<u>Advanced Algebra and Trigonometry</u>

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Number and Quantity

Domain	Cluster	Standard
Number Expressions (N-NE)	**Represent, interpret, compare, and simplify number expressions.**	1. Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them. 2. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.* 3. Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of pi and *e*. 4. Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not. 5. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Complex numbers (N-CN)	**Perform complex number arithmetic and understand the representation on the complex plane.**	1. Perform arithmetic operations with complex numbers expressing answers in the form a+bi. 2. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. 3. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. 4. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1+\sqrt{3}i)^3 = 8$ *because* $(-1+\sqrt{3}i)$ *has modulus* 2 *and argument* 120°. 5. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
	Use complex numbers in polynomial identities and equations.	6. Extend polynomial identities to the complex numbers. *For example, rewrite* x^2+4 *as* $(x+2i)(x-2i)$. 7. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

	Domain	Cluster	Standard
Number and Quantity	Vector and Matrix Quantities (N-VM)	Represent and model with vector quantities.	1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*). 2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. 3. Solve problems involving velocity and other quantities that can be represented by vectors.
		Understand the graphic representation of vectors and vector arithmetic.	4. Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. 5. Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple *c**v*** using ||*c**v***|| = |*c*|*v*. Compute the direction of *c**v*** knowing that when |*c*|*v* ≠ 0, the direction of *c**v*** is either along ***v*** (for *c* > 0) or against ***v*** (for *c* < 0). 6. Calculate and interpret the dot product of two vectors.
		Perform operations on matrices and use matrices in applications.	7. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. 8. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. 9. Add, subtract, and multiply matrices of appropriate dimensions. 10. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. 11. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. 12. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. 13. Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

	Domain	Cluster	Standard
Algebra	Sequences and Series (A-S)	Understand and use sequences and series.	1. Demonstrate an understanding of sequences by representing them recursively and explicitly. 2. Use sigma notation to represent a series; expand and collect expressions in both finite and ininite settings. 3. Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist. a. Determine whether a given arithmetic or geometric series converges or diverges. b. Find the sum of a given geometric series (both infinite and finite). c. Find the sum of a finite arithmetic series. 4. Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples. 5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
	Solve Equations and Inequalities (A-REI)	Solve systems of equations and nonlinear inequalities.	1. Represent a system of linear equations as a single matrix equation in a vector variable. 2. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). 3. Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology. 4. Solve systems of nonlinear inequalities by graphing.
	Conic Sections (A-C)	Understand the properties of conic sections and apply them to model real-world phenomena.*	1. Display all of the conic sections as portions of a cone. 2. From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics. 3. Transform equations of conic sections to convert between general and standard form.

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Functions	Building Functions (F-BF)	Build new functions from existing functions.	1. Understand how the algebraic properties of an equation transform the geometric properties of its graph. *For example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation (i.e., translations, stretches, and changes in periodicity and amplitude).* 2. Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions. 3. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* 4. Construct the difference quotient for a given function and simplify the resulting expression. 5. Find inverse functions (including exponential and logarithmic). a. Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, $\frac{1}{f(x)}$, and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain. Recognize a function is invertible if and only if it is one-to-one. 6. Explain why the graph of a function and its inverse are reflections of one another over the line y=x.
	Interpreting Functions (F-IF)	Analyze Functions using different representations.	1. Determine whether a function is even, odd, or neither. 2. Identify or analyze the distinguishing properties of exponential, polynomial, logarithmic, trigonometric, and rational functions from tables, graphs, and equations. 3. Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (polynomial, rational, exponential, logarithmic, and trigonometric). 4. Identify characteristics of graphs based on a set of conditions or on a general equation such as $y= ax^2+ c$. 5. Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals of concavity and increasing and decreasing. 6. Graph rational functions, identifying zeros, asymptotes (including slant), and holes when suitable factorizations are available, and showing end-behavior. 7. Solve real world problems that can be modeled using quadratic, exponential, or logarithmic functions★ (by hand and with appropriate technology).
	Trigonometric Functions (F-TF)	Extend the domain of trigonometric functions using the unit circle.	1. Convert from radians to degrees and from degrees to radians. 2. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number. 3. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
	Graphing Trigonometric Functions (G-GT)	Model periodic phenomena with trigonometric functions.★	1. Interpret transformations of trigonometric functions. 2. Match a trigonometric equation with its graph. 3. Determine the difference made by choice of units for angle measurement when graphing a trigonometric function. 4. Graph the sine, cosine, and tangent functions and identify characteristics such as period, amplitude, phase shift, and asymptotes.

