TN Department of Education | 2015 Summer Training

Math Grades 3-5

Participant Packet

Tennessee Department of Education | 2015 Summer Training

MATH

GRADES 3-5

Table of Contents

Module 1
Examining the Beliefs and Knowledge of Effective Mathematics Instruction

Guiding Principles

Across all grade bands and subjects there are common guiding principles for each TNCore training:

- All students are capable of achieving at a high level.
- Students rise to the level of expectation when challenged and supported appropriately.
- Students learn best when they are authentically engaged in their own learning.
- We must continuously improve our effectiveness as teachers and leaders in order to improve student success.
- We must make every minute with our students count with purposeful work and effective instruction.

Norms for Collaboration

- Keep students at the center
- Be present and engaged
- Monitor air time and share your voice
- Challenge with respect
- Stay solutions oriented
- Risk productive struggle
- Balance urgency and patience

Rationale

"All of us who are stakeholders have a role to play and important actions to take if we are finally to recognize our critical need for a world where the mathematics education of our students draws from research, is informed by common sense and good judgment, and is driven by a non-negotiable belief that we must develop mathematical understanding and self-confidence in all students."

- NCTM Principles to Action, 2014.

Goals

Participants will:

- Examine productive and unproductive beliefs about mathematics education
- Learn about the different types of knowledge effective mathematics teachers must have
- Connect work from previous TNCore trainings to NCTM's Teaching Practices

Session Activities

Participants will:

- Complete mathematics tasks across the grade band
- Read and discuss NCTM's productive and unproductive beliefs
- Define Mathematical Knowledge for Teaching using a Frayer Model

Overview

Module 1 focuses on the importance of productive vs. unproductive beliefs (NCTM, 2014) around teaching mathematics that teachers hold. Participants engage in high-level tasks and discuss teh different mathematical knowledge types (Ball, 2012) and mathematical teaching practices (NCTM, 2014) that support effective and rigorous mathematics instruction.

Practice Problems

1. There are 6 tables in Mrs. Potter's art classroom. There are 4 students sitting at each table. Each student has a box of 10 colored pencils.

A. How many colored pencils are at each table?

B. How many colored pencils do Mrs. Potter's students have in total?

https://www.illustrativemathematics.org/content-standards/3/NBT/A/3/tasks/1445

2. There are almost 40 thousand fourth graders in Mississippi and almost 400 thousand fourth graders in Texas. There are almost 4 million fourth graders in the United States.

We write 4 million as 4,000,000. How many times more fourth graders are there in Texas than in Mississippi? How many times more fourth graders are there in the United States than in Texas? Use the approximate populations listed above to solve.

There are about 4 thousand fourth graders in Washington, D.C. How many times more fourth graders are there in the United States than in Washington, D.C.?

https://www.illustrativemathematics.org/content-standards/4/NBT/A/1/tasks/1808

3. Jillian says:

I know that 20 times 7 is 140 and if I take away 2 sevens that leaves 126. So 126 ÷ 7 = 18.

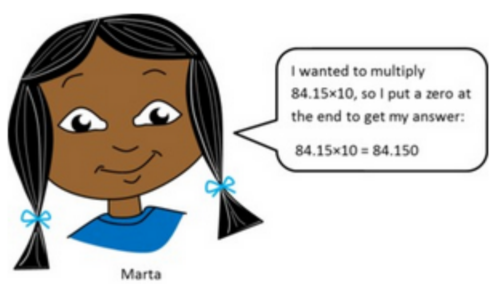
Is Jillian's calculation correct? Explain.

Draw a picture showing Jillian's reasoning.

Use Jillian's method to find 222 ÷ 6.

https://www.illustrativemathematics.org/content-standards/4/NBT/B/6/tasks/1774

4. Marta made an error while finding the product 84.15×10.



In your own words, explain Marta's misunderstanding. Please explain what she should do to get the correct answer and include the correct answer in your response.

https://www.illustrativemathematics.org/content-standards/5/NBT/A/2/tasks/1524

5. The table shows four people who earn the typical amount for their education level.

Name	Level of Education	Weekly Income
Miley	High School Dropout	$440.50
Niko	High School Graduate	$650.35
Taylor	2-Year College Graduate	$771.25
Pinky	4-Year College Graduate	$1,099.20

How much more does Niko earn than Miley in one week?
If Taylor and Miley both work for 2 weeks, how much more will Taylor earn?

How much money will Pinky earn in a month? About how long will Miley have to work to earn the same amount?

https://www.illustrativemathematics.org/contentstandards/5/NBT/B/7/tasks/1293

6. Select all the numbers that are equivalent to 0.4.

A $\frac{4}{10}$

B $\frac{4}{100}$

C $\frac{40}{100}$

D 0.04

E 0.40

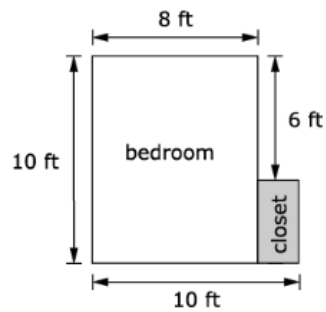
MICA

7. Select all the numbers that round to 300 when rounded to the **nearest** hundred.

- A 339
- B 245
- C 270
- D 305
- E 360

MICA

8. The shape of Rachel's bedroom and closet are shown. The measures of some of the walls are given.



Part A

Rachel plans to cover the floor of both the bedroom and the closet with carpet.
How much carpet, in square feet, will Rachel need?

MICA

NCTM Unproductive and Productive Beliefs

Beliefs About Teaching and Learning Mathematics	
Unproductive Beliefs	**Productive Beliefs**
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Independent/Small Group Time

- What are your biggest takeaways after reading the unproductive and productive beliefs?

- How can the differences in beliefs about teaching and learning mathematics impact students' opportunity to learn?

Mathematical Knowledge for Teaching

What do teachers need to know and be able to do in order to teach effectively? Or, what does effective teaching require in terms of content understanding?

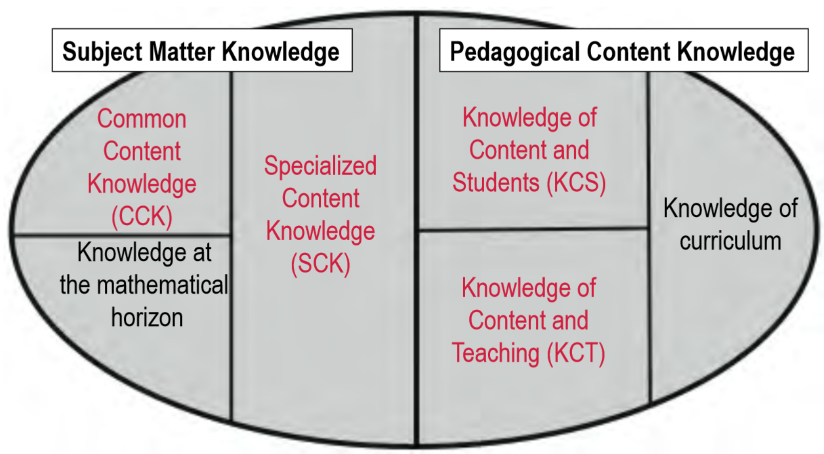
Mathematical knowledge for teaching (MKT) is a kind of professional knowledge of mathematics different from that demanded by other mathematically intensive occupations, such as engineering, physics, accounting, or carpentry. What distinguishes this sort of mathematical knowledge from other knowledge of mathematics is that it is subject matter knowledge needed by teachers for specific tasks of teaching, but still clearly subject matter knowledge. These tasks of teaching depend on mathematical knowledge, and, significantly, they have aspects that do not depend on knowledge of students or of teaching. These tasks require knowing how knowledge is generated and structured in the discipline and how such considerations matter in teaching, such as extending procedures and concepts of whole-number computation to the context of rational numbers in ways that preserve properties and meaning. These tasks also require a host of other mathematical knowledge and skill.

By "mathematical knowledge for teaching," we mean the mathematical knowledge needed to carry out the work of teaching mathematics. It is concerned with the tasks involved in teaching and the mathematical demands of these tasks. By "teaching," we mean everything that teachers must do to support the learning of their students. Clearly we mean the interactive work of teaching lessons in classrooms and all the tasks that arise in the course of that work. But we also mean planning for those lessons, evaluating students' work, writing and grading assessments, explaining the classwork to parents, making and managing homework, attending to concerns for equity, and dealing with the building principal who has strong views about the math curriculum. Each of these tasks, and many others as well, involve knowledge of mathematical ideas, skills of mathematical reasoning, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency.

- Kilpatrick, Swafford, & Findell, 2000.

MKT is divided into two domains: subject matter knowledge and pedagogical content knowledge. Each domain includes three categories. Subject matter knowledge includes: common content knowledge, specialized content knowledge, and knowledge of the mathematical horizon. Pedagogical content knowledge includes: knowledge of content and students, knowledge of the curriculum, and knowledge of content and teaching.

Mathematical Knowledge for Teaching



Subject Matter Knowledge

1) **Common Content Knowledge (CCK):** the mathematical knowledge and skill used in settings other than teaching. Teachers need to know the material they teach; they must recognize when their students give wrong answers, or when the textbook gives an inaccurate definition. When teachers write on the board, they need to use terms and notation correctly. In short, they must be able to do the work that they assign their students. But some of this requires mathematical knowledge and skill that others have as well- thus, it is not special to the work of teaching. By "common," however, we do not mean to suggest that everyone has this knowledge. Rather, we mean to indicate that this is knowledge of a kind used in a wide variety of settings-in other words, not unique to teaching.

2) **Knowledge at the Mathematical Horizon:** Horizon knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. First grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later. It also includes the vision useful in seeing connections to much later mathematical ideas. Having this sort of knowledge of the mathematical horizon can help in making decisions about how, for example, to talk about the number line.

3) **Specialized Content Knowledge (SCK):** the mathematical knowledge and skill unique to teaching. Close examination reveals that SCK is mathematical knowledge not typically needed for purposes other than teaching. In looking for patterns in student error or in sizing up whether a nonstandard approach would work in general, teachers have to do a kind of mathematical work that others do not. This work involves an uncanny kind of unpacking of mathematics that is not needed or even desirable in settings other than teaching. Many of the everyday tasks of teaching are distinctive to this special work.

Pedagogical Content Knowledge

4) **Knowledge of Content and Students (KCS):** the knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing. When choosing an example, teachers need to predict what students will find interesting and motivating. When assigning a task, teachers need to anticipate what students are likely to do with it and whether they will find it easy or hard. Teachers must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language. Each of these tasks requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking. Central to these tasks is knowledge of common student conceptions and misconception about particular mathematical content.

