability

- Counting hands of cards, code words, license plates, phone numbers, make-up of committees, etc.
- Probabilities associated with games (such as using games from *The Price is Right*)

Financial Math

- Amortization and loans (copayments, credit cards, loans, etc.)
- Compound interest; comparing payments, interest rates, length of loan period, investments, etc.
- Computing taxes
- Computing paychecks (deductions, social security payments, etc.)
- Comparing insurance plans (term vs. whole life)
- Annuities

Linear Programming

- Maximizing capacity while minimizing costs

Applied Mathematical Concepts includes the following domains and clusters:

- **Financial Mathematics**
  - Use financial mathematics to solve problems.
  - Use financial mathematics to make decisions.
  - Determine appropriate models to solve contextual problems.
- **Linear Programming**
  - Use linear programming techniques to solve real-world problems.
  - Solve real-world optimization problems.
- **Logic and Boolean Algebra**
  - Use logic and Boolean Algebra in real-world situations.
  - Apply Boolean Algebra to real-world network problems.
- **Problem Solving**
  - Apply problem solving techniques to real-world problems.
- **Investigate Logic**
  - Use logic to make arguments and solve problems.
  - Determine the validity of arguments.
- **Organize and Interpret Data**
  - Analyze data from multiple viewpoints and perspectives.
- **Counting and Combinatorial Reasoning**
  - Apply probability and counting principles to real-world situations.
  - Use combinatorial reasoning to solve real-world problems.
- **Normal Probability Distribution**
  - Work with the normal distribution in real-world situations.
- **Understand and Use Confidence Intervals**
  - Work with confidence intervals in real-world situations.

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Number and Quantity

Financial Mathematics (N.NQ)

Cluster Headings	Content Standards
A. Use financial mathematics to solve problems.	**AM.N.NQ.A.1** Define interest, compound interest, annuities, sinking funds, amortizations, annuities, future value, and present value. **AM.N.NQ.A.2** Recognize the importance of applying a financial model to business. **AM.N.NQ.A.3** Determine future value and present value of an annuity. **AM.N.NQ.A.4** Determine the amortization schedule for an annuity and a home mortgage.
B. Use financial mathematics to make decisions.	**AM.N.NQ.B.5** Apply financial mathematics to depreciation schedules. **AM.N.NQ.B.6** Solve contextual problems involving financial decision-making. **AM.N.NQ.B.7** Apply arithmetic and geometric sequences to simple and compound interest, annuities, loans, and amortization. **AM.N.NQ.B.8** Solve problems in mathematics of finance involving compound interest using exponential and logarithmic techniques.
C. Determine appropriate models to solve contextual problems.	**AM.N.NQ.C.9** Know when to use transcendental functions to accomplish various application purposes such as predicting population growth. **AM.N.NQ.C.10** Use orders of magnitude estimates for determining an appropriate model for a contextual situation.

Algebra

Linear Programming (A.LP)

Cluster Headings	Content Standards
A. Use linear programming techniques to solve real-world problems.	**AM.A.LP.A.1** Use mathematical models involving equations and systems of equations to represent, interpret, and analyze quantitative relationships, change in various contexts, and other real-world phenomena. **AM.A.LP.A.2** Read, interpret, and solve linear programming problems graphically and by computational methods.

Cluster Headings	Content Standards
B. Solve real-world optimization problems.	**AM.A.LP.B.3** Use linear programming to solve optimization problems. **AM.A.LP.B.4** Interpret the meaning of the maximum or minimum value in terms of the objective function.

Logic and Boolean Algebra (A.LB)

Cluster Headings	Content Standards
A. Use logic and Boolean Algebra in real-world situations.	**AM.A.LB.A.1** Develop the symbols and properties of Boolean algebra; connect Boolean algebra to standard logic. **AM.A.LB.A.2** Construct truth tables to determine the validity of an argument.
B. Apply Boolean Algebra to real-world network problems.	**AM.A.LB.B.3** Analyze basic electrical networks; compare the networks to Boolean Algebra configurations. **AM.A.LB.B.4** Develop electrical networks and translate them into Boolean Algebra equations.

