Teacher Training on the Revised Math Standards

Grades K–2

Welcome & Introductions

Agenda: Day 1

Time	Content
8–11:15 (includes break)	Part 1: The Standards ▪ M1: Standards Review Process ▪ M2: TN Academic Standards ▪ M3: Summary of Revisions
11:15–12:30	Lunch (on your own)
12:30–4 (includes break)	Part 2: Diving into the Standards ▪ M4: Know-Do-Understand Part 3: Instructional Shifts ▪ M5: Revisiting SMP's and Instructional Shifts ▪ M6: Literacy Skills for Mathematical Proficiency

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Agenda: Day 2

Time	Content
8–11:15 (includes break)	Part 4: Assessment ▪ M7: Connecting Standards and Assessment
11:15–12:30	Lunch (on your own)
12:30–4 (includes break)	▪ M8: Evaluating Instructional Materials Part 5: Putting it All Together ▪ M9: Instructional Planning

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Norms

- Keep students at the center.
- Be present and engaged.
- Be reflective and solutions oriented.
- Challenge ideas with respect.
- Monitor airtime.

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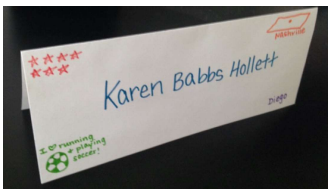
Introductions

- Make a name tent
  - Top left corner: # of ★ = how many years you have taught
  - Top right corner: where you're from
  - Bottom left corner: your favorite hobby
  - Bottom right corner: the name of a student who inspires you to learn and improve



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Today's Goals

- Review the standards revision process.
- Highlight changes/revisions to standards.
- Use a KUD exercise to deepen our understanding of the expectations of the standards.
- Discuss the instructional shifts and their relationship to the Standards for Mathematical Practice (SMPs).
- Explore the Literacy Skills for Mathematical Proficiency.

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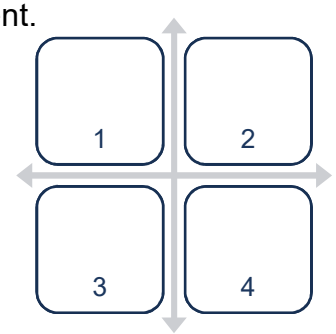
Appointment with Peers

- Create an appointment with four fellow participants by having them sign your manual.
- Participant partners will meet together during the training to discuss content.



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Key Ideas

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Key Ideas

Strong Standards

Standards are the bricks that should be masterfully laid through quality instruction to ensure that all students reach the expectation of the standards.

High Expectations

We have a continued goal to prepare students to be college and career ready.

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Key Ideas

Instructional Shifts

The instructional shifts are an essential component of the standards and provide guidance for how the standards should be taught and implemented.

Aligned Materials and Assessments

Educators play a key role in ensuring that our standards, classroom instructional materials and assessments are aligned.

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Part 1: The Standards

Key Idea

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Module 1: Standards Review Process

Standards Review Process

Public Comment Period – November 2014

Educator Advisory Teams Review – April 2015

Standards Review and Development Committee – Fall 2015

Revised Standards Released for Public Comment – October 2015

Standards Recommendation Committee – January 2016

State Board of Education Approval – April 2016

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Educator Advisory Team Members

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Standards and Assessment Timeline

Subject	Standards Proposed to State Board for Final Read	LEAs: New Standards in Classrooms	Aligned Assessment
Math	April 2016	2017-18	2017-18
ELA	April 2016	2017-18	2017-18
Science	October 2016	2018-19	2018-19
Social Studies	July 2017	2019-20	2019-20

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Standards Revision Key Points

- The instructional shifts remain the same and are still the focus of the standards.
- The revised standards represent a stronger foundation that will support the progression of rigorous standards throughout the grade levels.
- The revised standards improve connections:
  - within a single grade level, and
  - between multiple grade levels.

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Module 2: Tennessee Academic Standards for Mathematics

Goals

- Reinforce the continued expectations of the Tennessee Math Academic Standards.
- Revisit the three instructional expectations and their continued *and* connected role in the current standards.
- Review the overarching changes to the current Tennessee Math Academic Standards.

Setting the Stage

- Read and annotate the General Introduction to the TN Math Standards (page 1–2).
- After reading and annotating the two parts, write the sentence or phrase you felt was the most important in your participant manual on page.
- Be prepared to share with your colleagues.

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What Has NOT Changed

Overview of Similarities

- Students **prepared** for college and career

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Overview of Similarities

- Students **prepared** for college and career
- K–12 **learning progressions**

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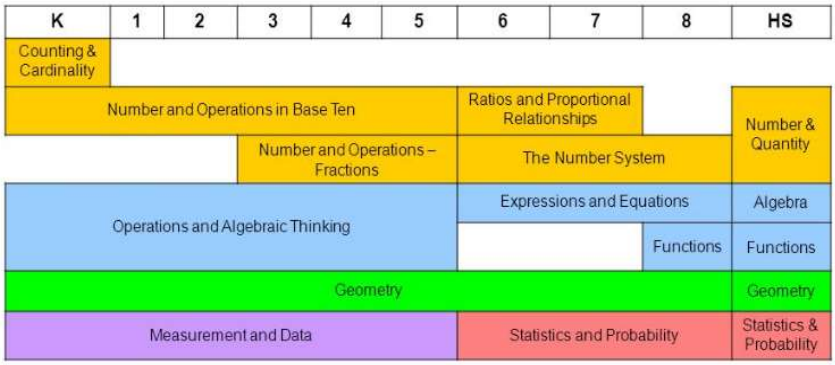
The Learning Progressions



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Overview of Similarities

- Students **prepared** for college and career
- K–12 **Learning Progressions**
- Traditional and integrated **pathways** (for high school)

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Overview of Similarities

- Students **prepared** for college and career
- K–12 **Learning Progressions**
- Traditional and Integrated **pathways**
- Standards for **Mathematical Practice**

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Standards for Mathematical Practice

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Overview of Similarities

- Students **prepared** for college and career
- K–12 **Learning Progressions**
- Traditional and Integrated **pathway**
- Standards for **Mathematical Practices**
- **Instructional shifts**

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Instructional Shifts

- **Focus**
  - The standards are focused on fewer topics so that students can dig deeper within the mathematics.
- **Coherence**
  - Topics within a grade are connected to support focus. Additionally, standards are linked across grades to ensure vertical coherence.
- **Rigor**
  - The standards set expectations for a balanced approach to pursuing conceptual understanding, procedural fluency, application, and modeling.