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Geometry	Applied Trigonometry (G-AT)	Use trigonometry to solve problems.★	1. Use the definitions of the basic trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles. 2. Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. 3. Derive and apply the formulas for the area of sector of a circle. 4. Calculate the arc length of a circle subtended by a central angle. 5. Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Geometry	Trigonometric Identities (G-TI)	Apply trigonometric identities to rewrite expressions and solve equations.★	1. Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, quotient, sum/difference, double-angle, and half-angle.
Statistics and Probability	Model with Data★ (S-MD)	Model data using regressions equations.	1. Create scatter plots, analyze patterns and describe relationships for bivariate data (linear, polynomial, trigonometric or exponential) to model real-world phenomena and to make predictions. 2. Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data. 3. Use a regression equation modeling bivariate data to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.

Bridge Math

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Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

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Category	Domain	Standard
Ways of Looking: Revisiting Concepts	Diagrammatic Mathematics (W-DM)	1. Identify the graph of a linear inequality on the number line. 2. Create and use absolute value functions to model and solve problems in common settings. 3. Given an equation of a line, write an accurate definition of a line by determining the unique characteristics that define it (i.e. slope and intercepts). 4. Compute the perimeter of simple composite geometric figures with unknown side lengths. 5. Apply a variety of strategies to determine the circumference and the area for circles. 6. Investigate the area of a sector and the arc length of a circle.
	Verbal Mathematics (W-VM)	1. Understand that a line parallel to one side of a triangle divides the other two sides proportionally, and conversely. 2. Apply similar triangles to solve problems, such as finding heights and distances. 3. Use several angle properties to find an unknown angle measure (i.e. supplementary, complementary, vertical, angles along a transversal, and sum of angles in a polygon). 4. Describe, compare, and contrast plane and solid figures using their attributes. 5. Multiply, divide and simplify radicals. 6. Use mathematical grammar and appropriate mathematical symbols to represent contextual situations.
	Symbolic Mathematics (W-SM)	1. Operate with numbers expressed in scientific notation. 2. Develop a thorough understanding of both rational and irrational numbers; make both historical and concrete connections between irrational numbers and the real world. 3. Use mathematical symbols and variables to express a relationship between quantities. 4. Model a variety of problem situations with expressions. 5. Skillfully manipulate formulas involving exponents. 6. Understand how mathematical properties yield equivalent equations and can be used in determining if two expressions are equivalent. 7. Perform polynomial arithmetic, including adding, subtracting, multiplying, dividing, factoring, and simplifying results. 8. Demonstrate fluency with techniques needed to simplify radical expressions and calculate with them, including addition, subtraction, and multiplication. 9. Rationalize denominators in order to perform division with radicals.
	Graphic Mathematics (W-GM)	1. Understand that a linear function models a situation in which a quantity changes at a constant rate, m, relative to another. 2. Graph quadratic equations and identify key characteristics of the graph. 3. Find the solution of a quadratic equation and/or zeros of a quadratic function. 4. Operate (add, subtract, multiply, and divide) with and evaluate rational expressions. 5. Operate (add, subtract, multiply, divide, simplify, powers) with radicals and radical expressions including radicands involving rational numbers and algebraic expressions. 6. Identify and calculate the measures of central tendency and spread in a set of data. 7. Understand the correlation coefficient and its role in measuring the goodness of fit for a model for a data set. 8. Analyze data to make predictions based on an understanding of the data set, for example, use a scatter plot to determine if a linear relationship exists and describe the association between the variables. 9. Use algebra and geometry to solve problems involving midpoints and distances (i.e. geometric figures).
	Numeric Mathematics (W-NM)	1. Understand that there are numbers that are not rational numbers, called irrational numbers, e.g., π, e, and $\sqrt{2}$, which together with the rational numbers form the real number system that satisfies the laws of arithmetic. 2. Apply and use elementary number concepts and number properties to model and solve non-routine problems that involve new ideas. 3. Determine if a data set represents a line through numerically analyzing slope calculations. If appropriate, interpret the slope in terms of a rate. 4. Find the probability of simple events, disjoint events, compound events, and independent events in various settings using a variety of counting techniques. 5. Develop fluency with the basic operations of complex numbers.