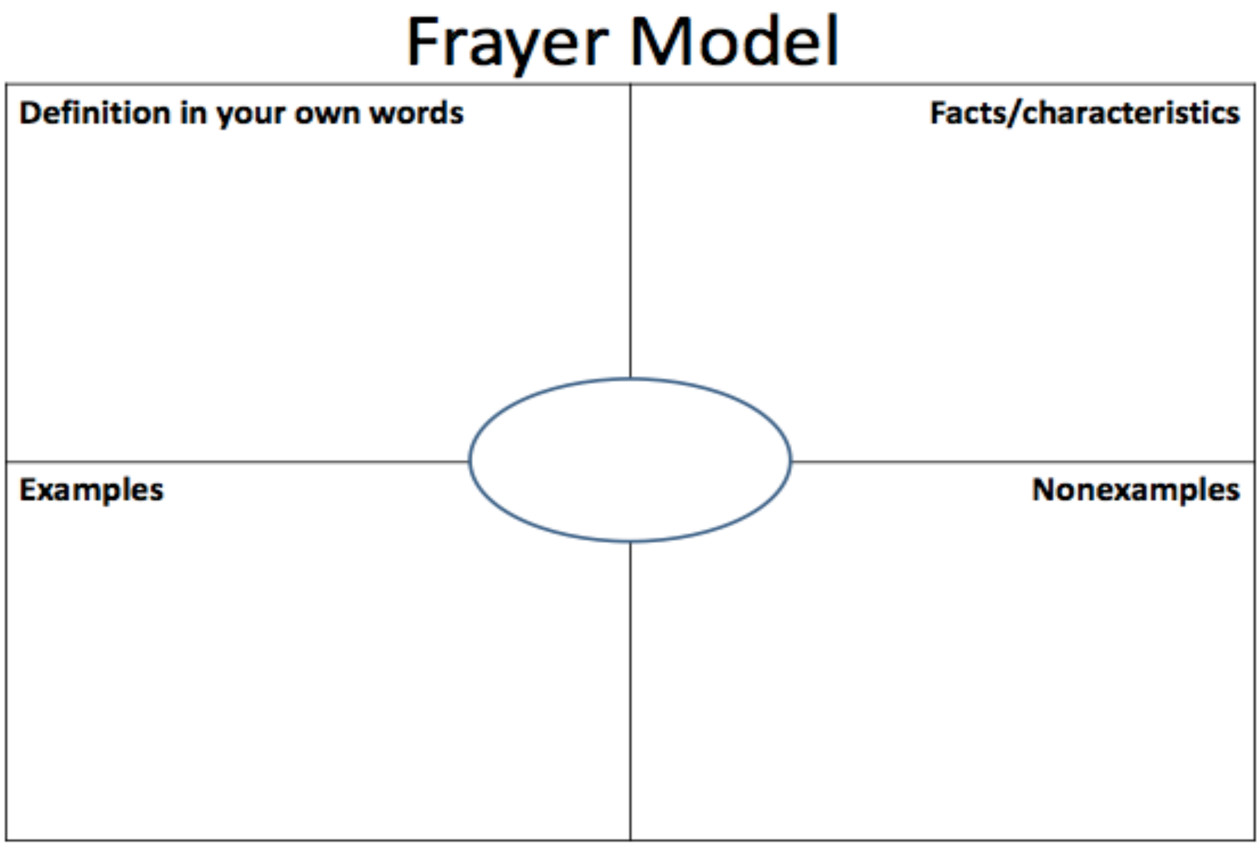
5) **Knowledge of Content and Teaching (KCT):** the knowledge that combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of the design of instruction. Teachers sequence particular content for instruction. They choose which examples to start with and which examples to use to take students deeper into the content. Teachers evaluate the instructional advantages and disadvantages of representations used to teach a specific idea and identify what different methods and procedures afford instructionally. Each of these tasks require an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning.

6) **Knowledge of Curriculum:** is "represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (Shulman, p. 10)."

-Ball, Thames and Phelps: Content Knowledge for Teaching, What Makes It Special? (2008).(n.d.).
- Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing Mathematics for Teaching: Who Knows Mathematics Well Enough To Teach Third Grade, and How Can We Decide?
-Shulman, L. S. (1986).Those who understand: Knowledge growth in teaching. *Educational Researcher, l* 5(2), 4-14.

MKT Frayer Model Activity

- Discuss the particular domain of MKT assigned to your group.
- With your group complete Frayer Model for your particular domain on chart paper.
- Be prepared to share your model.



Small Group Discussion

- What resonates with you about the idea of MKT?
- What did you find new or interesting?
- How do you as a professional mathematics teacher engage in increasing your levels of MKT?
- What will you think about doing differently as a result?

"Teachers who score higher on...measures of mathematical knowledge for teaching produce better gains in student achievement."

- Ball, et al., 2005.

Mathematics Teaching Practices

"Eight Mathematics Teaching Practices provide a framework for strengthening the teaching and learning of mathematics. This research-informed framework of teaching and learning reflects... knowledge of mathematics teaching that has accumulated over the last two decades. The list [below and on the next page] identifies these eight Mathematics Teaching Practices, which represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics"
(Principles to Actions, 2014, p. 9).

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	
Connections to previous trainings	*Tools and resources*
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	
Connections to previous trainings	*Tools and resources*
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.	
Connections to previous trainings	*Tools and resources*
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.	
Connections to previous trainings	*Tools and resources*

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.	
Connections to previous trainings	*Tools and resources*
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	
Connections to previous trainings	*Tools and resources*
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.	
Connections to previous trainings	*Tools and resources*
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.	
Connections to previous trainings	*Tools and resources*

Small Group Discussion

- What did you find affirming?
- What do you need to refine?
- How do these practices connect to the idea of MKT?

Think about it…

“Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels.”

-NCTM, Principles to Action Executive Summary.

Reflection

- Why is it necessary to consider productive vs. unproductive beliefs when designing lessons and implementing high-level tasks?

- How does having a deeper awareness of your own content and pedagogical knowledge impact your view of instruction?

- What will you do differently as a result?

- What are you still wondering about?

Module 2
Defining Mathematics Teaching and Learning Through Content Understanding

Rationale

"There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics."

- Lappan & Briars, 1995.

Goals

- Synthesize and refine our understanding of teaching and learning
- Consider the mathematical knowledge necessary to teach effectively
- Understand how deep understanding is necessary to support future learning

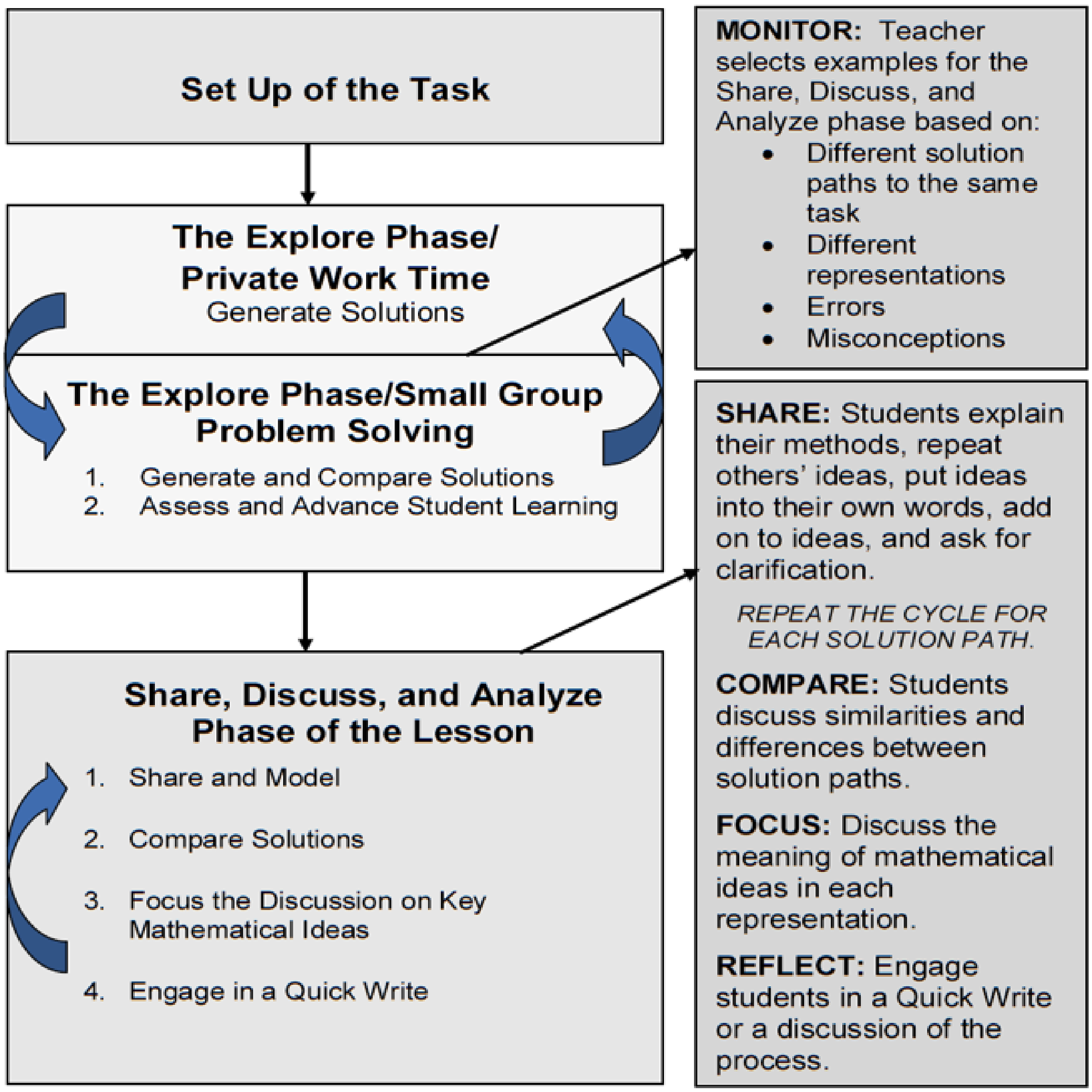
Session Activities

- Engage in a lesson
- Deepen our understanding of the mathematics in the NBT domain in grades 3-5

Overview

In Module 2, participants will engage in multiple content-rich tasks designed to further develop their content knowledge and conceptual understanding as teachers of mathematics. Participants will analyze their learning, and reflect on teh content through the lens of Mathematical Knowledge for Teaching (Ball, 2012), and the eight Mathematical Teaching Practices (NCTM, 2014).

Structures and Routines of a Lesson



Math Task

Independent Think Time

- Work on the following task privately

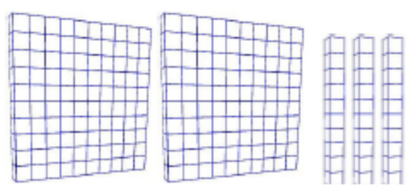
Small Group Time

- Compare your work with others in your group
- Consider the different ways to solve the task
- Place your group's thinking on chart paper

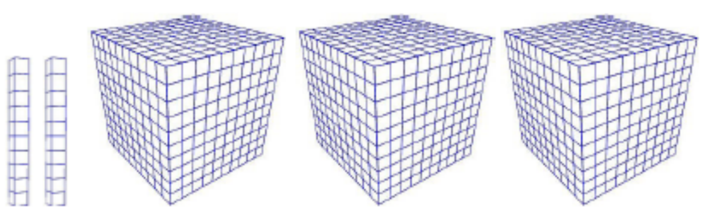
Task: Tree House Windows	**5th Grade**

Tom and Gina are thinking about the number 2.3 mils, which represents the thickness of the plastic sheeting used to cover windows of their tree house to keep out the wind in the winter. The teacher asks each of them to create a representation of the number 2.3 using place value blocks.

Tom chose a flat for his unit and represented 2.3 with two flats and 3 rods.



Gina chose a cube for her unit and represented 2.3 with two rods and 3 cubes.



a. Which do you think is the better representation for the number 2.3? Explain your reasoning.
b. Consider the representation you did not choose for part a. Explain why you don't think it is the best representation.
c. Create and draw your own representation of the number 2.3 using a rod for your unit. Explain why this representation is also appropriate.

Expectations for Whole Group Discussion

- Solution paths will be shared.
- Listen with the goals of:
  - putting the ideas into your own words;
  - adding on to the ideas of others;
  - making connections between solution paths; and
  - asking questions about the ideas shared.
- The goal is to understand the mathematics and to make connections among the various solution paths and representations.

Reflecting on Your Learning

- What about this lesson made it possible for you to learn?

- Which Standards for Mathematical Practice did you have the opportunity to use?

- Where did you see any of the eight teaching practices?