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What Has Changed

Overarching Revisions K–12

K–12

- Revised structure
- Literacy Skills for Mathematical Proficiency

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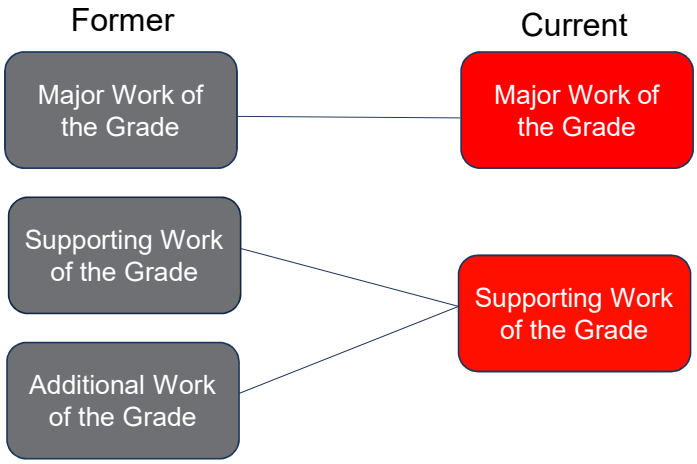
Category Change



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Revised Structure

Operations and Algebraic Thinking (OA)

Domain	Cluster	Cluster Headings	Content Standards
Operations and Algebraic Thinking	Use the four operations with whole numbers to solve problems.	**A. Use the four operations with whole numbers to solve problems.** (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations)	**4.OA.A.1** Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.A.2** Multiply or divide to solve contextual problems involving multiplicative comparison, and distinguish multiplicative comparison from additive comparison. *For example, school A has 300 students and school B has 600 students: to say that school B has two times as many students is an example of multiplicative comparison; to say that school B has 300 more students is an example of additive comparison.* **4.OA.A.3** Solve multi-step contextual problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
	Gain familiarity with factors and multiples.	**B. Gain familiarity with factors and multiples.**	**4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
	Generate and analyze patterns.	**C. Generate and analyze patterns.**	**4.OA.C.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Major Content | Supporting Content

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High School: Scope & Clarification

Cluster Headings	Content Standards	Scope & Clarifications
B. Solve equations and inequalities in one variable.	**A1.A.REI.B.3** Solve quadratic equations and inequalities in one variable. **a.** Use the method of completing the square to rewrite any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions	*For A1.A.REI.B.3b:* *Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions.* *Note: solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials. This is formally assessed in Algebra II.*
C. Solve systems of equations.	**A1.A.REI.C.4** Write and solve a system of linear equations in context.	*Solve systems both algebraically and graphically.* *Systems are limited to at most two equations in two variables.*

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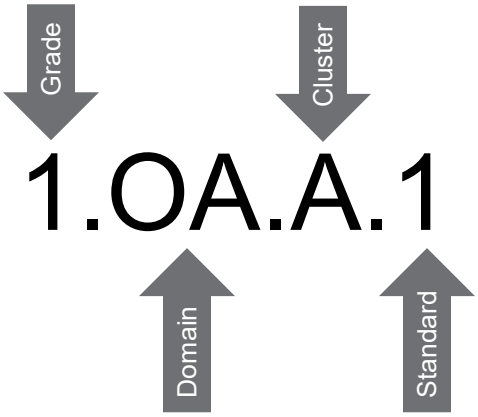
Nomenclature and Coding



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Literacy Skills for Mathematical Proficiency

Communication in mathematics requires literacy skills in reading, vocabulary, speaking, listening, and writing.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies.
2. Understand and use correct mathematical vocabulary.
3. Discuss and articulate mathematical ideas.
4. Write mathematical arguments.

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Module 2 Review

- Reinforce the continued expectations of the Tennessee Math Standards.
- Revisit the three instructional shifts and their continued *and* connected role in the revised standards.
- Review the overarching changes to the revised Tennessee Math Standards.

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10-Minute Break

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Module 3: Summary of Revisions by Grade Band

Goal

- Review a summary of the revisions to the math standards by grade band.
- Compare 2016–17 standards to 2017–18 standards.

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Turn & Talk

"To assess student achievement accurately, teachers and administrators must know and understand the content standards that their students are to master. Again, we cannot teach or assess achievement that we have not defined."

—S. Chappuis, Stiggins, Arter & J. Chappuis (2006)

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Revisions to Math Standards

Specific to K–5

- Refined for clarity
- Increased fluency expectations
- Revised examples

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K–5: Overarching Revisions

- **Supporting and additional work of the grade is combined as supporting work of the grade**
- Increased fluency expectations
- Added/shifted a small number of standards to strengthen coherence across grade levels
- Revised language to provide clarity and continuity
- Highlighted chart for grade-level mastery expectations for addition, subtraction, multiplication, and division

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K–5: Overarching Revisions

- Supporting and additional work of the grade is combined as supporting work of the grade
- **Increased fluency expectations**
- Added/shifted a small number of standards to strengthen coherence across grade levels
- Revised language to provide clarity and continuity
- Highlighted chart for grade-level mastery expectations for addition, subtraction, multiplication, and division

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Sample Revision: K–2

	Increased Fluency Expectations	
	Former Standard	Current Standard
Kindergarten	K. OA.5 Fluently add and subtract within 5.	K.OA.A.5 Fluently add and subtract within 10 using mental strategies.
First Grade	1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.	1.OA.C.6 Fluently add and subtract within 20 using mental strategies. By the end of Grade 1, know from memory all sums up to 10.
Second Grade	2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.	2.OA.B.2 Fluently add and subtract within 30 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers and related subtraction facts.