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Making Connections	Symbolic & Diagrammatic Mathematics (M-SD)	1. Use the laws of exponents to simplify and interpret expressions for exponential functions, recognizing positive rational exponents as indicating roots of the base and negative exponents as indicating the reciprocal of a power. 2. Solve a linear inequality and provide an interpretation of the solution. 3. Recognize special products and factors of polynomials to facilitate problem solving with polynomials; in particular, find the zeros of a quadratic polynomial function. 4. Investigate the properties of plane figures, developing precise mathematical descriptions of geometric shapes, both in the plane and in space. 5. Apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures in composite figures.
	Symbolic & Verbal Mathematics (M-SV)	1. Explain, solve, and/or draw conclusions for complex problems using relationships and elementary number concepts. 2. Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions. 3. Write ratios, proportions, and solve proportions in a contextual setting for an unknown value. 4. Solve literal equations for any variable; interpret the results based on units.
	Symbolic & Numeric Mathematics (M-SN)	1. In the context of exponential models, solve equations of the form $a \bullet b^{ct} = d$ where a, c, and d are specific numbers and the base b is 2, 10, or *e*. 2. Use the rules of exponents to develop an understanding of the difference between the rational and real numbers. 3. Recognize functions as mappings of an independent variable into a dependent variable. 4. Evaluate polynomial and exponential functions that use function notation. 5. Recognize composite functions as an application of substitution and use this understanding to write expressions for and evaluate composite functions.
	Symbolic & Graphic Mathematics (M-SG)	1. Graphically represent the solution to a linear equation and the solution to a system of linear equations in two variables. 2. Graphically represent the solution to a linear inequality and the solution to a system of linear inequalities in two variables. 3. Relate the basic definitions of the trigonometric ratios to the right triangle. 4. Identify the graphs of basic trigonometric functions and shifts of those graphs. 5. Solve a simple system consisting of one linear equation and one quadratic equation in two variables; for example, find points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. Illustrate the solution graphically.
	Numeric & Graphic Mathematics (M-NG)	1. Given a variety of appropriate information, determine the equation of a line. 2. Use appropriate technology to generate the equation of a line from a set of data and if appropriate, use it to make a prediction. 3. Use appropriate technology to find the mathematical model for a set of non-linear data. 4. Compare measures of central tendency and spread for a single data set along with its graph and summary statistics. 5. Compare data sets using graphs and summary statistics, and measures of central tendency and spread. 6. Examine radical and rational functions, both graphically and numerically, to determine restrictions on the domain. 7. Apply special right-triangle properties and the Pythagorean Theorem to solve congruent, similar shape, and contextual problems.
	Numeric & Diagrammatic Mathematics (M-ND)	1. Understand and use basic counting techniques in contextual settings. 2. Use counting techniques to calculate probabilities for conditional and independent events. 3. Compare a theoretical probability model to the outcomes of a probability experiment for the same process.

Category	Domain	Standard
Applications: Ways of Looking at the World	Applications with Numbers (A-AN)	1. Solve problems using scientific notation. 2. Solve problems involving percent of increase or decrease, for example mark-ups and mark-downs. 3. Solve rate, distance, and work problems using proportions and percentages. 4. Solve problems involving evaluation of exponential functions, for example applications involving simple and compound interest.
	Applications with Geometry (A-AG)	1. Solve problems involving ratios in geometric settings, such as similar figures and right triangle distance problems. 2. Solve problems involving finding missing dimensions given area or perimeter of the figure. 3. Solve problems involving surface areas and volumes of 3-dimensional figures, including maximization, scale, and increment problems. 4. Solve problems involving angles of elevation and angles of declination. 5. Solve problems requiring the interpretation of polynomial, rational, and exponential graphs that depict real-world phenomena, including identification of max/min and end behavior of functions.
	Applications with Functions (A-AF)	1. Solve problems involving applications of linear equations. 2. Solve problems involving direct and inverse variations, such as frequency, interest, and pressure. 3. Solve problems involving systems of equations such as mixture problems. 4. Solve problems involving quadratic equations such as area and gravity; additionally examine the fact that quadratic functions have maximum or minimum values and can be used to model problems with optimum solutions. 5. Solve problems involving radical equations, such as wind chill and body mass index. 6. Solve problems involving rational equations such as work problems. 7. Solve problems involving exponential applications such as half-life and continuous interest.
	Applications with Data (A-AD)	1. Solve problems involving constructing and interpreting pie charts. 2. Solve problems that use the construction and interpretation of Venn diagrams to analyze the attributes of a set of data, for example logic and counting problems. 3. Solve problems involving geometric probabilities.