Problem Solving (A.PS)

Cluster Headings	Content Standards
A. Apply problem solving techniques to real-world situations.	**AM.A.PS.A.1** Apply problem solving strategies to real-world situations. *Strategies include, but are not limited to: making orderly lists or tables, drawing diagrams, considering simpler problems, looking for patterns, working backwards, guess and check, using logical reasoning, etc.*

Geometry and Measurement

Investigate Logic (G.L)

Cluster Headings	Content Standards
A. Use logic to make arguments and solve problems.	**AM.G.L.A.1** Define the order of operations for the logical operators. **AM.G.L.A.2** Define conjunction, disjunction, negation, conditional, and biconditional. **AM.G.L.A.3** Solve a variety of logic puzzles.

Cluster Headings	Content Standards
A. Use logic to make arguments and solve problems.	**AM.G.L.A.4** Construct and use a truth table to draw conclusions about a statement.
B. Determine the validity of arguments.	**AM.G.L.B.5** Apply the laws of logic to judge the validity of arguments. **AM.G.L.B.6** Give counterexamples to disprove statements. **AM.G.L.B.7** Analyze arguments with quantifiers through the use of Venn diagrams. **AM.G.L.B.8** Represent logical statements with networks.

Data Analysis, Statistics, and Probability

Organize and Interpret Data (D.ID)

Cluster Headings	Content Standards
A. Analyze data from multiple viewpoints and perspectives.	**AM.D.ID.A.1** Organize data for problem solving. **AM.D.ID.A.2** Use a variety of counting methods to organize information, determine probabilities, and solve problems. **AM.D.ID.A.3** Translate from one representation of data to another, e.g., a bar graph to a circle graph. **AM.D.ID.A.4** Calculate and interpret statistical problems using measures of central tendency and graphs. **AM.D.ID.A.5** Calculate expected value, e.g., to determine the fair price of an investment. **AM.D.ID.A.6** Analyze survey data using Venn diagrams. **AM.D.ID.A.7** Evaluate and compare two investments or strategies, where one investment or strategy is safer but has lower expected value. Include large and small investments and situations with serious consequences.

Counting and Combinatorial Reasoning (D.CR)

Cluster Headings	Content Standards
A. Apply probability and counting principles to real-world situations.	**AM.D.CR.A.1** Use permutations, combinations, and the multiplication principle to solve counting problems. **AM.D.CR.A.2** Design and interpret simple experiments using tree-diagrams, permutations, and combinations. **AM.D.CR.A.3** Apply counting principles to probabilistic situations involving equally likely outcomes. **AM.D.CR.A.4** Solve counting problems by using Venn diagrams and show relationships modeled by the Venn diagram. **AM.D.CR.A.5** Use permutations and combinations to compute probabilities of compound events and solve problems.
B. Use combinatorial reasoning to solve real-world problems.	**AM.D.CR.B.6** Apply the Law of Large numbers to contextual situations.
	AM.D.CR.B.7 Discuss the various examples and consequences of innumeracy; consider poor estimation, improper experimental design, inappropriate comparisons, and scientific notation comparisons.
	AM.D.CR.B.8 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **a.** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **b.** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
	AM.D.CR.B.9 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **AM.D.CR.B.10** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Normal Probability Distribution (D.ND)

Cluster Headings	Content Standards
A. Work with the normal distribution in real-world situations.	**AM.D.ND.A.1** Calculate the mean (expected value) and standard deviation of both a random variable and a linear transformation of a random variable.