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K–5: Overarching Revisions

- Supporting and additional work of the grade is combined as supporting work of the grade
- Increased fluency expectations
- **Added/shifted a small number of standards to strengthen coherence across grade levels**
- Revised language to provide clarity and continuity
- Highlighted chart for grade-level mastery expectations for addition, subtraction, multiplication, and division

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Sample Revision: K–2

	Former Standard	Current Standard
Kindergarten	No Past Standard	K.MD.B.3 Identify the penny, nickel, dime, and quarter and recognize the value of each.
First Grade	No Past Standard	1.MD.B.4 Count the value of a set of like coins less than one dollar using the ¢ symbol only.
Second Grade	2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.	2.MD.C.8 Solve contextual problems involving dollar bills, quarters, dimes, nickels, and pennies using ¢ and $ symbols appropriately.

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Sample Revision: 3–5

	Former Standard	Current Standard
Fourth Grade	4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),…	4.MD.A.1 Measure and estimate to determine relative sizes of measurement units within a single system of measurement involving length, liquid volume, and mass/weight of objects using customary and metric units.
Fifth Grade	5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	5.MD.A.1 Convert customary and metric measurement units within a single system by expressing measurements of a larger unit in terms of a smaller unit. Use these conversions to solve multi-step real world problems involving distances, intervals of time, liquid volumes, masses of objects, and money (including problems involving simple fractions or decimals). For example, 3.6 liters and 4.1 liters can be combined as 7.7 liters or 7700 milliliters.

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K–5: Overarching Revisions

- Supporting and additional work of the grade is combined as supporting work of the grade
- Increased fluency expectations
- Added/shifted a small number of standards to strengthen coherence across grade levels
- **Revised language to provide clarity and continuity**
- Highlighted chart for grade-level mastery expectations for addition, subtraction, multiplication, and division

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Sample Revision: K–2

Former Standard

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Current Standard

2.NBT.A.3 Read and write numbers to 1000 using **standard form**, **word form**, and expanded form.

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Sample Revision: 3–5

Former Standard

4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place.

Current Standard

4.NBT.A.3 Round multi-digit whole numbers to any place **(up to and including the hundred-thousand place)** using understanding of place value.

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K–5: Overarching Revisions

- Supporting and additional work of the grade is combined as supporting work of the grade
- Increased fluency expectations
- Added/shifted a small number of standards to strengthen coherence across grade levels
- Revised language to provide clarity and continuity
- **Highlighted chart for grade-level mastery expectations for addition, subtraction, multiplication, and division**

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Addition and Subtraction Chart

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ? **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5 **One-Step Problem** **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ? **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3 **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3 **One-Step Problem** **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ? **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ? **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2 **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1st)**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? 5 – 3 = ? ? + 3 = 5 **One-Step Problem** **(2nd)**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ? **(1st)**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ? **One-Step Problem** **(2nd)**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1st)**

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.
2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

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Multiplication and Division Chart

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

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Focusing on Fluency in Grades K–5

One-minute Free Write:

What is fluency?

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Fluency

"All students should be able to recall and use their math education when the need arises. That is, a student should know certain math facts and concepts such as the multiplication table, how to add, subtract, multiply, and divide basic numbers, how to work with simple fractions and percentages, etc. There is a level of procedural fluency that a student's K–12 math education should provide him or her along with conceptual understanding so that this can be recalled and used throughout his or her life."

–Tennessee Math Standards

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What is Fluency?

- The ability to apply procedures accurately, efficiently, and flexibly.
- Recognizing when one strategy or procedure is more appropriate to apply than another.
- Having opportunities to justify both informal strategies and commonly used procedures through distributed practice.
- Procedural fluency includes computational fluency with the four arithmetic operations. In the early grades, students are expected to develop fluency with whole numbers in addition, subtraction, multiplication, and division.

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Definition of Fluency

Computational fluency refers to having **efficient and accurate methods** for computing. Students exhibit computational fluency when they demonstrate **flexibility** in the computational methods they choose, **understand** and **can explain** these methods, and **produce accurate answers efficiently**.

The computational **methods** that a student uses should be **based** on mathematical **ideas** that the student **understands well**, including the structure of the base-ten number system, properties of multiplication and division, and number relationships.

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Fluency Progression Chart

- Examine the fluency standards for each. How do the ideas build upon one another? Or, in other words, what is the vertical alignment?

Focus	K	1	2	3	4	5
Fluency						

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Fluency Standards: K–2

- K.OA.A.5 Fluently add and subtract within 10 using mental strategies.
- 1.OA.C.6 Fluently add and subtract within 20 using mental strategies. By the end of 1st grade, know from memory all sums up to 10.
- 2.OA.B.2 Fluently add and subtract within 30 using mental strategies. By the end of 2nd grade, know from memory all sums of two one-digit numbers and related subtraction facts.
- 2.NBT.B.5 Fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction.

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Fluency Standards: 3

- 3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of 3rd grade, know from memory all products of two one-digit numbers and related division facts.
- 3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

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Fluency Standards: 4–5

- 4.NBT.B.4 Fluently add and subtract within 1,000,000 using appropriate strategies and algorithms.
- 5.NBT.B.5 Fluently multiply multi-digit whole numbers (up to three-digit by four-digit factors) using appropriate strategies and algorithms.

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Fluency Progression Across Grades K–5

<table>
<tr><th></th><th>Kindergarten</th><th>1st Grade</th><th>2nd Grade</th><th>3rd Grade</th><th>4th Grade</th><th>5th Grade</th></tr>
<tr><td rowspan="4">Fluency Expectations</td><td rowspan="4">Mental addition and subtraction within 10</td><td>Mental addition and subtraction within 20</td><td>Mental addition and subtraction within 30</td><td>Addition and subtraction within 1000 using strategies, algorithms, properties, and relationships</td><td rowspan="4">Addition and subtraction within 1,000,000 using strategies and algorithms</td><td rowspan="4">Multiplication of multi-digit whole numbers (up to three-digit by four-digit factors) using strategies and algorithms</td></tr>
<tr><td rowspan="3">Memorization of all sums up to 10</td><td rowspan="2">Addition and subtraction within 100 using properties, strategies, and relationships.</td><td>Memorization of all products of two one-digit numbers and related division facts</td></tr>
<tr><td rowspan="2">Multiplication and division within 100 using strategies, relationships, or properties</td></tr>
<tr><td>Memorization of all sums of two one-digit numbers and related subtraction facts</td></tr>
</table>

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Strategies for Teaching Fluency

- **Technology Use**: use of technology such as mobile math apps have been shown to improve fluency (Shin, S., & Kwon, J., 2014, p. 1).
- **Self-Management Techniques**: Students who were taught self-management techniques such as asking questions like, "Am I paying attention?" were shown to have increased mathematical fluency (McDougall & Brady, 1998).
- (If you must use timed practice) **Distributed, Rather than Explicit, Timed Practice**: In a study on 3rd grade students in the Midwest, students gained more fluency with basic math facts when timed practice was distributed across the day rather than all at once.
- **Games**: Using games such as 24 can increase procedural fluency (Suh, J.).