Setting Up The Task

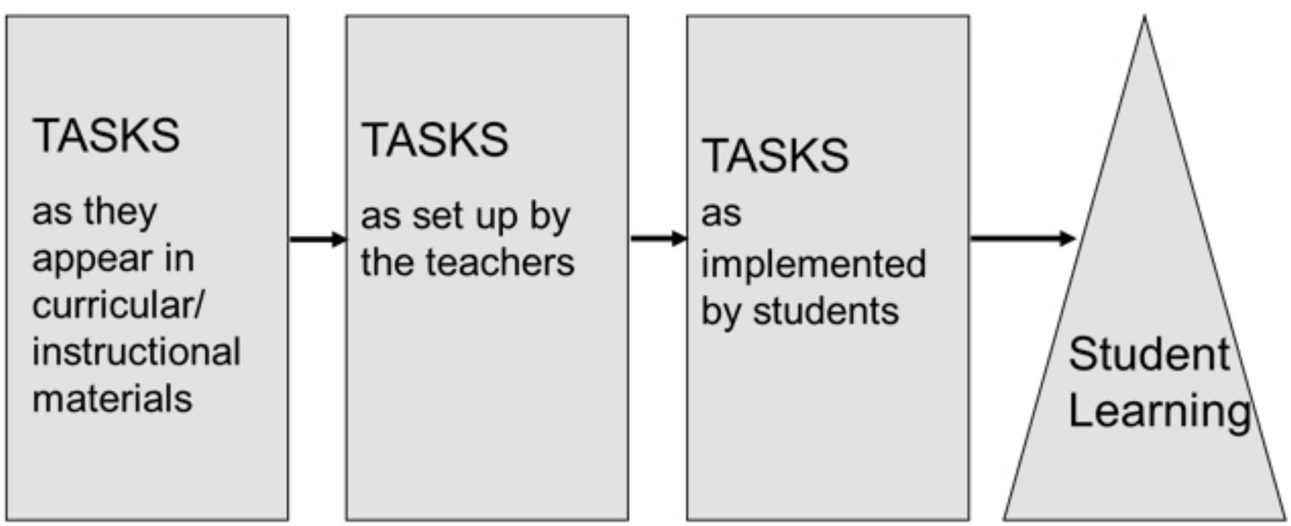
"A task's setup impacts both what students and the teacher are able to achieve during a lesson."

The set-up phase should orient the students to the task in order to ensure all students can engage productively. Here are some key features of the set-up phase:

1. Discuss contextual features.
2. Introduce key mathematical ideas.
3. Establish common language for discussing the mathematics in the task.
4. Maintain the demands of the task.

- Jackson, K., Shahan, E., Gibbons, L., & Cobb, P. (2012). Launching Complex Tasks*. Mathematics Teaching in the Middle School. 18*(1). 24-29.

The Mathematical Tasks Framework



-Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E.A. (2000). Implementing standards-based mathematics instruction: A casebook for professional development (p. 4). New York, NY: Teachers College Press.

Factors Associated with Maintenance and Decline of High-Level Cognitive Demands

"Higher- achieving countries implemented a greater percentage of high-level tasks in ways that maintained the demadns of the task."

-Stiegler & Hiebert, 2004.

Maintenance

- Scaffolds of student thinking and reasoning provided
- A means by which students can monitor their own progress is provided
- High-level performance is modeled
- A press for justifications, explanations through questioning and feedback
- Tasks build on students' prior knowledge
- Frequent conceptual connections are made
- Sufficient time to explore

Decline

- Problematic aspects of the task become routinized
- Understanding shifts to correctness, completeness
- Insufficient time to wrestle with the demanding aspects of the task
- Classroom management problems
- Inappropriate task for a given group of students
- Accountability for high-level products or processes not expected

Thinking Deeply about the Mathematics

Work on the Number Line Task privately for a few minutes before discussing with your group.

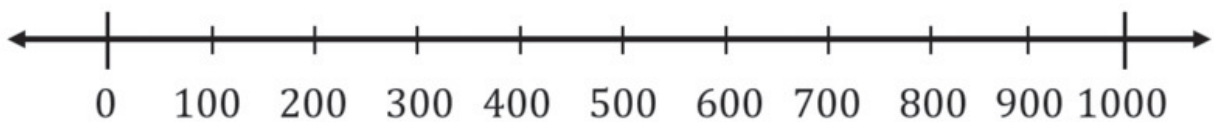
The Number Line Task- Task A

Plot the following numbers on the number line:

80

328

791



a. Round each number to the nearest 100. How can you see this on the number line?

b. Round each number to the nearest 1000. How can you see this on the number line?

c. Using what you know about place value, how can you always determine where a number should be placed on a number line?

Multiplying and Dividing by 10- Task B

Solve the equations. Draw models to represent your work.

a. 70 x 10 = ______________________

b. 600 ÷ 60 = _____________________

1. Explain how your models differ between each problem. Why is this?

2. Using your understanding of place value, multiplication and division, explain mathematically how these problems could be solved without the use of a model.

3. What does this tell you about the relationship between the digits in whole numbers?

Multiplying and Dividing by 10- Task C

Solve the following equations:

a. $7.2 \times 10 =$ ______________

b. $12.0 \times 10 =$ ____________

c. $364.1 \times 10 =$ ___________

d. $7.2 \div 10 =$ ______________

e. $12.0 \div 10 =$ ______________

f. $364.1 \div 10 =$ ____________

1. What patterns do you see in the products above?

2. Explain why these patterns are occurring mathematically.

3. Given what you know about place value, explain how multiplying and dividing numbers by ten impacts the digits within the number.

Multiplication and Division- Task D

Use the computation below to find the products.

$$\begin{array}{r} 189 \\ 16\overline{)3024} \\ \underline{16} \\ 142 \\ \underline{128} \\ 144 \\ \underline{144} \\ 0 \end{array}$$

1. 189×16

2. 80×16

3. 9×16

https://www.illustrativemathematics.org/content-standards/6/NS/B/2/tasks/270

Task Reflection

- What mathematical understandings from grades 3-5 did you need to have in order to solve these tasks? Be specific. (Use the Grades 3-5 Number and Operations Base Tens (NBT) Standards to help).

3rd Grade NBT Standards

3.NBT.A.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

4th Grade NBT Standards

4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division*.

4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place.

4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5th Grade NBT Standards

5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

5.NBT.A.3 Read, write, and compare decimals to thousandths.

- a.) Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
- b.) Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.A.4 Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Reflection

- In what ways did you think differently about the teaching and learning of mathematics?

- What new insights about the NBT Standards do you have?

- Which domain(s) of MKT did you feel increased and why is this important?

Module 3
Maintaining a Focus on Cognitive Rigor in Instruction

Rationale

Students who performed the best on project-based measures of reasoning and problem-solving were in classrooms in which tasks were more likely to be set up and enacted at high levels of cognitive demand.

-Stein & Lane, 1996; Stein, Lane & Silver, 1996.

The success of students was due in part to the high cognitive demand of the curriculum and the teachers' ability to maintain the level of demand during enactment through questioning.

-Boaler & Staples, 2008.

Goals

- Engage NCTM's Teaching Practices with an eye toward maintaining the cognitive rigor of the task
- Consider the importance of teaching as the intersection of content, pedagogy, and student learning

Session Activities

- Review tools that support rigorous task-based instruction
- Work on NCTM's Teaching Practices with student work
- Gallery walk and refine our ideas

Overview

Module 3 focuses participatns back into the importance of intentional instruction through the analysis of a task using tools from previous summer trainings (structures and routines of a lesson, assessing and advancing questions, accountable talk, ect.). Student work will be analyzed, and teachers will react to teh work using instructional strategies and effective teaching practices, all while working to maintain the cognitive demand of the task.

Tools to Support Task-Based Instruction

- On the next several pages are tools from previous summers that support selecting and enacting tasks at high levels of cognitive demand.
- In your small group, review your assigned tool and be prepared to give highlights to the whole group.

(Listen as other groups share for new insights if you are familiar or new learnings if you aren't familiar with these tools.)

Tools	New Insights or New Learnings
Task Analysis Guide	
Assessing and Advancing Questions	
Connections Between Representations	
Productive Discussions Through Selecting and Sequencing Student Work	
Accountable Talk Features and Indicators	

Task Analysis Guide

"If we want students to develop the capacity to think, reason, and problem-solve then we need to start with high-level, cognitively complex tasks."

-Stein, M. K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. Educational Research and Evaluation, 2 (4), 50-80.

Lower-Level Tasks	Higher-Level Tasks
Memorization Tasks • Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	Procedures With Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	Doing Mathematics Tasks • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Characteristics of Assessing and Advancing Questions

"Asking questions that assess student understanding of mathematical ideas, strategies or representations provides teachers with insights into what students know and can do. The insights gained from these questions prepare teachers to then ask questions that advance student understanding of mathematical concepts, strategies or connections between representations" (NCTM, 2000).

Assessing Questions

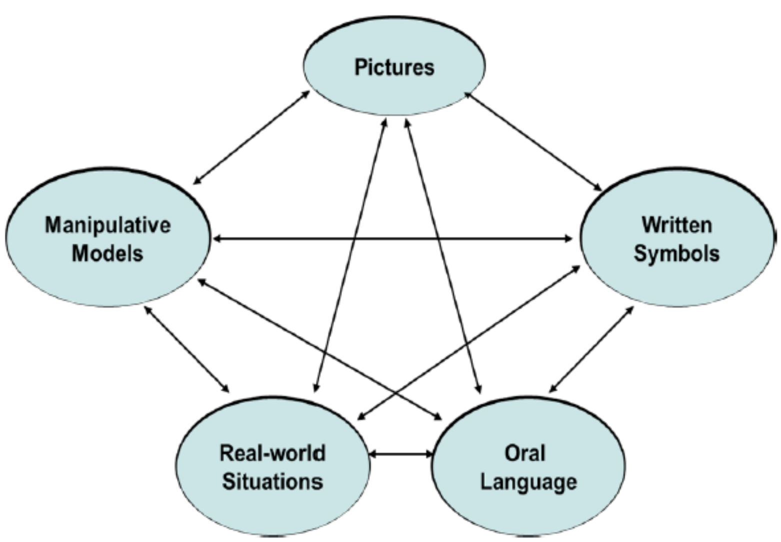
- Based closely on the work the student has produced.
- Clarify what the student has done and what the student understands about what s/he has done.
- Provide information to the teacher about what the student understands.

Advancing Questions

- Use what students have produced as a basis for making progress toward the target goal.
- Move students beyond their current thinking by pressing students to extend what they know to a new situation.
- Press students to think about something they are not currently thinking about.

Connections Between Representation

"Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling."



- Adapted from Lesh, Post, & Behr, 1987.

Orchestrating Productive Discussions

Teachers must "decide what aspects of a task to highlight, how to organize and orchestrate the work of students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge" (NCTM, 2000, p. 19).

Rules of Thumb for Selecting and Sequencing Student Solutions

Based on level of student understanding:

- It is okay to begin by showing incomplete work or work that is not completely clear in order to engage the class in a discussion regarding what else needs to happen to complete or clarify the solution strategy.
- Arrange solutions in order of increasing difficulty with the most complex methods presented last; may be a movement from concrete to abstract.
- Select solutions that illustrate both efficient and inefficient methods so that you can discuss circumstances in which one may be preferable over the other.
- Share at least one completely correct response.
- Don't be afraid to address misconceptions if they are critical to the mathematics being discussed, but stay away from responses that show profound misunderstandings or that do not advance the mathematical discussion.
- Consider individual accomplishments of students (e.g., Is there a student who has not presented in a few days? Is there a student who has done something that is quite unique that would give that student a chance to shine in front of his/her peers?).