Cluster Headings	Content Standards
A. Work with the normal distribution in real-world situations.	**AM.D.ND.A.2** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Understand and Use Confidence Intervals (D.CI)

Cluster Headings	Content Standards
A. Work with confidence intervals in real-world situations.	**AM.D.CI.A.1** Understand the meaning of confidence level, of confidence intervals, and the properties of confidence intervals. **AM.D.CI.A.2** Construct and interpret a large sample confidence interval for a proportion and for a difference between two proportions. **AM.D.CI.A.3** Construct the confidence interval for a mean and for a difference between two means.

Calculus | C

Calculus is designed for students interested in STEM-based careers and builds on the concepts studied in precalculus. The study of calculus on the high school level includes a study of limits, derivatives, and an introduction to integrals.

Calculus includes the following domains and clusters:

- **Limits of Functions**
  - Understand the concept of the limit of a function.
- **Behavior of Functions**
  - Describe the asymptotic and unbounded behavior of functions.
- **Continuity**
  - Develop an understanding of continuity as a property of functions.
- **Understand the Concept of the Derivative**
  - Demonstrate an understanding of the derivative.
  - Understand the derivative at a point.
- **Computing and Applying Derivatives**
  - Apply differentiation techniques.
  - Use first and second derivatives to analyze a function.
  - Apply derivatives to solve problems.
- **Understanding Integrals**
  - Demonstrate understanding of a definite integral.
  - Understand and apply the Fundamental Theorem of Calculus.
- **Calculate and Apply Integrals**
  - Apply techniques of antidifferentiation.
  - Apply integrals to solve problems.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Functions, Graphs, and Limits

Limits of Functions (F.LF)

Cluster Headings	Content Standards
A. Understand the concept of the limit of a function.	**C.F.LF.A.1** Calculate limits (including limits at infinity) using algebra. **C.F.LF.A.2** Estimate limits of functions (including one-sided limits) from graphs or tables of data. Apply the definition of a limit to a variety of functions, including piece-wise functions. **C.F.LF.A.3** Draw a sketch that illustrates the definition of the limit; develop multiple real-world scenarios that illustrate the definition of the limit.

Behavior of Functions (F.BF)

Cluster Headings	Content Standards
A. Describe the asymptotic and unbounded behavior of functions.	**C.F.BF.A.1** Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. **C.F.BF.A.2** Discuss the various types of end behavior of functions; identify prototypical functions for each type of end behavior.

Continuity (F.C)

Cluster Headings	Content Standards
A. Develop an understanding of continuity as a property of functions	**C.F.C.A.1** Define continuity at a point using limits; define continuous functions. **C.F.C.A.2** Determine whether a given function is continuous at a specific point. **C.F.C.A.3** Determine and define different types of discontinuity (point, jump, infinite) in terms of limits. **C.F.C.A.4** Apply the Intermediate Value Theorem and Extreme Value Theorem to continuous functions.

Derivatives

Understand the Concept of the Derivative (D.CD)

Cluster Headings	Content Standards
A. Demonstrate an understanding of the derivative.	**C.D.CD.A.1** Represent and interpret the derivative of a function graphically, numerically, and analytically. **C.D.CD.A.2** Interpret the derivative as an instantaneous rate of change. **C.D.CD.A.3** Define the derivative as the limit of the difference quotient; illustrate with the sketch of a graph. **C.D.CD.A.4** Demonstrate the relationship between differentiability and continuity.
B. Understand the derivative at a point.	**C.D.CD.B.5** Interpret the derivative as the slope of a curve (which could be a line) at a point, including points at which there are vertical tangents and points at which there are no tangents (i.e., where a function is not locally linear). **C.D.CD.B.6** Approximate both the instantaneous rate of change and the average rate of change given a graph or table of values. **C.D.CD.B.7** Write the equation of the line tangent to a curve at a given point. **C.D.CD.B.8** Apply the Mean Value Theorem. **C.D.CD.B.9** Understand Rolle's Theorem as a special case of the Mean Value Theorem.