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Revisions to Math Standards

Specific to 6–8

- Refined major work of the grade
- Revised supporting work of the grade, especially in statistics and probability

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Grades 6–8: Overarching Revisions

- **Slight revisions made to geometry in grade 8**
- Supporting and additional work of the grade is combined as supporting work of the grade
- Revised language to provide clarity and continuity
- Revised a small number of standards to strengthen coherence by condensing, expanding, and removing standards
- Revised a small number of statistics and probability standards

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Grades 6–8: Overarching Revisions

- Slight revisions made to geometry in grade 8
- **Supporting and additional work of the grade is combined as supporting work of the grade**
- Revised language to provide clarity and continuity
- Revised a small number of standards to strengthen coherence by condensing, expanding, and removing standards
- Revised a small number of statistics and probability standards

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Grades 6–8: Overarching Revisions

- Slight revisions made to geometry in grade 8
- Supporting and additional work of the grade is combined as supporting work of the grade
- **Revised language to provide clarity and continuity**
- Revised a small number of standards to strengthen coherence by condensing, expanding, and removing standards
- Revised a small number of statistics and probability standards

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Grades 6–8: Sample Revision

Former Standard

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

Current Standard

6.SP.A.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center **(mean, median, mode)**, spread **(range)**, and overall shape.

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Grades 6–8: Overarching Revisions

- Slight revisions made to geometry in grade 8
- Supporting and additional work of the grade is combined as supporting work of the grade
- Revised language to provide clarity and continuity
- **Revised a small number of standards to strengthen coherence by condensing, expanding, and removing standards**
- Revised a small number of statistics and probability standards

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Grades 6–8: Sample Revision

Former Standard

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another. *For example, Susan is putting money in her savings account by depositing a set amount each week (50). Represent her savings account balance with respect to the number of weekly deposits (*s = 50w*, illustrating the relationship between balance amount* s *and number of weeks* w*).* Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

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Grades 6–8: Sample Revision

Current Standard

6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another. *For example, Susan is putting money in her savings account by depositing a set amount each week (50). Represent her savings account balance with respect to the number of weekly deposits (*s = 50w*, illustrating the relationship between balance amount* s *and number of weeks* w*).*

a. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.

b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

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Grades 6–8: Sample Revision

Former Standard

7.G.3 Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids

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Grades 6–8: Overarching Revisions

- Slight revisions made to geometry in grade 8
- Supporting and additional work of the grade is combined as supporting work of the grade
- Revised language to provide clarity and continuity
- Revised a small number of standards to strengthen coherence by condensing, expanding, and removing standards
- **Revised a small number of statistics and probability standards**

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Grades 6–8: Sample Revision

Former Standard

6.SP.5c Summarize numerical data sets in relation to their context, such as by: c. Giving quantitative measures of center (median and/or mean) and variability **(interquartile range and/or mean absolute deviation)**, as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Current Standard

6.SP.B.5c Summarize numerical data sets in relation to their context, such as by: c. Giving quantitative measures of center (median and/or mean) and variability **(range)**, as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

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Revisions to Math Standards

Specific to 9–12

- Revised scope and clarifications
- Revisions for Algebra II and Integrated III
- Restructured additional Mathematics courses to reflect college and career readiness

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Grades 9–12: Overarching Revisions

- **Supporting and additional work of the grade is combined as supporting work of the grade**
- Removed or shifted a small number of standards to the major work of the grade to streamline vertical progression
- Revised language and examples to provide clarity and continuity
- Shifted a small number of supporting work of the grade standards to the additional mathematics courses
- Restructured additional courses to reflect college and career readiness

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Grades 9–12: Overarching Revisions

- Supporting and additional work of the grade is combined as supporting work of the grade
- **Removed or shifted a small number of standards to the major work of the grade to streamline vertical progression**
- Revised language and examples to provide clarity and continuity
- Shifted a small number of supporting work of the grade standards to the additional mathematics courses
- Restructured additional courses to reflect college and career readiness

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Grades 9–12: Overarching Revisions

- Supporting and additional work of the grade is combined as supporting work of the grade
- Removed or shifted a small number of standards to the major work of the grade to streamline vertical progression
- **Revised language and examples to provide clarity and continuity**
- Shifted a small number of supporting work of the grade standards to the additional mathematics courses
- Restructured additional courses to reflect college and career readiness

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Grades 9–12: Sample Revisions

Former Standard

G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Current Standard

G.SRT.C.8 ***Know and*** use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

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Grades 9–12: Overarching Revisions

- Supporting and additional work of the grade is combined as supporting work of the grade
- Removed or shifted a small number of standards to the major work of the grade to streamline vertical progression
- Revised language and examples to provide clarity and continuity
- **Shifted a small number of supporting work of the grade standards to the additional mathematics courses**
- Restructured additional courses to reflect college and career readiness

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Grades 9–12: Sample Revisions

Moved Standard

From:

A2.F.TF.5

M3.F.TF.5

To:

P.F.TF.A.4 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

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Grades 9–12: Overarching Revisions

- Supporting and additional work of the grade is combined as supporting work of the grade
- Removed or shifted a small number of standards to the major work of the grade to streamline vertical progression
- Revised language and examples to provide clarity and continuity
- Shifted a small number of supporting work of the grade standards to the additional mathematics courses
- **Restructured additional courses to reflect college and career readiness**

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Additional Mathematics Courses Rationale

- **High expectations**
- Retention of **rigorous standards**
- Clearly **defined** and **coherent** pathways
- **Equity** and **opportunity**
- Aligned with **student interest** in postsecondary fields
- Shift to a **discipline- and career-based pathway**

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Additional Mathematics Courses

- **Advanced Algebra and Trigonometry**
- **Discrete Math**
- **Finite Math**
- Bridge Math
- Pre-Calculus
- Statistics
- Calculus

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Additional Mathematics Courses

- **Applied Mathematical Concepts (New Course)**
- Bridge Math
- Pre-Calculus
- Statistics
- Calculus

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Standards Comparison Activity

- Compare the 2016-17 standards to the revised standards for your assigned grade or course.
- Highlight any changes in the 2017-18 column.