Based on the diversity of/similarity of the answers within the classroom:

- Show most frequently used solution methods first to provide entry to all (or the majority) of students.
- Present solutions that show a range of representations (e.g., graphs, tables, equations, diagrams).
- Order solutions (or pair them) so that each solution builds (to the extent possible) on the solution that preceded it.

Based on reaching your mathematical goals:

- Keep the goals and essential understandings in mind and build the discussion so that more students can access these concepts and leave with rich understandings.
- Know that there is more than one way to go about presenting the solutions—have a reason for what you are doing and a goal that your sequencing will target.
- Make sure you get to the generalization (if there is one).

Accountable Talk® Features and Indicators

"Mathematics reform calls for teachers to engage students in discussing, explaining, and justifying their ideas. Although teachers are asked to use students' ideas as the basis for instruction, they must also keep in mind the mathematics that the class is expected to explore" (Sherin, 2000, p. 125).

Accountability to the Learning Community

- Active participation in classroom talk
- Listen attentively
- Elaborate and build on each other's ideas
- Work to clarify or expand a proposition

Accountability to Knowledge

- Specific and accurate knowledge
- Appropriate evidence for claims and arguments
- Commitment to getting it right

Accountability to Rigorous Thinking

- Synthesize several sources of information
- Construct explanations and test understanding of concepts
- Formulate conjectures and hypotheses
- Employ generally accepted standards of reasoning
- Challenge the quality of evidence and reasoning

Think About It…

- Teacher A says, "My students work on problems so they know how to answer these types of problems."

- Teacher B says, "My students work on problems so they can learn the mathematics in these types of problems."

What is the difference between the goals of teacher A and teacher B and why is this significant?

Task-Based Instruction: True or False?

- All tasks must be high-level.
- Accountable talk® is only used during a high-level task.
- All high level instructional tasks must have a context.
- TN State Standards require task-based instruction.
- Students never need to engage in low-level tasks.
- Tasks are most effective when they are used to solidify learning.

Effective Uses of High Level Tasks	Ineffective Uses of High Level Tasks

Rounding Decimals Task

When Jamir goes to the store, he notices that a gallon of milk is also labeled 3.79 liters.

a. Write the number 3.79 in word form.

Jamir claims that one gallon of milk is close to 3.8 liters. His dad says it is close to 4 liters.

b. Explain how Jamir and his dad can round 3.79 in different ways to get their answers.

Small Group Discussion

- Discuss your responses and possible student responses to the task.

Analyzing Student Work

Independent Think Time

- Carefully examine each student's work.
- What do the students know? Not know? What is the evidence?

Student A

a. Write the number 3.79 in word form.

three and seventy-nine hunredths

Jamir claims that one gallon of milk is close to 3.8 liters. His dad says it is close to 4 liters.

b. Explain how Jamir and his dad can round 3.79 in different ways to get their answers.

Jamir can round his to the nearest tenth, but his dad rounded it to the nearest whole

Student B

a. Write the number 3.79 in word form.

Three and Seventy-Nine hundredths.

b. Explain how Jamir and his dad can round 3.79 in different ways to get their answers.

For 3.8, you would round 3.79 to the nearest tenth, which would become 3.8. For 4, you would round 3.79 to the nearest whole, which would become 4.

Student C

a. Write the number 3.79 in word form.

three point seventy-nine hundreths

b. Explain how Jamir and his dad can round 3.79 in different ways to get their answers.

His dad round by the whole number. Jamir did the tenth.

Student D

a. Write the number 3.79 in word form.

3 and 79 hundredths

b. Explain how Jamir and his dad can round 3.79 in different ways to get their answers.

3.79 = 3.80

the nine makes it go to 80

3.79 = 4.00

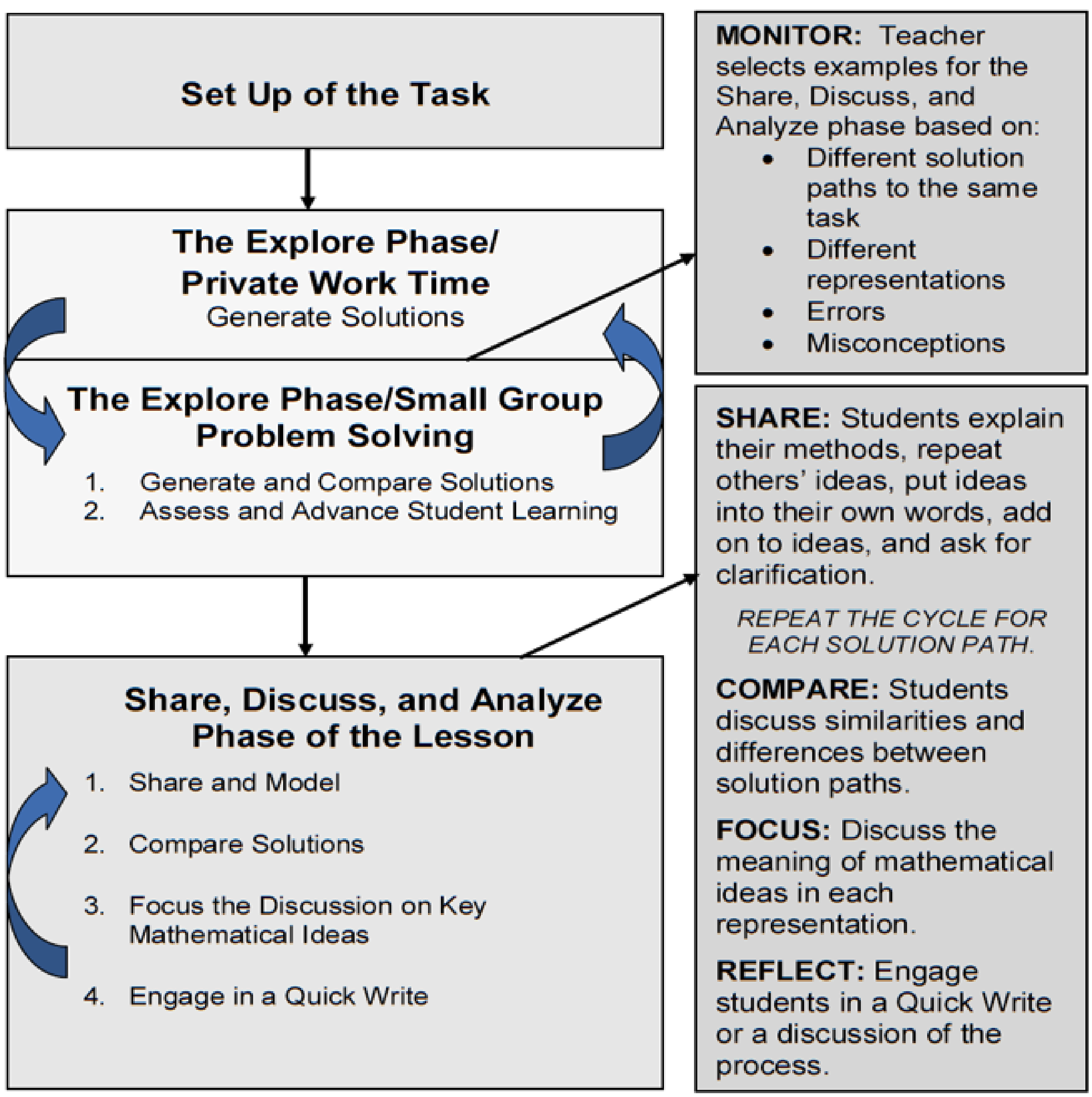
the seven makes it go to four

Engaging with the Teaching Practices

With your group, discuss the following questions to complete the chart on the next page.

- What would be your mathematical goal for students as they complete this task?
- For each piece of student work, write one assessing and one advancing question.
- What are the representations that you want to come out in the discussion to support the mathematics?
- Sequence the student work in the order you think it should be shared with the class. What questions might you ask to bring out and connect in support of your goal?
- Are there any other ways to support productive struggle? (i.e. scaffolds, manipulatives)

Structure and Routines of a Lesson



Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
What are the mathematical goals for this task?
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Write one assessing and advancing question for each student.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Sequence the student work for share-discuss-analyze phase of the lesson. *What questions might you ask to problematize the discussion?*
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Which representations are critical for understanding the mathematics? *How will you ask students to make connections among these representations?*
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Are there any other ways to support productive struggle? (i.e. scaffolds, manipulatives)

Establish mathematics goals to focus learning	
What are *teachers* doing?	**What are *students* doing?**
• Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. • Identifying how the goals fit within a mathematics learning progression. • Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. • Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	• Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) • Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. • Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. • Assessing and monitoring their own under-standing and progress toward the mathematics learning goals.
Pose purposeful questions	
What are *teachers* doing?	**What are *students* doing?**
• Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. • Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. • Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. • Allowing sufficient wait time so that more students can formulate and offer responses.	• Expecting to be asked to explain, clarify, and elaborate on their thinking. • Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. • Reflecting on and justifying their reasoning, not simply providing answers. • Listening to, commenting on, and questioning the contributions of their classmates.
Facilitate meaningful mathematical discourse	
What are *teachers* doing?	**What are *students* doing?**
• Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. • Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. • Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. • Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	• Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. • Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. • Seeking to understand the approach-es used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. • Identifying how different approaches to solving a task are the same and how they are different.

Use and connect mathematical representations	
What are *teachers* doing?	**What are *students* doing?**
• Selecting tasks that allow students to decide which representations to use in making sense of the problems. • Allocating substantial instructional time for students to use, discuss, and make connections among representations. • Introducing forms of representations that can be useful to students. • Asking students to make math drawings or use other visual supports to explain and justify their reasoning. • Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. • Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	• Using multiple forms of representations to make sense of and understand mathematics. • Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. • Making choices about which forms of representations to use as tools for solving problems. • Sketching diagrams to make sense of problem situations. • Contextualizing mathematical ideas by connecting them to real-world situations. • Considering the advantages or suitability of using various representations when solving problems.

Support productive struggle in learning mathematics	
What are *teachers* doing?	**What are *students* doing?**
• Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. • Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. • Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. • Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	• Struggling at times with mathematics tasks but knowing that breakthroughs of-ten emerge from confusion and struggle. • Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. • Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. • Helping one another without telling their classmates what the answer is or how to solve the problem.

Gallery Walk

- As a group, examine the work of the other groups.
- Leave targeted, actionable feedback.
- When you return to your group's work, consider revising your work based on the feedback.
- Be prepared to share your thinking with the whole group.

Reflection

- What are some new understandings you have regarding the tools to support task-based instruction?

- What do you believe to be the most critical considerations when reviewing student work? Why?

- Why is it important to consider the mathematical teaching practices when planning to implement a high-level task in your classroom?

- In what ways has this module increased your knowledge in any of the domains of MKT?

Module 4
Implications on Planning and Instruction

Rationale

"The move toward rigor places students squarely at the center of the classroom, where they will grapple with challenging content individually and collaboratively, and where they will be expected to actively demonstrate their learning."

-Marzano and Toth, 2014.

Goals

- Learn about TNReady design for mathematics and instructional implications
- Consider the importance of coherence for learning mathematics at the right grain size
- Examine how student learning is developed through a unit of study

Session Activities

- View blueprints, item types, and calculator policy of TNReady math assessment
- "Unpack" blueprints to consider instructional implications for 2015-2016
- Discuss how the concepts of coherence and grain size are connected to task arc creation and implementation
- Discuss planning units of study as they relate to coherence and the progression of student understanding

Overview

Participants in Module 4 will review the structure and design of the TNReady assessment. Activities in Module 4 engage participants in the process of focusing on the instructional and planning implications of the new assessment, while maintain a strong emphasis on intentionality and coherence and the impact of this focus on unit and lesson planning.