Computing and Applying Derivatives (D.AD)

Cluster Headings	Content Standards
A. Apply differentiation techniques.	**C.D.AD.A.1** Describe in detail how the basic derivative rules are used to differentiate a function; discuss the difference between using the limit definition of the derivative and using the derivative rules. **C.D.AD.A.2** Calculate the derivative of basic functions (power, exponential, logarithmic, and trigonometric). **C.D.AD.A.3** Calculate the derivatives of sums, products, and quotients of basic functions. **C.D.AD.A.4** Apply the chain rule to find the derivative of a composite function.

Cluster Headings	Content Standards
A. Apply differentiation techniques.	**C.D.AD.A.5** Implicitly differentiate an equation in two or more variables. **C.D.AD.A.6** Use implicit differentiation to find the derivative of the inverse of a function.
B. Use first and second derivatives to analyze a function.	**C.D.AD.B.7** Relate the increasing and decreasing behavior of *f* to the sign of *f'* both analytically and graphically. **C.D.AD.B.8** Use the first derivative to find extrema (local and global). **C.D.AD.B.9** Analytically locate the intervals on which a function is increasing, decreasing, or neither. **C.D.AD.B.10** Relate the concavity of *f* to the sign of *f"* both analytically and graphically. **C.D.AD.B.11** Use the second derivative to find points of inflection as points where concavity changes. **C.D.AD.B.12** Analytically locate intervals on which a function is concave up, concave down, or neither. **C.D.AD.B.13** Relate corresponding characteristics of the graphs of *f*, *f'*, and *f"*. **C.D.AD.B.14** Translate verbal descriptions into equations involving derivatives and vice versa.
C. Apply derivatives to solve problems.	**C.D.AD.C.15** Model rates of change, including related rates problems. In each case, include a discussion of units. **C.D.AD.C.16** Solve optimization problems to find a desired maximum or minimum value. **C.D.AD.C.17** Use differentiation to solve problems involving velocity, speed, and acceleration. **C.D.AD.C.18** Use tangent lines to approximate function values and changes in function values when inputs change (linearization).

Integrals

Understanding Integrals (I.UI)

Cluster Headings	Content Standards
A. Demonstrate understanding of a definite integral.	**C.I.UI.A.1** Define the definite integral as the limit of Riemann sums and as the net accumulation of change. **C.I.UI.A.2** Correctly write a Riemann sum that represents the definition of a definite integral. **C.I.UI.A.3** Use Riemann sums (left, right, and midpoint evaluation points) and trapezoid sums to approximate definite integrals of functions represented graphically, numerically, and by tables of values.
B. Understand and apply the Fundamental Theorem of Calculus.	**C.I.UI.B.4** Recognize differentiation and antidifferentiation as inverse operations. **C.I.UI.B.5** Evaluate definite integrals using the Fundamental Theorem of Calculus. **C.I.UI.B.6** Use the Fundamental Theorem of Calculus to represent a particular antiderivative of a function and to understand when the antiderivative so represented is continuous and differentiable. **C.I.UI.B.7** Apply basic properties of definite integrals (e.g. additive, constant multiple, translations).

Calculate and Apply Integrals (I.AI)

Cluster Headings	Content Standards
A. Apply techniques of antidifferentiation.	**C.I.AI.A.1** Develop facility with finding antiderivatives that follow directly from derivatives of basic functions (power, exponential, logarithmic, and trigonometric). **C.I.AI.A.2** Use substitution of variables to calculate antiderivatives (including changing limits for definite integrals). **C.I.AI.A.3** Find specific antiderivatives using initial conditions.

Cluster Headings	Content Standards
B. Apply integrals to solve problems.	**C.I.AI.B.4** Use a definite integral to find the area of a region. **C.I.AI.B.5** Use a definite integral to find the volume of a solid formed by rotating a region around a given axis. **C.I.AI.B.6** Use integrals to solve a variety of problems (e.g., distance traveled by a particle along a line, exponential growth/decay).