Calculus

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1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

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5. Use appropriate tools strategically.

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	Domain	Cluster	Standards
Functions, Graphs, and Limits	Limits of Functions (F-LF)	Understand the concept of the limit of a function.	1. Calculate limits (including limits at infinity) using algebra. 2. Estimate limits of functions (including one-sided limits) from graphs or tables of data. Apply the definition of a limit to a variety of functions, including piece-wise functions. 3. Draw a sketch that illustrates the definition of the limit; develop multiple real world scenarios that illustrate the definition of the limit.
	Behavior of Functions (F-BF)	Describe the asymptotic and unbounded behavior of functions.	1. Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. 2. Discuss the various types of end behavior of functions; identify prototypical functions for each type of end behavior.
	Continuity (F-C)	Develop an understanding of continuity as a property of functions	1. Define continuity at a point using limits; define continuous functions. 2. Determine whether a given function is continuous at a specific point. 3. Determine and define different types of discontinuity (point, jump, infinite) in terms of limits. 4. Apply the Intermediate Value Theorem and Extreme Value Theorem to continuous functions.

Derivatives

Domain	Cluster	Standards
Understand the Concept of the Derivative (D-CD)	Demonstrate an understanding of the derivative	1. Represent and interpret the derivative of a function graphically, numerically, and analytically. 2. Interpret the derivative as an instantaneous rate of change. 3. Define the derivative as the limit of the difference quotient; illustrate with the sketch of a graph. 4. Demonstrate the relationship between differentiability and continuity.
	Understand the derivative at a point	1. Interpret the derivative as the slope of a curve (which could be a line) at a point, including points at which there are vertical tangents and points at which there are no tangents (i.e., where a function is not locally linear). 2. Approximate both the instantaneous rate of change and the average rate of change given a graph or table of values. 3. Write the equation of the line tangent to a curve at a given point. 4. Apply the Mean Value Theorem. 5. Understand Rolle's Theorem as a special case of the Mean Value Theorem.
Computing and Applying Derivatives (D-AD)	Apply differentiation techniques	1. Describe in detail how the basic derivative rules are used to differentiate a function; discuss the difference between using the limit definition of the derivative and using the derivative rules. 2. Calculate the derivative of basic functions (power, exponential, logarithmic, and trigonometric). 3. Calculate the derivatives of sums, products, and quotients of basic functions. 4. Apply the chain rule to find the derivative of a composite function. 5. Implicitly differentiate an equation in two or more variables. 6. Use implicit differentiation to find the derivative of the inverse of a function.
	Use first and second derivatives to analyze a function	1. Relate the increasing and decreasing behavior of f to the sign of f' both analytically and graphically. 2. Use the first derivative to find extrema (local and global). 3. Analytically locate the intervals on which a function is increasing, decreasing or neither. 4. Relate the concavity of f to the sign of f'' both analytically and graphically. 5. Use the second derivative to find points of inflection as points where concavity changes. 6. Analytically locate intervals on which a function is concave up, concave down or neither. 7. Relate corresponding characteristics of the graphs of f, f', and f''. 8. Translate verbal descriptions into equations involving derivatives and vice versa.
	Apply derivatives to solve problems	1. Model rates of change, including related rates problems. In each case, include a discussion of units. 2. Solve optimization problems to find a desired maximum or minimum value. 3. Use differentiation to solve problems involving velocity, speed, and acceleration. 4. Use tangent lines to approximate function values and changes in function values when inputs change (linearization).

	Domain	Cluster	Standards
Integrals	Understanding Integrals (I-UI)	Demonstrate understanding of a Definite Integral	1. Define the definite integral as the limit of Riemann sums and as the net accumulation of change. 2. Correctly write a Riemann sum that represents the definition of a definite integral. 3. Use Riemann sums (left, right, and midpoint evaluation points) and trapezoid sums to approximate definite integrals of functions represented graphically, numerically, and by tables of values.
		Understand and apply the fundamental Theorem of Calculus	1. Recognize differentiation and antidifferentiation as inverse operations. 2. Evaluate definite integrals using the Fundamental Theorem of Calculus. 3. Use the Fundamental Theorem of Calculus to represent a particular antiderivative of a function and to understand when the antiderivative so represented is continuous and differentiable. 4. Apply basic properties of definite integrals (e.g. additive, constant multiple, translations).
	Calculate and Apply Integrals (I-AI)	Apply techniques of antidifferentiation	1. Develop facility with finding antiderivatives that follow directly from derivatives of basic functions (power, exponential, logarithmic, and trigonometric). 2. Use substitution of variables to calculate antiderivatives (including changing limits for definite integrals). 3. Find specific antiderivatives using initial conditions.
		Apply integrals to solve problems	1. Use a definite integral to find the area of a region. 2. Use a definite integral to find the volume of a solid formed by rotating a region around a given axis. 3. Use integrals to solve a variety of problems (e.g., distance traveled by a particle along a line, exponential growth/decay).