TNReady Note Table

	Noticings	Wonderings	Impact on Planning and Instruction
Math Priorities			
Calculator Policy			
Item Types			
Fluency			
Blueprints			

TNReady Overview

Beginning with the 2015-16 school year, TNReady will provide students, teachers, and parents with more detailed, accurate, and authentic information about each student's progress and achievement in the classroom.

TNReady is more than just a new "TCAP." It is a new way to assess what our students know and what we can do to help them succeed in the future.

TNReady Math Priorities

- Grades 3-8: Focus on fewer concepts – assess those topics in a range of ways
- High School: Strengthen coherence – assess topics in connected ways
- Include authentic assessment of real-life situations
- Support alignment with ACT
- Include calculator-permitted and calculator-prohibited sections at every grade level

Calculator Policy

Two central beliefs:

- Calculators are important tools for college and career readiness.
- Students must be able to demonstrate many skills without reliance on calculators.

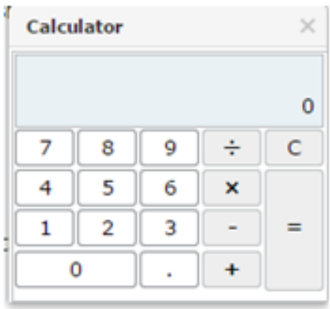
At all grade levels and in all courses, **TNReady will include both calculator permitted and calculator prohibited sections.**

Examples of permitted and non-permitted calculators, consistent with ACT and other benchmark assessments.

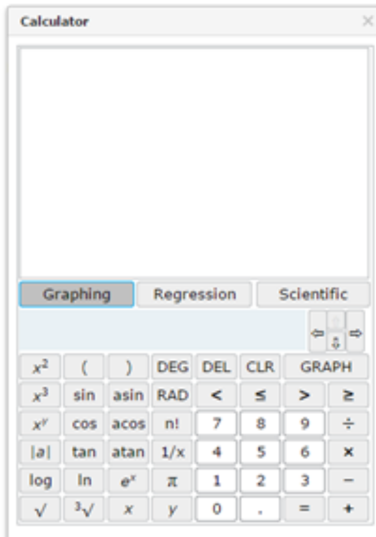
Handhelds are permitted with online testing.

Calculator Types

Basic



Graphing



Scientific

Calculator Policy – Think About it…

- How do the changes in calculator policy impact classroom instruction?

- What considerations will have to be made to ensure students are prepared for this transition?

Math Item Types Overview

There are 6 types of items in Mathematics:

1. Equation
2. Graphic
3. Multiple Choice
4. Multiple Select
5. Performance Tasks (for grade 3-8 only)
6. Technology Enhanced Items

Fluency

Grade	Standard	Expected Fluency
3	**3.**OA.C.7 **3.**NBT.A.2	Multiply/Divide within 100 (Know single digit products from memory) Add/Subtract within 1000
4	4.NBT.B.4	Add/Subtract within 1,000,000
5	5.NBT.B.5	Multi-digit multiplication
6	6.NS.B.2 6.NS.B.3	Multi-digit division Multi-digit decimal operations

Grades 3-6 Fluency standards will only be assessed on Part II of the TNReady assessment. Calculators will not be allowed on fluency items.

Math Blueprints

Blueprint Summary Includes:

- Range of number of items for each part
- Range of percentage for each part
- Total range of number of items
- Total range of percentage for each part
- Percentage of test derived from each cluster

Each blueprint also includes a table that shows:

  - What standards are assessed on Part I
  - What standards are assessed on Part II
  - An overall table for both Part I and Part II

Each of these tables also includes a range of number of items and a range of score points

Grades 3-8 Math Blueprints

- 100% of the content on Part I of the math section will be drawn from the major work of the grade
- 40-60% of the content on Part II of the math section will be drawn from the major work of the grade
- Across both Part I and Part II, 65-75% of the content of the math section will be drawn from the major work of the grade

Content drawn from the major work of the grade is bolded.

High School Math Blueprints

- Clusters have been grouped by category
- Part I includes items that are:
    - Best assessed through equation, graphic and performance tasks
    - Topics that are widely recognized prerequisites for college readiness
    - Topics that need to be treated in a coherent way
- Part II includes all standards with continued focus on questions that draw on the coherence of the standards
- Standards information is available by part

Blueprint Activity

Small Group

- Review the blueprint for your grade level or course.
- Look for the following ideas:
    - Which standards are only assessed in either Part I or Part II?
    - Which standards are assessed on both Part I and Part II?
    - Which sections are assessed most heavily? Least?

Whole Group Discussion

- How will this inform or modify instructional decisions for 2015-2016?
- What activities should we work to engage in to best prepare our students for success?

Grain Size

"Each discipline has a granularity at which its truth is clearest, most coherent. To depart from this grain size in either direction is to depart from the truth."

-Aristole, Ethics.

"[When considering mathematics] proper grain size is the unit at which it makes most sense to organize mathematics for learning."

-Daro, 2013.

Small Group Discussion

- Discuss your noticings and wonderings from the video with your small group.

Grain Size

- Mathematics is simplest at the right grain size
- "Strands" are too big, vague e.g. "numbers"
- Lessons are too small: too many small pieces scattered over the floor, what if some are missing or broken?
- Units are the right size (8-12 a year)
- Stop managing lessons
- Start managing units

Coherence

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations.

-CCSSM K-8 Publishers Criteria, 2012.

Small Group Discussion

- How are the concepts of coherence and grain size connected?

Whole Group Discussion

- Why is it important to consider coherence across standards and concepts when applying the ideas of “grain size” to our planning?

Think About It...

- How do we provide coherent structures around mathematical concepts to ensure students make connections?

Task Arc

[Materials] are coherent if they are: articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential and hierarchical nature of the disciplinary content from which the subject matter derives.

- Schmidt & Houang, 2012, "Curricular Coherence and the Common Core State Standards for Mathematics," Educational Researcher, http://edr.sagepub.com/content/41/8/294, p. 295.

Small Group Discussion

Review the task arc located in your handouts, specifically the arc preview and standards alignment pages. Discuss the following ideas with your small group:

- How does the structure of the task arc support instruction and student learning?
- How do the task arcs support the idea of grain size discussed in the video?
- How could task arcs be utilized in the planning of a unit of study?

Record your group take-aways on chart paper.

Private Think Time

- How do task arcs support the instruction of a group of standards?

Small Group Discussion

- How does the progression of standards within the task arc support student learning? Why is this progression important?

Unit Planning

It is the nature of mathematics that much new learning is about extending knowledge from prior learning to new situations. For this reason, teachers need to understand the progressions in the standards so they can see where individual students and groups of students are coming from, and where they are heading.

- Daro, McCallum, Zimba, 2012.

Private Think Time

- How do you, or teachers in your building, currently work to plan mathematic units of study?

- Do these practices align with the ideas of coherence, grain size, and learning progressions? How or how not?

Small Group Discussion

- How can you utilize the ideas of coherence and grain size to support your own unit planning?
- What resources and tools do you have to support unit planning?
- How can you utilize these tools and resources to ensure student learning develops throughout the unit?
- How do the teaching practices and MKT domains impact your ability to provide coherence and support student learning in your unit plan?

Record your groups thinking on chart paper.

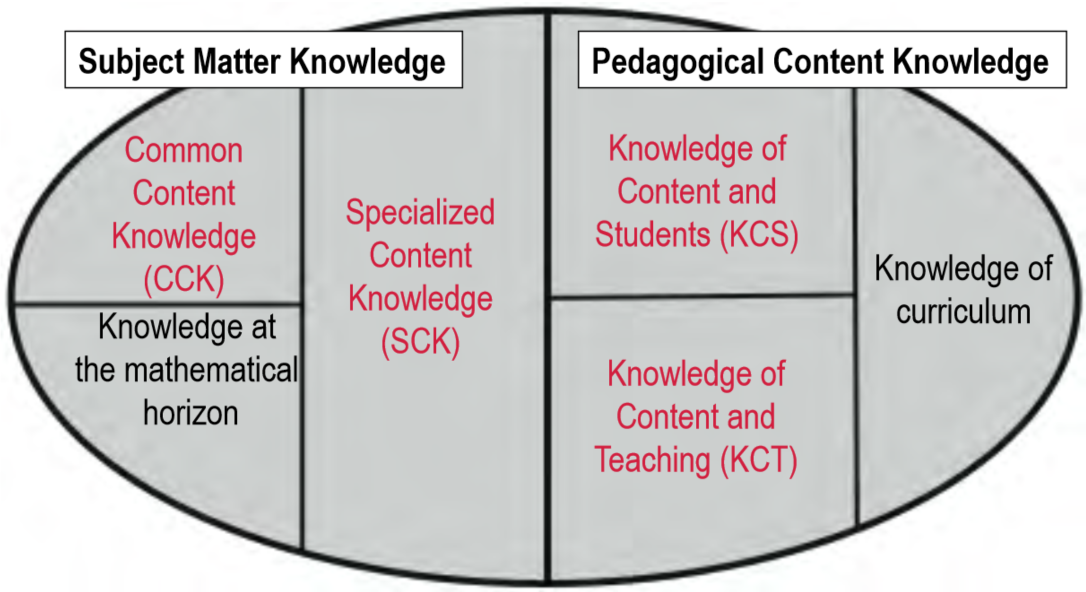
Gallery Walk

- Review other groups ideas on unit planning. Record noticings and wonderings as you walk.

Whole Group Discussion

- How do we work to ensure units of study support student progression through the content in a coherent manner?

Mathematical Knowledge for Teaching



Think About it…

- How does increasing the six domains of subject matter and pedagogical content knowledge areas of MKT support your implementation of the 8 teaching practices as you think about planning, instruction, and assessment?

- Why is it important to consider all six domains when planning a unit of study?

Reflection

- How has this module changed the way you think about planning and implementing units of study?
- What connections can you make between the four modules we have completed?
- What are you still wondering about?

Appendix

Appendix

Tennessee State Standards

Appendix A: Standards for Mathematical Practice

Appendix B: Academic Standards

TNCore Tools

Appendix C: Structure and Routines of a Lesson

Appendix D: Math Accountable Talk® Academic Discussion

Appendix E: Task Analysis Guide

Appendix F: Connections between Representations

Appendix G: Strategies for Modifying Textbook Tasks

TNReady Blueprint

Appendix H: TNReady Blueprints

Appendix I: Fluency

Appendix J: Item Types

Appendix K: Calculator Policies

Appendix L: Practice Tools

Task Resourcces

Appendix M: Task Packet

Appendix N: Task Arc

Progressions

Appendix O: K-5, Number and Operations in Base Ten

Tennessee State Standards

Appendix A Standards for Math Practice

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Appendix B
Academic Standards

Tennessee's State Mathematics Standards | Grade 3

In Grade 3, instructional time should focus on four critical areas:

(1) developing understanding of multiplication and division and strategies for multiplication and division within 100;
(2) developing understanding of fractions, especially unit fractions (fractions with numerator 1);
(3) developing understanding of the structure of rectangular arrays and of area; and
(4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Adopted by the Tennessee State Board of Education, July 2010

Domain	Cluster	Standard
Operations and Algebraic Thinking	**Represent and solve problems involving multiplication and division.**	1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
		2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
		3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1
		4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*
	Understand properties of multiplication and the relationship between multiplication and division.	5. Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3× 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* (Students need not use formal terms for these properties.)
		6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
	Multiply and divide within 100.	7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
	Solve problems involving the four operations, and identify and explain patterns in arithmetic.	8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)
		9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Domain	Cluster	Standard
Number and Operations in Base Ten	**Use place value understanding and properties of operations to perform multi-digit arithmetic.** (A range of algorithms may be used.)	1. Use place value understanding to round whole numbers to the nearest 10 or 100.
		2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
		3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
Number and Operations—Fractions*	**Develop understanding of fractions as numbers.**	1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
		2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
		3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 =2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. *Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Domain	Cluster	Standard
Measurement and Data	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
		2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).6 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
	Represent and interpret data.	3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
		4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	5. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
		6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
		7. Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
Geometry	Reason with Shapes and their attributes.	1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
		2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