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Standards Comparison Activity

- If you had to summarize the revisions to these selected standards in twenty words or less, what would you say?
- Write your response to the question in the participant manual on page 61.

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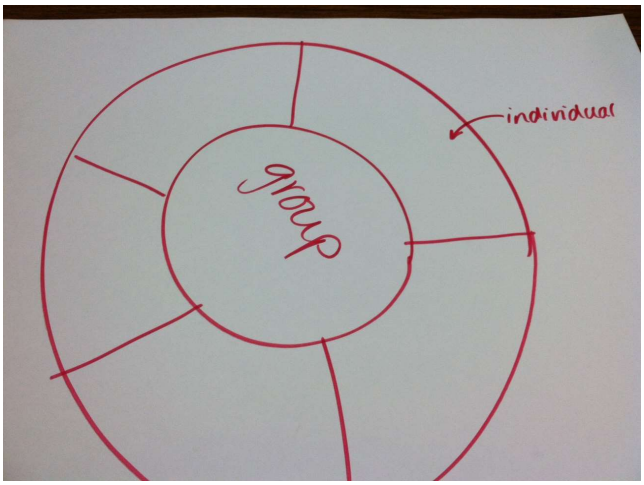
Consensus Circle



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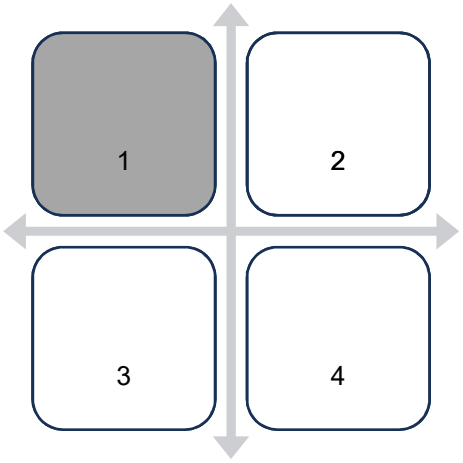
Appointment with Peers



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Module 3 Review

- The instructional shifts remain the same and are still the focus of the standards.
- The revised standards represent a stronger foundation that will support the progression of rigorous standards throughout the grade levels.
- The revised standards improve connections:
  - within a single grade level, and
  - between multiple grade levels.

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Part One

Strong Standards

Standards are the bricks that should be masterfully laid through quality instruction to ensure that all students reach the expectation of the standards.

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Lunch Break: 1 hour 15 minutes

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Please Sign In!

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Part 2:
Diving into the Standards

Key Ideas

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Module 4: A Deeper Dive into Grades K–2 Math

Goals

- Concisely describe a course based on its introduction.
- Develop a means for deconstructing standards to determine the mathematical emphasis of the standard—its intent and purpose.
- Use the KUD approach to guide planning, instruction, and assessment.

Seeing the Big Picture

Closer Look

Take a few minutes to read the Overview page for your grade level (the outline only) and think about how this relates to the overarching revisions we have just seen.

Grade K	-	p. 15
Grade 1	-	p. 21
Grade 2	-	p. 28

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What Would You Tweet?

Summarize your course in 140 characters. Write a tweet that will inform others regarding what is included in your course.

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Progression of Learning

- Examine the major foci for each grade. How do the ideas build upon one another? Or, in other words, what is the vertical alignment? Here is one example.

Focus	K	1	2
Fact fluency	Within 10	Within 20	Within 100

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Intent and Purpose

Intent and Purpose

"With my ears to the ground, listening to my students, my eyes are focused on the mathematical horizon."

—Ball (1993)

Clarify Student Learning

- Consider the problem 3 + 1 = ?
- Think/Share/Compare Activity
  - What must a child know in order to solve this problem?
  - What mathematical understandings are required?

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Analyzing Standards

- K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality.
  a. When counting objects, say the number names in the standard order, using one-to-one correspondence.
  b. Recognize that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
  c. Recognize that each successive number name refers to a quantity that is one greater.

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Analyzing Standards

- K.CC.B.4 **Understand** the relationship between numbers and quantities; **connect** counting to cardinality.

  a. When counting objects, **say** the number names in the standard order, using one-to-one correspondence.

  b. **Recognize** that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

  c. **Recognize** that each successive number name refers to a quantity that is one greater.

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The Verbs Lead to the Nouns (or noun phrases)

- As noted, the verbs raise the questions of "what," leading one to next highlight the associated nouns.
- One may wish to highlight what might be termed a noun phrase instead of trying to be exact on the object of the verb. Remember that we are trying to understand the intent and purpose of the standard.
- There is also a temptation to almost highlight everything! Let's limit ourselves to the verbs and the nouns connected to them.

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Analyzing Standards

- K.CC.B.4 **Understand** the **relationship between numbers and quantities**; **connect counting to cardinality**.

  a. When counting objects, **say** the **number names in the standard order**, using one-to-one correspondence.

  b. **Recognize** that the **last number name said tells the number of objects counted**. The number of objects is the same regardless of their arrangement or the order in which they were counted.

  c. **Recognize** that **each successive number name refers to a quantity** that is **one greater**.

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From Standard to Instruction: KUD

Clarify Student Learning

- What knowledge do students need? (What do they need to **know**?)
- What do they need to **understand** to successfully meet this standard? ("I can" statements or essential questions)
- What will students **do** to let you know they understand? (What learning opportunities will you provide?)