Discrete Math

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Number and Quantity	Election theory (N-ET)	1. Use election theory techniques to analyze election data. 2. Investigate and describe weighted voting and the results of various election methods. Both very standard and less well-known techniques will be studied and compared; these may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners. 3. Use fair division techniques to solve apportionment problems.
	Number bases (N-NB)	1. Understand various bases as used in computer science and numerical data transmission, especially base 2, base 8, and base 12. 2. Expand the understanding of place value to include numbers written in various numerical systems and in various bases. 3. Use base 2 arithmetic to understand checksums in data transmission. 4. Convert numbers between bases, especially multi-digit numbers. 5. Compare ancient numeral systems in various bases to base 10 and base 8 numerals. 6. Perform familiar arithmetic processes in base 2, base 8, and base 12.
Algebra	Binomial theorem and Pascal's Triangle (A-BP)	1. Use the binomial theorem to expand powers of binomials 2. Build the binomial theorem using graphics/pyramid design; interpret it for both a multivariable binomial expansion and a variable and numeric binomial expansion. 3. Represent, apply, and describe relationships among the binomial theorem, Pascal's triangle, and combinations. 4. Construct and describe patterns in Pascal's triangle.
	Logic and Boolean algebra (A-LB)	1. Develop the symbols and properties of Boolean algebra; connect Boolean algebra to standard logic. 2. Construct truth tables to determine the validity of an argument. 3. Analyze basic electrical networks; compare the networks to Boolean Algebra configurations. 4. Develop electrical networks and translate them into Boolean Algebra equations.
Functions	Iterative and recursive functions (F-IR)	1. Represent and analyze functions by using iteration and recursion. 2. Use iteration and recursion to model and solve problems. 3. Use iterative and recursive thinking to solve a variety of contextual problems. 4. Create and analyze iterative geometric patterns, including fractals. 5. Describe, analyze, and create iterative procedures and recursive formulas by using technology such as computer software and graphing calculators.

Category	Domain	Standard
Geometry	Estimation (G-E)	1. Put numbers in perspective through estimation, comparison, and scaling. 2. Apply estimation techniques in solving Fermi-type problems. 3. Apply estimation techniques to data given in a variety of ways.
	Graph theory (G-GT)	1. Use graph theory to model and solve contextual problems. 2. Use vertex-edge graphs to model and solve a variety of problems related to paths, circuits, networks, and relationships among a finite number of objects. 3. Apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. 4. State a variety of map color programs. Develop solutions to examples of maps and discuss the final resolution of the four-color problem. 5. Discuss the different interpretations of the four-color problem and the validity of a computer proof. 6. Interpret the Binomial Theorem to solve coloring problems and numerical problems.
Data Analysis, Statistics and Probability	Permutations and combinations (D-PC)	1. Represent, analyze, and apply permutations and combinations. 2. Design and interpret simple experiments using tree-diagrams, permutations, and combinations. 3. Use reasoning and formulas to solve counting problems in which repetition is not allowed and in which ordering does not matter.
	Counting and combinatorial reasoning (D-CR)	1. Apply counting principles to probabilistic situations involving equally likely outcomes. 2. Solve counting problems by using Venn diagrams and show relationships modeled by the Venn diagram. 3. Apply the Law of Large numbers to contextual situations. 4. Recognize the difference between continuous and discrete situations. 5. Apply appropriate counting techniques in discrete situations. 6. Derive basic combinatorics identities by counting the same sets two different ways to get a basic identity. 7. Use combinatorial reasoning to construct proofs as well as solve a variety of problems. 8. Informally prove the classical identity $C(n,k)=C(n-1,k-1)+C(n-1,k)$ for integers n and k with $0<k<n$. 9. Connect Pascal's triangle and probability to solve problems. 10. Discuss the various examples and consequences of innumeracy; consider poor estimation, improper experimental design, inappropriate comparisons, and scientific notation comparisons. 11. Prove the sum of the first n integers adds up to n(n+1)/2 in three different manners.