	Major Content		**Supporting Content**		**Additional Content**

Tennessee's State Mathematics Standards | Grade 4

In Grade 4, instructional time should focus on three critical areas:

(1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends;
(2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers;
(3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Domain	Cluster	Standard
Operations and Algebraic Thinking	Use the four operations with whole numbers to solve problems.	1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
		2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
		3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
	Gain familiarity with factors and multiples.	4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
	Generate and analyze patterns.	5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*
Number and Operations in Base Ten*	Generalize place value understanding for multi-digit whole numbers.	1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
		2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
		3. Use place value understanding to round multi-digit whole numbers to any place.
	Use place value understanding and properties of operations to perform multi-digit arithmetic.	4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
		5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
		6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. *Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Domain	Cluster	Standard
Number and Operations—Fractions**	**Extend understanding of fraction equivalence and ordering.**	1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
		2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* *3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
		4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
	Understand decimal notation for fractions, and compare decimal fractions.	5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.4 *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)
		6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
		7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. **Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Domain	Cluster	Standard
Measurement and Data	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*
		2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
		3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
	Represent and interpret data.	4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
	Geometric measurement: understand concepts of angle and measure angles.	5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
		6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
		7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
Geometry	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
		2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
		3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

	Major Content		Supporting Content		Additional Content

Tennessee's State Mathematics Standards | Grade 5

In Grade 5, instructional time should focus on three critical areas:

(1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions);
(2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and
(3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Adopted by the Tennessee State Board of Education, July 2010

Domain	Cluster	Standard
Operations and Algebraic Thinking	**Write and interpret numerical expressions.**	1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
		2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
	Analyze patterns and relationships.	3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*
Number and Operations in Base Ten	**Understand the place value system.**	1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
		2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
		3. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
		4. Use place value understanding to round decimals to any place.
	Perform operations with multi-digit whole numbers and with decimals to hundredths.	5. Fluently multiply multi-digit whole numbers using the standard algorithm.
		6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
		7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)

Domain	Cluster	Standard
Number and Operations – Fractions	**Use equivalent fractions as a strategy to add and subtract fractions.**	1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. *(In general,* $a/b + c/d = (ad + bc)/bd$.*)*
		2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $2/5 + 1/2 = 3/7$, *by observing that* $3/7 < 1/2$.
	Apply and extend previous understandings of multiplication And division to multiply and divide fractions.	3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
		4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(2/3) \times 4 = 8/3$, *and create a story context for this equation. Do the same with* $(2/3) \times (4/5) = 8/15$. *(In general,* $(a/b) \times (c/d) = ac/bd$.*)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
		5. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
		6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
		7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $(1/3) \div 4$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $(1/3) \div 4 = 1/12$ *because* $(1/12) \div 4 = 1/3$. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div (1/5)$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div (1/5) = 20$ *because* $20 \times (1/5) = 4$. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* [1]Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Domain	Cluster	Standard
Measurement and Data	Convert like measurement units within a given measurement system.	1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
	Represent and interpret data.	2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
		4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
		5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
Geometry	Graph points on the coordinate plane to solve real-world and mathematical problems.	1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
		2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
	Classify two-dimensional figures into categories based on their properties.	3. Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
		4. Classify two-dimensional figures in a hierarchy based on properties.

	Major Content		**Supporting Content**		**Additional Content**

TNCore Tools

Appendix C
Structure and Routines of a Lesson

The Structure and Routines of a Lesson

Set Up of the Task

The Explore Phase/
Private Work Time
Generate Solutions

The Explore Phase/Small Group Problem Solving

1. Generate and Compare Solutions
2. Assess and Advance Student Learning

Share, Discuss, and Analyze Phase of the Lesson

1. Share and Model
2. Compare Solutions
3. Focus the Discussion on Key Mathematical Ideas
4. Engage in a Quick Write

MONITOR: Teacher selects examples for the Share, Discuss, and Analyze phase based on:

- Different solution paths to the same task
- Different representations
- Errors
- Misconceptions

SHARE: Students explain their methods, repeat others' ideas, put ideas into their own words, add on to ideas, and ask for clarification.

REPEAT THE CYCLE FOR EACH SOLUTION PATH.

COMPARE: Students discuss similarities and differences between solution paths.

FOCUS: Discuss the meaning of mathematical ideas in each representation.

REFLECT: Engage students in a Quick Write or a discussion of the process.

Appendix D
Math Accountable Talk® Academic Discussion

institute for learning

TNCore

Accountable Talk® Features and Indicators

Accountability to the Learning Community

- Active participation in classroom talk
- Listen attentively
- Elaborate and build on each other's ideas
- Work to clarify or expand a proposition

Accountability to Knowledge

- Specific and accurate knowledge
- Appropriate evidence for claims and arguments
- Commitment to getting it right

Accountability to Rigorous Thinking

- Synthesize several sources of information
- Construct explanations and test understanding of concepts
- Formulate conjectures and hypotheses
- Employ generally accepted standards of reasoning
- Challenge the quality of evidence and reasoning

Accountable Talk® Moves

Talk Move	Function	Example
To Ensure Purposeful, Coherent, and Productive Group Discussion		
Marking	Direct attention to the value and importance of a student's contribution.	That's an important point.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: Is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	S: 4 + 4 + 4. You said three groups of four.
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?

institute for learning

TNCore

Accountable Talk® Moves

Talk Move	Function	Example
	To Ensure Purposeful, Coherent, and Productive Group Discussion	
Marking	Direct **attention** to the value and importance of a student's contribution.	It is important to say describe to compare the size of the pieces and then to look at how many pieces of that size.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: Is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	You said 3, yes there are three columns and each column is 1/3 of the whole
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?
	To Support Accountability to Community	
Keeping the Channels Open	Ensure that students can hear each other, and remind them that they must hear what others have said.	Say that again and louder. Can someone repeat what was just said?
Keeping Everyone Together	Ensure that everyone not only heard, but also understood, what a speaker said.	Can someone add on to what was said? Did everyone hear that?
Linking Contributions	Make explicit the relationship between a new contribution and what has gone before.	Does anyone have a similar idea? Do you agree or disagree with what was said? Your idea sounds similar to his idea.
Verifying and Clarifying	Revoice a student's contribution, thereby helping both speakers and listeners to engage more profitably in the conversation.	So are you saying..? Can you say more? Who understood what was said?
	To Support Accountability to Knowledge	
Pressing for Accuracy	Hold students accountable for the accuracy, credibility, and clarity of their contributions.	Why does that happen? Someone give me the term for that.
Building on Prior Knowledge	Tie a current contribution back to knowledge accumulated by the class at a previous time.	What have we learned in the past that links with this?
	To Support Accountability to Rigorous Thinking	
Pressing for Reasoning	Elicit evidence to establish what contribution a student's utterance is intended to make within the group's larger enterprise.	Say why this works. What does this mean? Who can make a claim and then tell us what their claim means?
Expanding Reasoning	Open up extra time and space in the conversation for student reasoning.	Does the idea work if I change the context? Use bigger numbers?

Appendix E
Task Analysis Guide

The Mathematical Task Analysis Guide

Lower-Level Demands

Memorization Tasks

- Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.

Procedures Without Connections Tasks

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers rather than developing mathematical understanding.
- Require no explanations, or explanations that focus solely on describing the procedure that was used.

Higher-Level Demands

Procedures With Connections Tasks

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

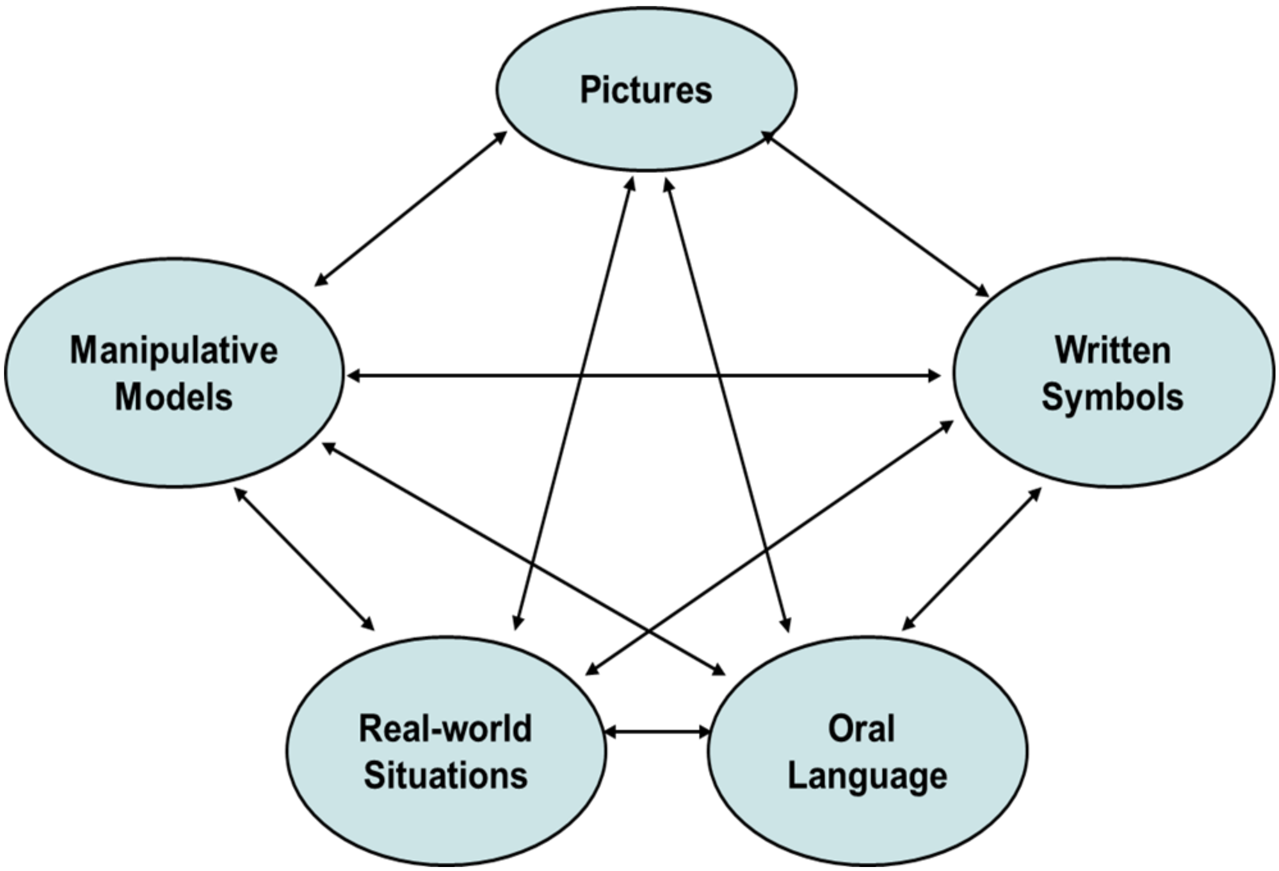
Doing Mathematics Tasks

- Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships.
- Demands self-monitoring or self-regulation of one's own cognitive processes.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Mathematics Teaching in the Middle School. Also in: Stein, Smith, Henningsen, & Silver (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 16. New York: Teachers College Press.