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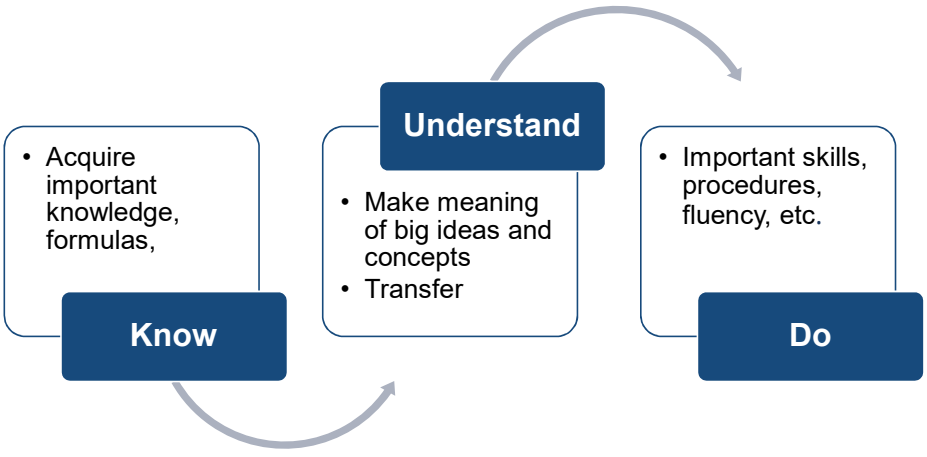
KUD



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Summary of the Intent and Purpose of the Standard

- Write a brief description of each part of KUD with respect to this standard.
- What does the math look like? What vocabulary stands out? Any symbolism to be understood or other literacy skills needed?
- Can you now write an "I can" statement and/or an essential question based on our understanding of the standard and the focus that KUD gives us?

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What Do Students *Understand*?

- I can count saying the numbers in the correct order.
- I can count one and only one object each time I say a number name.
- I can tell the number of objects in a set because I know the last number I say when I count is the number of objects in a set.
- I can tell how many things when they are in lines, rows, or circles.
- I can tell the number of things in a set are the same no matter which order they are counted.

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Grade-specific Examples

You try it!
You will be given a standard for your grade level.

K.OA.A.1

1.OA.A.1

2.OA.A.1

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Analyzing Your Standard

- First, highlight verbs that speak to what the student will do.
- Second, highlight the nouns (noun phrases) connected to the verbs.
- Third, KUD, jot down what the students are to know, understand, and do (brief descriptions will do).
- Fourth, determine what the math will look like and touch on vocabulary and literacy skills needed.
- Now, write an I CAN statement and/or essential question that will be used in determining student mastery.

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Gallery Walk

Please provide any comments or questions on Post-Its and put them on the chart paper.

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Gallery Walk Debrief

- Small Group Discussion
  - Review the feedback you were given.
  - What are your key takeaways?

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Module 4 Review

- Concisely describe a course based on it's introduction.
- Develop a means for deconstructing standards to determine the mathematical emphasis of the standard – it's intent and purpose.
- Use the KUD approach to guide planning, instruction, and assessment.

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Part Two

High Expectations

We have a continued goal to prepare students to be college and career ready.

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10-Minute Break

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Part 3: Instructional Shifts

Key Idea

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Module 5: Revisiting the Shifts & Standards for Mathematical Practice

Goals

- Revisit the concepts of focus, coherence, and rigor and how they play out in instruction.
- Discuss the purpose and place of the content and practice standards.
- Focus on two areas of mindset: praise and mistakes.
- Share instructional strategies related to the Standards for Mathematical Practice.
- Discuss research on the influence of mindsets in the math classroom.

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The Standards for Mathematical Practice

- **Why do we have the standards?**
- Tell us what students should know and be able to do
- So, what should students know and do?
  - Content Standards
  - Mathematics Practice Standards
  - Literacy Skills
- **Knowing that these are *what* students need to learn, teachers determine *how* to teach these.**

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Standards for Mathematical Practice

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Mindset

- The TN Academic Standards for Mathematics may seem challenging for students whose mindset has been fixed by their past experiences in mathematics classrooms.
- As teachers, we are best positioned to influence students' mathematical mindsets through our actions/practices in the mathematics classroom.

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Fixed Mindset vs. Growth Mindset

- **Fixed mindset**

  Intelligence is a fixed trait. You cannot change it.

- **Growth mindset**

  You can grow your intelligence through effort.

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Study of Praise and Mindsets

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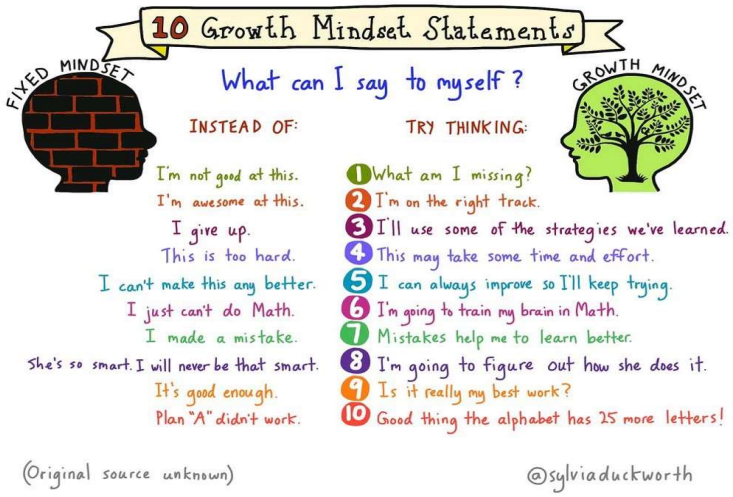
TrainUgly.com

Fixed vs. Growth Mindset



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Why Address Mindsets?

"If there's a threat of being wrong every time I raise my hand, and being wrong is a bad thing, then very quickly I decide math isn't for me, I don't like this, I'm not a smart person".

—Noah Heller, Harvard Graduate School of Education

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Setting Up Positive Norms in the Math Classroom

- Everyone can learn math to the highest levels
- Mistakes are valuable
- Questions are important
- Math is about creativity and making sense
- Math is about connections and communicating
- Math class is about learning not preforming
- Depth is more important than speed

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Instructional Shifts

- **Focus**
  - The standards are focused on fewer topics so that students can dig deeper within the mathematics.
- **Coherence**
  - Topics within a grade are connected to support focus. Additionally, standards are linked across grades to ensure vertical coherence.
- **Rigor**
  - The standards set expectations for a balanced approach to pursuing conceptual understanding, procedural fluency, and application.