Senior Finite Math

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Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Category	Domain	Standards
Number and Quantity	Matrix operations (N-M)	1. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. 2. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. 3. Add, subtract, and multiply matrices of appropriate dimensions.
	Financial mathematics (N-NQ)	1. Define interest, compound interest, annuities, sinking funds, amortizations, annuities, future value and present value. 2. Recognize the importance of applying a financial model to business. 3. Determine future value and present value of an annuity. 4. Determine the amortization schedule for an annuity and a home mortgage. 5. Apply financial mathematics to depreciation schedules. 6. Solve contextual problems involving financial decision-making. 7. Apply arithmetic and geometric sequences to simple and compound interest, annuities, loans, and amortization. 8. Solve problems in mathematics of finance involving compound interest using exponential and logarithmic techniques. 9. Know when to use transcendental functions to accomplish various application purposes such as predicting population growth. 10. Use orders of magnitude estimates for determining an appropriate model for a contextual situation.
Algebra	Exponential and logarithmic expressions (A-EL)	1. Define logarithms. 2. Use properties of logarithms to expand and condense logarithmic expressions. 3. Understand the relationships between exponential and logarithmic expressions. 4. Use exponential and logarithmic relationships to model, predict, and solve contextual problems.
	Linear systems, matrices and their applications (A-LM)	1. Find and use the inverse of a matrix to solve a contextual problem. 2. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3x3 or greater). 3. Use matrices to solve systems of linear equations, including the echelon method and the Gauss-Jordan method. 4. Recognize the existence of infinite solutions to systems of linear equations; express these solutions in parametric form. 5. Identify and write the general solution to a system of linear equations; in the case of an infinite solution set, select various particular solutions given specific properties. 6. Use Markov Chains to solve problems.
	Linear programming (A-LP)	1. Use mathematical models involving equations and systems of equations to represent, interpret, and analyze quantitative relationships, change in various contexts, and other real-world phenomena. 2. Read, interpret, and solve linear programming problems graphically and by computational methods. 3. Use linear programming to solve optimization problems. 4. Interpret the meaning of the maximum or minimum value in terms of the objective function.

Category	Domain	Standards
Functions	Transcendental Functions (F-TF)	1. Describe transcendental functions through various representations, including words, equations, tables, and graphs; discuss how they are used in modeling contextual situations. 2. Use the language and notation of functions to develop models of real-world phenomena. 3. Analyze the effect of changing various parameters on transcendental functions and their graphs.
Geometry and Measurement	Set Theory (G-ST)	1. Define sets, subsets, proper subsets and empty sets with correct notation. 2. Analyze and compare sets, including using Venn Diagrams. 3. Perform set operations such as union, intersection, complement, and set difference. 4. Operate with sets and use set theory to solve problems.
	Investigate logic (G-L)	1. Define the order of operations for the logical operators. 2. Define conjunction, disjunction, negation, conditional, and biconditional. 3. Solve a variety of logic puzzles. 4. Construct and use a truth table to draw conclusions about a statement. 5. Apply the laws of logic to judge the validity of arguments. 6. Give counterexamples to disprove statements. 7. Analyze arguments with quantifiers through the use of Venn diagrams. 8. Represent logical statements with networks.
	Apportionment (G-A)	1. Understand the mathematical basis of apportionment principles and paradoxes. 2. Discuss the differences between two different types of apportionment and construct an example illustrating them. 3. Compare apportionments between states and the validity of the resulting representations.

Category	Domain	Standards
Data Analysis, Statistics, and Probability	Develop concepts in probability (D-CP)	1. Differentiate between permutations and combinations. 2. Evaluate expressions indicating permutations or combinations. 3. Define the relationship between permutations and the multiplication principle. 4. Use permutations and combinations to compute probabilities of compound events and solve problems. 5. Understand and apply the relationship between conditional probabilities and the probabilities of the individual events. 6. Calculate conditional probabilities using Bayes Theorem.
	Organize and Interpret data (D-ID)	1. Organize data for problem solving. 2. Use a variety of counting methods to organize information, determine probabilities, and solve problems. 3. Analyze survey data using Venn diagrams. 4. Calculate and interpret statistical problems using measures of central tendency and graphs. 5. Translate from one representation of data to another, e.g., a bar graph to a circle graph. 6. Calculate expected value, e.g., to determine the fair price of an investment. 7. Evaluate and compare two investments or strategies, where one investment or strategy is safer but has lower expected value. Include large and small investments and situations with serious consequences.