***Appendix* F**
Connections Between Representations

Linking to Research/Literature
Connections between Representations



Adapted from Lesh, Post, & Behr, 1987

Appendix G
Strategies for Modifying Textbook Tasks

Strategies for Modifying Textbook Tasks

Compare your list of task modifications with the list of task modification strategies identified by others. How is your list similar? Different?

- Ask students to create real-world stories for “naked number” problems.
- Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, with a context).
- Include a prompt that requires students to make a generalization.
- Use a task “out of sequence” before students have memorized a rule or have practiced a procedure that can be routinely applied.
- Eliminate components of the task that provide too much scaffolding.
- Adapt a task so as to provide more opportunities for students to think and reason – let students figure things out for themselves.
- Create a prompt that asks students to write about the meaning of the mathematics concept.
- Add a prompt that asks students to make note of a pattern or to make a mathematical conjecture and to test their conjecture.
- Include a prompt that requires students to compare solution paths or mathematical relationships and write about the relationship between strategies or concepts.
- Select numbers carefully so students are more inclined to note relationships between quantities (e.g., two tables can be used to think about the solutions to the four, six or eight tables).

TNReady Blueprints

***Appendix* H**
TNReady Blueprints

TNReady 3rd Grade Math Blueprint

	Part I		Part II		Total # of Items	Total # of score points	% of Test
	# of items	% of PT 1	# of items	% of PT 2			
Major Work of the Grade	**25-29**	**100%**	**22-26**	**60-65%**	**47-55**	**47-65**	**75-80%**
• **Solve problems involving multiplication and division**	**3-5**	**14-16%**	**2-4**	**6-8%**	**5-9**	**5-11**	**9-11%**
• **Understand multiplication and the relationship between multiplication and division**	**2-4**	**12-14%**	**2-4**	**7-9%**	**4-8**	**4-10**	**9-11%**
• **Multiply and divide within 100**	**0**	**0%**	**3-5**	**9-11%**	**3-5**	**3-7**	**5-7%**
• **Solve problems and identify and explain patterns in arithmetic**	**3-5**	**14-16%**	**2-4**	**7-9%**	**5-9**	**5-11**	**9-11%**
• **Develop understanding of fractions as numbers**	**5-7**	**22-24%**	**4-6**	**11-13%**	**9-13**	**9-15**	**15-17%**
• **Solve problems involving measurement and estimation**	**4-6**	**17-19%**	**2-4**	**6-8%**	**6-10**	**6-12**	**11-13%**
• **Geometric measurement: understand concepts of area**	**4-6**	**17-19%**	**2-4**	**6-8%**	**6-10**	**6-12**	**11-13%**
Additional and Supporting	0	0%	16-20	35-40%	16-20	16-26	20-25%
• Use place value understanding and properties of operations to perform multi-digit arithmetic	0	0%	4-6	12-14%	4-6	4-8	8-10%
• Represent and interpret data	0	0%	3-5	9-11%	3-5	3-7	5-7%
• Geometric measurement: recognize perimeter and distinguish between linear and area measures	0	0%	2-4	6-8%	2-4	2-6	3-5%
• Reason with shapes and their attributes	0	0%	4-6	11-13%	4-6	4-8	6-8%
Total	**25-29**	**100%**	**38-46**	**100%**	**63-75**	**63-85**	**100%**

Additional Notes:

*****On Part I, 100% of the content in grades 3-8 mathematics will be drawn from the clusters designated as major work of the grade. The math standards in grades 3-8 are coherent and the connections between major work and the additional and supporting clusters are important throughout the year. Part II questions measure major, additional, and supporting topics, including the integration of the key ideas at each grade level.

*****The total number of score points does not match the total number of items. This is because some items may be worth more than one point.

*****Clusters drawn from the major work of the grade are bolded throughout this document.

Part I – Calculator Allowed

100% of the content in Part I is drawn from the major work

Cluster	Standards		# of Items
3.OA.A - Represent and solve problems involving multiplication and division.	3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.	3-5
	3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.	
	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	
	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	
3.OA.B- Understand properties of multiplication and the relationship between multiplication and division.	3.OA.B.5	Apply properties of operations as strategies to multiply and divide.	2-4
	3.OA.B.6	Understand division as an unknown-factor problem.	
3.OA.D- Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)	3-5
	3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.	
3.NF.A- Develop understanding of fractions as numbers.	3.NF.A.1	Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.	5-7
	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	

	3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 =2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
3.MD.A- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	4-6
	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	
3.MD.C- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	3.MD.C.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	4-6
	3.MD.C.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	
	3.MD.C.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	

Part II – Calculator and Non-Calculator Portions

60-65% of the content in Part II is drawn from the major work

Cluster	Standards		# of Items
3.OA.A - Represent and solve problems involving multiplication and division.	3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.	2-4
	3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.	
	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	
	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	
3.OA.B- Understand properties of multiplication and the relationship between multiplication and division.	3.OA.B.5	Apply properties of operations as strategies to multiply and divide.	2-4
	3.OA.B.6	Understand division as an unknown-factor problem.	
3.OA.C- Multiply and divide within 100.	3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	3-5
3.OA.D- Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)	2-4
	3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.	

3.NF.A- Develop understanding of fractions as numbers.	3.NF.A.1	Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.	4-6
	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.	
	3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 =2/4, 4/6 = 2/3~~)~~. Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
3.MD.A- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	2-4
	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	
3.MD.C- Geometric measurement: understand concepts of area.	3.MD.C.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	2-4
	3.MD.C.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	

	3.MD.C.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	
3.NBT.A – Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)	3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.	4-6
	3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	
	3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.	
3.MD.B- Represent and interpret data.	3.MD.B.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.	3-5
	3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters	
3.MD.D- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	3.MD.D.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	2-4
3.G.A- Reason with shapes and their attributes.	3.G.A.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	4-6
	3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.	

Overall Blueprint (Includes Part I and Part II)

Cluster	Standards		# of Items	% of Test
3.OA.A - Represent and solve problems involving multiplication and division.	3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.	5-9	9-11%
	3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.		
	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.		
	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers.		
3.OA.B- Understand properties of multiplication and the relationship between multiplication and division.	3.OA.B.5	Apply properties of operations as strategies to multiply and divide.	4-8	9-11%
	3.OA.B.6	Understand division as an unknown-factor problem.		
3.OA.C- Multiply and divide within 100.	3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	3-5	5-7%
3.OA.D- Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)	5-9	9-11%
	3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.		

3.NF.A- Develop understanding of fractions as numbers.	3.NF.A.1	Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.	9-13	15-17%
	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.		
	3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 =2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.		
3.MD.A- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	6-10	11-13%
	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.		
3.MD.C- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	3.MD.C.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	6-10	11-13%
	3.MD.C.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).		

	3.MD.C.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.		
3.NBT.A – Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)	3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.	4-6	8-10%
	3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.		
	3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.		
3.MD.B- Represent and interpret data.	3.MD.B.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.	3-5	5-7%
	3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters		
3.MD.D- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	3.MD.D.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	2-4	3-5%
3.G.A- Reason with shapes and their attributes.	3.G.A.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	4-6	6-8%
	3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.		

TNReady 4th Grade Math Blueprint

	Part I		Part II		Total # of Items	Total # of score points	% of Test
	# of items	% of PT 1	# of items	% of PT 2			
Major Work of the Grade	25-29	100%	21-25	55-60%	45-53	45-63	75-80%
• **Use the four operations with whole numbers to solve problems**	**2-4**	**12-14%**	**2-4**	**7-9%**	**4-8**	**4-10**	**8-10%**
• **Generalize place value understanding**	**4-6**	**19-21%**	**3-5**	**9-11%**	**7-11**	**7-13**	**12-14%**
• **Perform multi-digit arithmetic**	**4-6**	**17-19%**	**4-6**	**11-13%**	**8-12**	**8-14**	**14-16%**
• **Extend understanding of fraction equivalence and ordering**	**4-6**	**17-19%**	**3-5**	**9-11%**	**7-11**	**7-13**	**12-14%**
• **Build fractions from unit fractions**	**4-6**	**19-21%**	**3-5**	**9-11%**	**7-11**	**7-13**	**12-14%**
• **Understand decimal notation and compare decimal fractions**	**2-4**	**12-14%**	**2-4**	**7-9%**	**4-8**	**4-10**	**8-10%**
Additional and Supporting	0	0%	15-19	40-45%	15-19	15-23	20-25%
• Gain familiarity with factors and multiples	0	0%	2-4	7-9%	2-4	2-6	3-5%
• Generate and analyze patterns	0	0%	1-3	4-6%	1-3	1-5	2-4%
• Solve problems involving measurement and conversion of measurements	0	0%	4-6	12-14%	4-6	4-8	6-8%
• Represent and interpret data	0	0%	1-3	4-6%	1-3	1-5	2-4%
• Geometric measurement: understand concepts of angle	0	0%	1-3	4-6%	1-3	1-5	2-4%
• Draw and identify lines and angles, and classify shapes	0	0%	2-4	6-8%	2-4	2-6	3-5%
Total	24-30	100%	32-44	100%	60-72	60-86	100%

Additional Notes:

***On Part I, 100% of the content in grades 3-8 mathematics will be drawn from the clusters designated as major work of the grade. The math standards in grades 3-8 are coherent and the connections between major work and the additional and supporting clusters are important throughout the year. Part II questions measure major, additional, and supporting topics, including the integration of the key ideas at each grade level.

***The total number of score points does not match the total number of items. This is because some items may be worth more than one point.

***Clusters drawn from the major work of the grade are bolded throughout this document.

Part I – Calculator Allowed

100% of the content in Part I is drawn from the major work

Cluster	Standards		# of Items
4.OA.A – Use the four operations with whole numbers to solve problems.	4.OA.A.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	2-4
	4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	
	4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
4.NBT.A – Generalize place value understanding for multi-digit whole numbers.	4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	4-6
	4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
	4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.	
4.NBT.B – Use place value understanding and properties of operations to perform multi-digit arithmetic.	4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	4-6
	4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
	4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
4.NF.A -Extend understanding of fraction equivalence	4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	4-6

and ordering.	4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
4.NF.B- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	4.NF.B.3	Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	4-6
	4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.	
4.NF.C – Understand decimal notation for fractions, and compare decimal fractions.	4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)	2-4
	4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100.	
	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	

<u>Part II – Calculator and Non-Calculator Portions</u>

55-60% of the content in Part II is drawn from the major work

Cluster	Standards		
4.OA.A – Use the four operations with whole numbers to solve problems.	4.OA.A.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	2-4
	4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	
	4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
4.NBT.A – Generalize place value understanding for multi-digit whole numbers.	4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	3-5
	4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
	4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.	
4.NBT.B – Use place value understanding and properties of operations to perform multi-digit arithmetic.	4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	4-6
	4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
	4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
4.NF.A -Extend understanding of fraction equivalence	4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	3-5

and ordering.	4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
4.NF.B- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	4.NF.B.3	Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	3-5
	4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.	
4.NF.C – Understand decimal notation for fractions, and compare decimal fractions.	4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)	2-4
	4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100.	
	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	
4.OA.B – Gain familiarity with factors and multiples.	4.OA.B.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	2-4
4.OA.C – Generate and analyze patterns.	4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	1-3

4.MD.A – Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table.	4-6
	4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	
	4.MD.A.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	
4.MD.B – Represent and interpret data.	4.MD.B.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	1-3
4.MD.C – Geometric measurement: understand concepts of angle and measure angles.	4.MD.C.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **a.** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	1-3
	4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	
	4.MD.C.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	
4.G.A – Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	4.G.A.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	2-4
	4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	
	4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	

Overall Blueprint (includes Part I and Part II)

Cluster	Standards		# of Items	% of Test
4.OA.A – Use the four operations with whole numbers to solve problems.	4.OA.A.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	4-8	8-10%
	4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.		
	4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.		
4.NBT.A – Generalize place value understanding for multi- digit whole numbers.	4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	7-11	12-14%
	4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.		
	4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.		
4.NBT.B – Use place value understanding and properties of operations to perform multi-digit arithmetic.	4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	8-12	14-16%
	4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		
	4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		
4.NF.A - Extend understandin g	4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	7-11	12-14%

of fraction equivalence and ordering.	4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.		
4.NF.B- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	4.NF.B.3	Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	7-11	12-14%
	4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of $1/b$. b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.		
4.NF.C – Understand decimal notation for fractions, and compare decimal fractions.	4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)	4-8	8-10%
	4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100.		
	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.		
4.OA.B – Gain familiarity with factors and multiples.	4.OA.B.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	2-4	3-5%

4.OA.C – Generate and analyze patterns.	4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	1-3	2-4%
4.MD.A – Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table.	4-6	6-8%
	4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.		
	4.MD.A.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems.		
4.MD.B – Represent and interpret data.	4.MD.B.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	1-3	2-4%
4.MD.C – Geometric measurement: understand concepts of angle and measure angles.	4.MD.C.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	1-3	2-4%
	4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.		
	4.MD.C.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.		
4.G.A – Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	4.G.A.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two- dimensional figures.	2-4	3-5%
	4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.		