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Shift One: Focus

- In your participant manual turn to page ____.
- Examine the table. Two of the foci listed represent major areas of focus for the indicated grade. One is an imposter. Which one doesn't belong?
- In your grade-level groups, discuss ways you could respond if someone asks you the following question, "Why focus? There's so much math that students could be learning. Why limit them?"

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Shift One: Focus

K	Compare numbers	Use tally marks	Understand the meaning of addition and subtraction
1	Add and subtract within 20	Measure lengths indirectly and by iterating length units	Create and extend patterns and sequences
2	Represent and solve problems involving addition and subtract	Understand place value	Identify line of symmetry in two dimensions

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Shift Two: Coherence

- In the participant manual, copy all of the standards related to the Operations and Algebraic Thinking Domain and note how coherence is evident in these standards. Note also standards that may be outside of the Operations and Algebraic Thinking Domain but are related to, or in support of, the standards related to Operation and Algebraic Thinking Domain.

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Shift Two: Coherence

K.OA.A.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.
K.OA.A.2	Add and subtract within 10 to solve contextual problems using objects or drawings to represent the problem.
K.OA.A.3	Decompose numbers less than or equal to 10 into addend pairs in more than one way (e.g., 5 = 2 + 3 and 5 = 4 + 1) by using objects or drawings. Record each decomposition using a drawing or writing an equation.
K.OA.A.4	Find the number that makes 10, when added to any given number, from 1 to 9 using objects or drawings. Record the answer using a drawing or writing an equation.
K.OA.A.5	Fluently add and subtract within 10 using mental strategies.

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Shift Three: Rigor

- *Make a true statement: Rigor* = _____+____+ ____
- In your groups, discuss ways to respond to one of the following comments: “These standards are expecting that we just teach rote memorization. Seems like a step backwards to me.” Or “I’m not going to spend time on fluency—it should just be a natural outcome of conceptual understanding.”
- The shift towards rigor is required by the standards. Find and copy in the space below standards from grade 1 that specifically set expectations for each component of rigor.

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Shift Three: Rigor

Grade 1 standards that require **fluency**:

Grade 1: 1.OA.C.6; 1.NBT.C.5

Grade 1 standards that require **deep conceptual understanding**:

Grade 1: 1.OA.A.7, 1.OA.A.7, 1.OA.B.3; 1.NBT.C.4, 1.NBT.C.6;

Grade 1 standards that require **application**:

Grade 1: 1.OA.A.1

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Turn & Talk

- What do these instructional shifts look like in the classroom?

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Module 5 Review

- We connected the instructional shifts to the standards and our classroom practices.
- We explored students' mathematical mindsets.
- We shared instructional strategies related to the Standards for Mathematical Practice.

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Module 6: Literacy Skills for Mathematical Proficiency

Goal

- Develop a better understanding of the Literacy Skills for Mathematical Proficiency.

Literacy in your Math Classroom

Reflect on ways literacy skills are already present in your mathematics classroom.

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Literacy Skills for Math Proficiency

Communication in mathematics requires literacy skills in reading, vocabulary, speaking, listening, and writing.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies.
2. Understand and use correct mathematical vocabulary.
3. Discuss and articulate mathematical ideas.
4. Write mathematical arguments.

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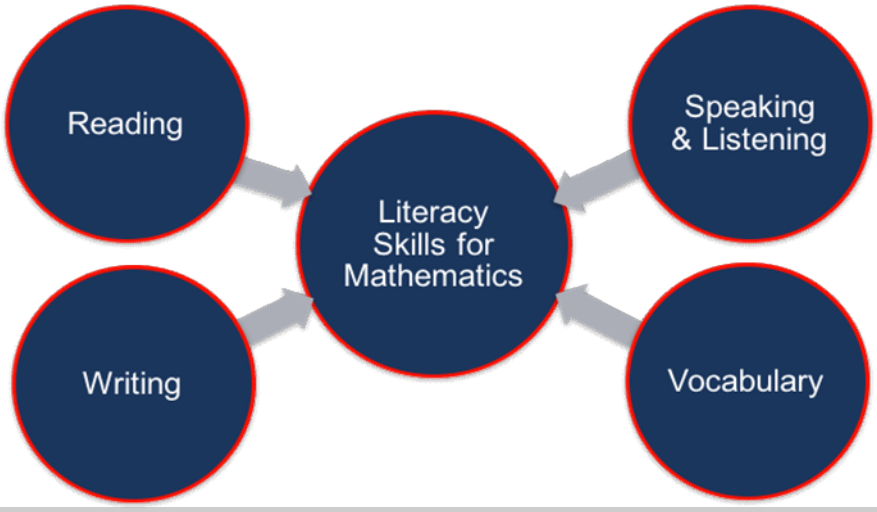
Literacy Skills in the Math Classroom



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Group Activity

- Divide into groups.
- Each group will be assigned one literacy topic.
- Each group will read and annotate the paragraph on that topic in the Math Standards Document (pg. 13–14).
- Use chart paper to create a supporting document to share the ideas from your topic with the group.

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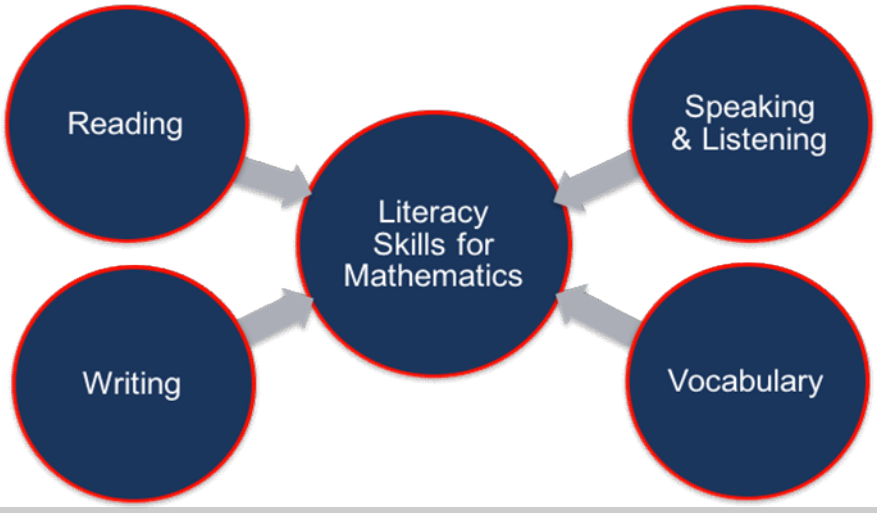
Literacy Skills in the Math Classroom



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Vocabulary

"Students must be taught that the language we read and speak in mathematics class is actually a technical jargon, even though it may look and sound like regular English."