Precalculus

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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	Domain	Cluster	Standard
Number and Quantity	Number Expressions (N-NE)	Represent, interpret, compare, and simplify number expressions	1. Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them. 2. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.★ 3. Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of pi and e. 4. Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not. 5. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
	Complex numbers (N-CN)	Perform complex number arithmetic and understand the representation on the complex plane.	1. Perform arithmetic operations with complex numbers expressing answers in the form a+bi. 2. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. 3. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. 4. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°. 5. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
		Use complex numbers in polynomial identities and equations.	6. Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. 7. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Number and Quantity

Domain	Cluster	Standard
Vector and Matrix Quantities (N-VM)	**Represent and model with vector quantities.**	1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v). 2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. 3. Solve problems involving velocity and other quantities that can be represented by vectors.
	Understand the graphic representation of vectors and vector arithmetic.	4. Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. 5. Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lVert c\boldsymbol{v}\rVert = \lvert c\rvert v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$). 6. Calculate and interpret the dot product of two vectors.
	Perform operations on matrices and use matrices in applications.	7. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. 8. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. 9. Add, subtract, and multiply matrices of appropriate dimensions. 10. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. 11. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. 12. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. 13. Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

	Domain	Cluster	Standard
Algebra	**Sequences and Series (A-S)**	**Understand and use sequences and series.**	1. Demonstrate an understanding of sequences by representing them recursively and explicitly. 2. Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings. 3. Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist. a. Determine whether a given arithmetic or geometric series converges or diverges. b. Find the sum of a given geometric series (both infinite and finite). c. Find the sum of a finite arithmetic series. 4. Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples. 5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
	Reasoning with Equations and Inequalities(A-REI)	**Solve systems of equations and nonlinear inequalities.**	1. Represent a system of linear equations as a single matrix equation in a vector variable. 2. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). 3. Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology. 4. Solve systems of nonlinear inequalities by graphing.
	Parametric Equations (A-PE)	**Describe and use parametric equations.★**	1. Graph curves parametrically (by hand and with appropriate technology). 2. Eliminate parameters by rewriting parametric equations as a single equation.
	Conic Sections (A-C)	**Understand the properties of conic sections and apply them to model real-world phenomena.***	1. Display all of the conic sections as portions of a cone. 2. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. 3. From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics. 4. Transform equations of conic sections to convert between general and standard form.

	Domain	Cluster	Standard
Functions	Building Functions (F-BF)	Build new functions from existing functions.	1. Understand how the algebraic properties of an equation transform the geometric properties of its graph. *For example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation (i.e., translations, stretches, reflections and changes in periodicity and amplitude).* 2. Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions. 3. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* 4. Construct the difference quotient for a given function and simplify the resulting expression. 5. Find inverse functions (including exponential, logarithmic and trigonometric). a. Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, $\frac{1}{f(x)}$, and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain. 6. Explain why the graph of a function and its inverse are reflections of one another over the line y=x.
	Interpreting Functions (F-IF)	Analyze Functions using different representations.	1. Determine whether a function is even, odd, or neither. 2. Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real world problems that can be modeled with these functions (by hand and with appropriate technology).★ 3. Identify or analyze the distinguishing properties of exponential, polynomial, logarithmic, trigonometric, and rational functions from tables, graphs, and equations. 4. Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational). 5. Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$. 6. Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur. 7. Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior.

	Domain	Cluster	Standard
Functions	**Trigonometric Functions (F-TF)**	**Extend the domain of trigonometric functions using the unit circle.**	1. Convert from radians to degrees and from degrees to radians. 2. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number. 3. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Functions	**Graphing Trigonometric Functions (F-GT)**	**Model periodic phenomena with trigonometric functions.★**	1. Interpret transformations of trigonometric functions. 2. Determine the difference made by choice of units for angle measurement when graphing a trigonometric function. 3. Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes. 4. Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions. 5. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. 6. Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions. 7. Graph the inverse trigonometric functions and identify their key characteristics. 8. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Geometry	**Applied Trigonometry (G-AT)**	**Use trigonometry to solve problems.★**	1. Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles. 2. Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. 3. Derive and apply the formulas for the area of sector of a circle. 4. Calculate the arc length of a circle subtended by a central angle. 5. Prove the Laws of Sines and Cosines and use them to solve problems. 6. Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

	Domain	Cluster	Standard
Geometry	Trigonometric Identities (G-TI)	Apply trigonometric identities to rewrite expressions and solve equations.★	1. Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. 2. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Geometry	Polar Coordinates (G-PC)	Use polar coordinates.	1. Graph functions in polar coordinates. 2. Convert between rectangular and polar coordinates. 3. Represent situations and solve problems involving polar coordinates.★
Statistics and Probability	Model with Data★ (S-MD)	Model data using regressions equations.	1. Create scatter plots, analyze patterns and describe relationships for bivariate data (linear, polynomial, trigonometric or exponential) to model real-world phenomena and to make predictions. 2. Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data. 3. Use a regression equation modeling bivariate data to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.