—Hersh, 1997

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Mathematics Vocabulary

- Student achievement is dependent upon students' reading comprehension and content area learning.
- Math vocabulary is decontextualized because they are not in everyday conversations.
- Mathematical terms can have specific meanings (i.e., average, reflection).
- Students need to develop a conceptual meaning in order to read and use the words accurately.

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Four Stages of Word Knowledge

1. Students have never encountered the word before.
2. Students have seen/heard the word but do not know the definition.
3. Students know the word but rely on context to define it.
4. Students know the word and can use it comfortably.

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Vocabulary

- Symbols are used as language.
- Many mathematical terms have different meanings in everyday use.
- Can you think of any examples?

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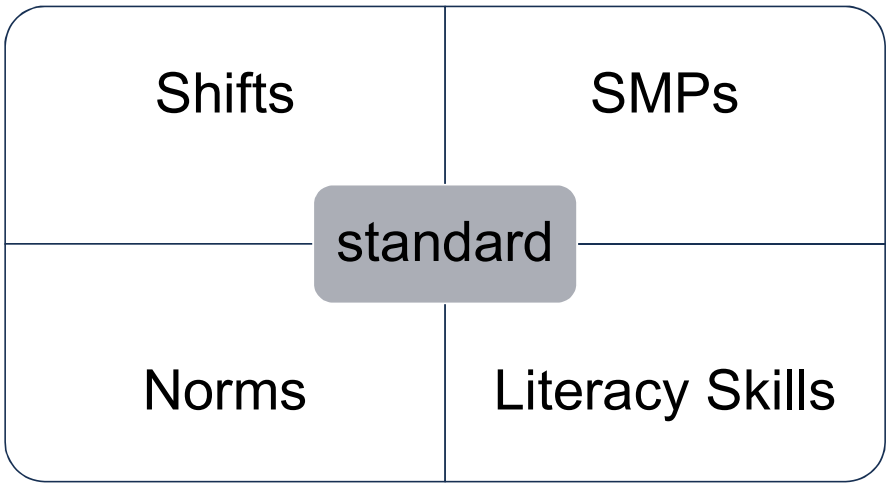
Making Connections



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Module 6 Review

- Literacy skills in the math classroom will support students' understanding of the content standards.
- When students can read, write, and speak about math ideas, connections are made between concepts.

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Part Three

Instructional Shifts

The instructional shifts are an essential component of the standards and provide guidance for how the standards should be taught and implemented.

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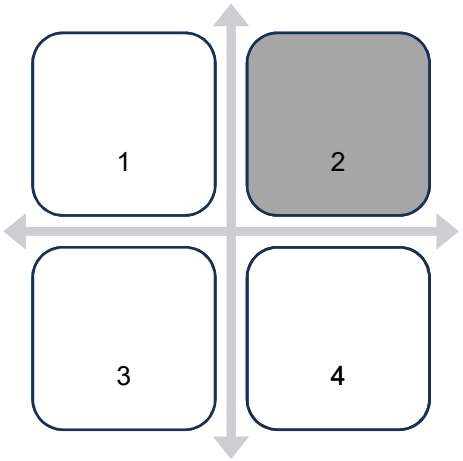
Appointment with Peers



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What to Expect: Day 2

- Dig more deeply into assessments and question development.
- Explore what it means to assure alignment of curriculum to standards.
- Explore how to begin to develop instructional planning.

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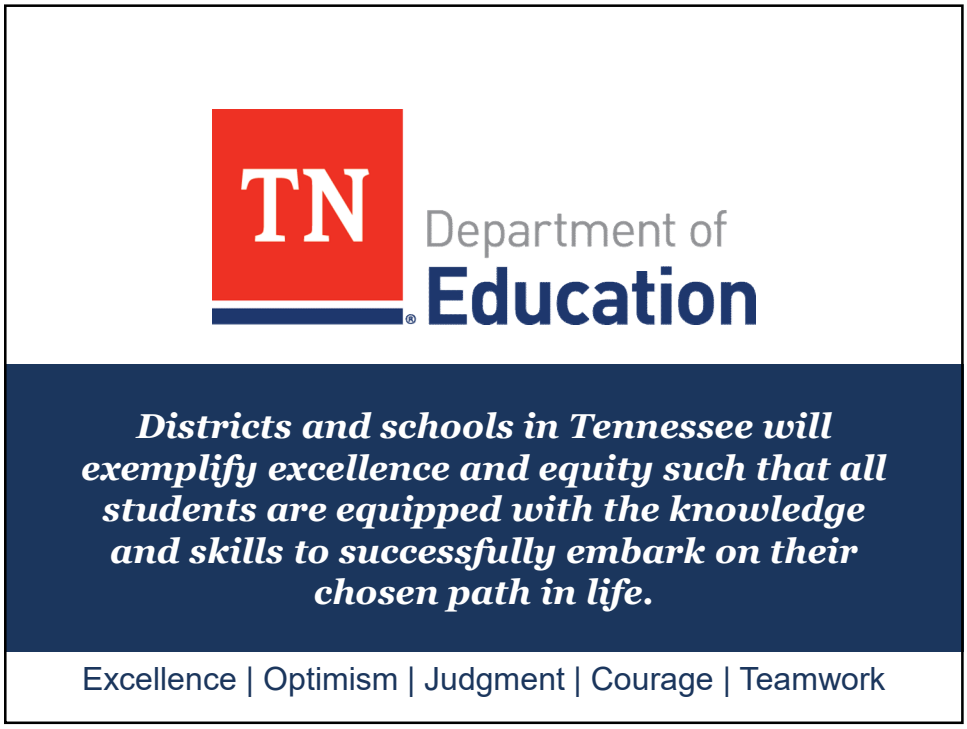
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Education
Districts and schools in Tennessee will exemplify excellence and equity such that all students are equipped with the knowledge and skills to successfully embark on their chosen path in life.
Excellence | Optimism | Judgment | Courage | Teamwork