Statistics

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

	Domain	Cluster	Standard
Exploring Data	Interpreting Categorical and Quantitative Data (S-ID)	Understand, represent, and use univariate data.	1. Understand the term 'variable' and differentiate between the data types: measurement, categorical, univariate and bivariate. 2. Understand histograms, parallel box plots, and scatterplots, and use them to display and compare data. 3. Summarize distributions of univariate data. 4. Compute basic statistics and understand the distinction between a statistic and a parameter. 5. For univariate measurement data, be able to display the distribution, describe its shape; select and calculate summary statistics. 6. Recognize how linear transformations of univariate data affect shape, center, and spread. 7. Analyze the effect of changing units on summary measures. 8. Construct and analyze frequency tables and bar charts. 9. Describe individual performances in terms of percentiles, z-scores, and t- scores.
		Understand, represent, and use bivariate data.	10. Explore categorical data. 11. Display and discuss bivariate data where at least one variable is categorical. 12. For bivariate measurement data, be able to display a scatterplot and describe its shape; use technological tools to determine regression equations and correlation coefficients. 13. Identify trends in bivariate data; find functions that model the data and that transform the data so that they can be modeled.
Probability	Conditional Probability and the Rules of Probability (S-CP)	Understand and apply basic concepts of probability.	1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). 2. Use permutations and combinations to compute probabilities of compound events and solve problems. 3. Demonstrate an understanding of the Law of Large Numbers (Strong and Weak).
		Use the rules of probability to compute probabilities of compound events in a uniform probability model.	4. Demonstrate an understanding of the addition rule, the multiplication rule, conditional probability, and independence. 5. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

	Domain	Cluster	Standard
Probability Distributions	Using Probability to Make Decisions (S-MD)	Understand and use discrete probability distributions.	1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. 2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. 3. Design a simulation of random behavior and probability distributions. 4. Analyze discrete random variables and their probability distributions, including binomial and geometric. 5. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* 6. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* 7. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* 8. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). 9. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
		Understand the normal probability distribution.	10. Calculate the mean (expected value) and standard deviation of both a random variable and a linear transformation of a random variable. 11. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Conceptual Category	Domain	Cluster	Standard
Sampling and Experimentation	Making Inferences and Justifying Conclusions (S-IC)	Know the characteristics of well-designed studies.	1. Understand the differences among various kinds of studies and which types of inferences can be legitimately drawn from each. 2. Compare census, sample survey, experiment, and observational study. 3. Describe the role of randomization in surveys and experiments. 4. Demonstrate an understanding of bias in sampling. 5. Describe the sampling distribution of a statistic and define the standard error of a statistic. 6. Demonstrate an understanding of the Central Limit Theorem.
		Design and conduct a statistical experiment to study a problem, then interpret and communicate the outcomes.	7. Select a method to collect data and plan and conduct surveys and experiments. 8. Compare and use sampling methods, including simple random sampling, stratified random sampling, and cluster sampling. 9. Test hypotheses using appropriate statistics. 10. Analyze results and make conclusions from observational studies, experiments, and surveys. 11. Evaluate reports based on data.
		Make inferences about population parameters based on a random sample from that population.	12. Develop and evaluate inferences and predictions that are based on data. 13. Use properties of point estimators, including biased/unbiased, and variability.
		Understand and use confidence intervals.	14. Understand the meaning of confidence level, of confidence intervals, and the properties of confidence intervals. 15. Construct and interpret a large sample confidence interval for a proportion and for a difference between two proportions. 16. Construct the confidence interval for a mean and for a difference between two means.
		Use distributions to make inferences about a data set.	17. Apply the properties of a Chi-square distribution in appropriate situations in order to make inferences about a data set. 18. Apply the properties of the normal distribution in appropriate situations in order to make inferences about a data set. 19. Interpret the t-distribution and determine the appropriate degrees of freedom.