	4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.		

<u>TNReady 5th Grade Math Blueprint</u>

	Part I		Part II		Total # of items	Total # of score points	% of Test
	# of items	% of PT I	# of items	% of PT II			
Major Work of the Grade	**25-29**	**100%**	**20-24**	**60-65%**	**45-53**	**45-60**	**75-80%**
• **Place value system**	**5-7**	**22-24%**	**3-5**	**10-12%**	**8-12**	**8-14**	**15-17%**
• **Operations with multi-digit numbers and decimals**	**4-6**	**17-19%**	**4-6**	**12-14%**	**8-12**	**8-14**	**14-16%**
• **Use equivalent fractions to add and subtract**	**4-6**	**17-19%**	**3-5**	**10-12%**	**7-11**	**7-13**	**14-16%**
• **Multiply and divide fractions**	**6-8**	**26-28%**	**5-7**	**13-15%**	**11-15**	**11-17**	**18-20%**
• **Measurement: Volume**	**3-5**	**13-15%**	**2-4**	**6-8%**	**5-9**	**5-11**	**9-11%**
Additional and Supporting	0	0	16-20	35-40%	16-20	16-26	20-25%
• Write and interpret numerical expressions	0	0	2-4	6-8%	2-4	2-6	3-5%
• Analyze patterns and relationships	0	0	1-3	4-6%	1-3	1-5	2-4%
• Convert measurement units	0	0	3-5	8-10%	3-5	3-7	5-7%
• Represent and interpret data	0	0	3-5	8-10%	3-5	3-7	5-7%
• Graph points on the coordinate plane	0	0	3-5	8-10%	3-5	3-7	5-7%
• Classify two-dimensional figures	0	0	1-3	4-6%	1-3	1-5	2-4%
Total	**25-29**	**100%**	**36-44**	**100%**	**61-73**	**61-86**	**100%**

<u>Additional Notes</u>:

*****On Part I, 100% of the content in grades 3-8 mathematics will be drawn from the clusters designated as major work of the grade. The math standards in grades 3-8 are coherent and the connections between major work and the additional and supporting clusters are important throughout the year. Part II questions measure major, additional, and supporting topics, including the integration of the key ideas at each grade level.

*****The total number of score points does not match the total number of items. This is because some items may be worth more than one point.

*****Clusters drawn from the major work of the grade are bolded throughout this document.

Part I – Calculator Allowed

100% of the content in Part I is drawn from the major work

Cluster	Standards		# of Items
5.NBT.A - Understand the place value system	5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	5-7
	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	
	5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
	5.NBT.A.4	Use place value understanding to round decimals to any place.	
5.NBT.B - Perform operations with multi-digit whole numbers and with decimals to hundredths	5.NBT.B.5	Fluently multiply multi-digit whole numbers using the standard algorithm.	4-6
	5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	
5.NF.A - Use equivalent fractions as a strategy to add and subtract fractions	5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	4-6
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	
5.NF.B - Apply and extend previous understandings of multiplication and division to multiply and divide fractions	5.NF.B.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	6-8
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x $q \div b$. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	
	5.NF.B.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given	

		number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	
	5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. b. Interpret division of a whole number by a unit fraction, and compute such quotients. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.	
5.MD.C - Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	5.MD.C.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	3-5
	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	
	5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	

Part II – Calculator and Non-Calculator Portions

60-65% of the content in Part II is drawn from the major work

Cluster	Standards		# of Items
5.NBT.A - Analyze patterns and relationships	5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	3-5
	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	
	5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
	5.NBT.A.4	Use place value understanding to round decimals to any place.	
5.NBT.B - Perform operations with multi-digit whole numbers and with decimals to hundredths	5.NBT.B.5	Fluently multiply multi-digit whole numbers using the standard algorithm.	4-6
	5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	
5.NF.A - Use equivalent fractions as a strategy to add and subtract fractions	5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	3-5
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	
5.NF.B - Apply and extend previous understandings of multiplication and division to multiply and divide fractions	5.NF.B.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	5-7
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	
	5.NF.B.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a	

		product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	
	5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. b. Interpret division of a whole number by a unit fraction, and compute such quotients. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.	
5.MD.C - Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	5.MD.C.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	2-4
	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	
	5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	
5.OA.A - Write and interpret numerical expressions	5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	2-4
	5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them	
5.OA.B - Analyze patterns and relationships	5.OA.A.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	1-3
5.MD.A - Convert like measurement units within a given measurement system	5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	3-5
5.MD.B - Represent and interpret data	5.MD.B.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.	3-5
5.G.A - Graph points on the coordinate plane to solve real-world and mathematical	5.G.A.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number	3-5

problems		indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	
	5.G.A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	
5.G.B - Classify two-dimensional figures into categories based their properties	5.G.B.3	Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category.	1-3
	5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.	

Overall Blueprint (Includes Part I and Part II)

Cluster	Standards		# of Items	% of Test
5.NBT.A - Understand the place value system	5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	8-12	15-17%
	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.		
	5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.		
	5.NBT.A.4	Use place value understanding to round decimals to any place.		
5.NBT.B - Perform operations with multi-digit whole numbers and with decimals to hundredths	5.NBT.B.5	Fluently multiply multi-digit whole numbers using the standard algorithm.	8-12	14-16%
	5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		
	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.		
5.NF.A - Use equivalent fractions as a strategy to add and subtract fractions	5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	7-11	14-16%
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.		
5.NF.B - Apply and extend previous understandings of multiplication and division to multiply and divide fractions	5.NF.B.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	11-15	18-20%
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.		
	5.NF.B.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1		

		results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.		
	5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.		
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. b. Interpret division of a whole number by a unit fraction, and compute such quotients. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.		
5.MD.C - Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	5.MD.C.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	5-9	9-11%
	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.		
	5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.		
5.OA.A - Write and interpret numerical expressions	5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	2-4	3-5%
	5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them		
5.OA.B - Analyze patterns and relationships	5.OA.A.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	1-3	2-4%
5.MD.A - Convert like measurement units within a given measurement system	5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	3-5	5-7%
5.MD.B - Represent and interpret data	5.MD.B.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.	3-5	5-7%
5.G.A - Graph points	5.G.A.1	Use a pair of perpendicular number lines, called axes, to define a	3-5	5-7%

on the coordinate plane to solve real-world and mathematical problems		coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).		
	5.G.A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.		
5.G.B - Classify two-dimensional figures into categories based their properties	5.G.B.3	Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category.	1-3	2-4%
	5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.		

***Appendix* I**
Fluency

TNCore

Tennessee State Standards, Fluency in Mathematics

Fluency is not meant to come at the expense of conceptual understanding. Rather, it should be an outcome resulting from a progression of learning and thoughtful practice. It is important to provide the conceptual building blocks that develop understanding along with skill toward developing fluency; the roots of this conceptual understanding often occur one or more grades earlier in the standards than the grade when fluency is expected.

"Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate *flexibility* in the computational methods they choose, *understand* and can explain these methods, and produce accurate answers *efficiently*. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships."

NCTM, Principles and Standards for School Mathematics, p. 152 (2000)

<u>Fluency Expectations, K-8</u>

Grade	Standard	Expected Fluency
K	K.OA.A.5	Add/Subtract within 5
1	1.OA.C.6	Add/Subtract within 10
2	2.OA.B.2 2.NBT.B.5	Add/Subtract within 20 (Know single digit sums from memory) Add/Subtract within 100
3	3.OA.C.7* 3.NBT.A.2*	Multiply/Divide within 100 (Know single digit products from memory) Add/Subtract within 1000
4	4.NBT.B.4*	Add/Subtract within 1,000,000
5	5.NBT.B.5*	Multi-digit multiplication
6	6.NS.B.2* 6.NS.B.3*	Multi-digit division Multi-digit decimal operations
7	7.NS.A.1,2 7.EE.B.3 7.EE.B.4	Fluency with rational number arithmetic Solve multistep problems with positive and negative rational numbers in any form Solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$ fluently
8	8.EE.C.7 8.G.C.9	Solve one-variable linear equations, including cases with infinitely many solutions or no solutions Solve problems involving volumes of cones, cylinders, and spheres together with previous geometry work, proportional reasoning and multi-step problem solving in grade 7

*These fluency standards will be assessed on TNReady. Students will not have access to a calculator for fluency items on TNReady.

TNCore

Tennessee State Standards, Fluency in Mathematics

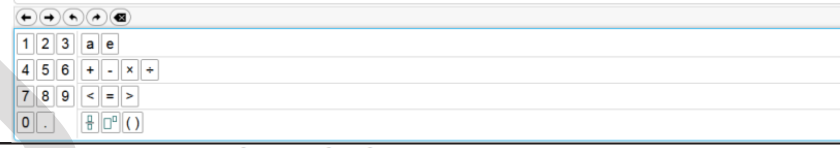
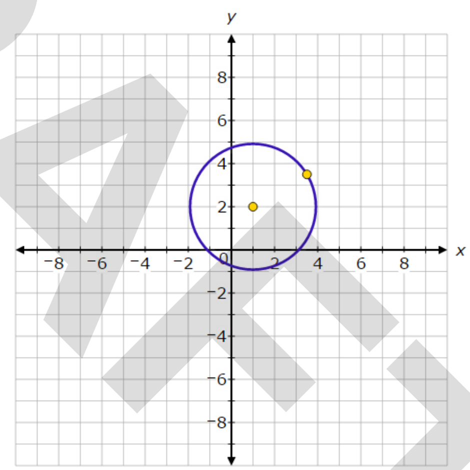
The high school standards do not set explicit expectations for fluency, but fluency is important in high school mathematics. Fluency in algebra can help students get past the need to manage computational and algebraic manipulation details so that they can observe structure and patterns in problems. Such fluency can also allow for smooth progress toward readiness for further study/careers in science, technology, engineering, and mathematics (STEM) fields. These fluencies are highlighted to stress the need to provide sufficient supports and opportunities for practice to help students gain fluency. Fluency is not meant to come at the expense of conceptual understanding. Rather, it should be an outcome resulting from a progression of learning and thoughtful practice. It is important to provide the conceptual building blocks that develop understanding along with skill toward developing fluency

Fluency Recommendations, High School

Course	Standard	Recommended Fluency
Algebra I	A/G A-APR.A.1 A-SSE.A.1b	Solving characteristic problems involving the analytic geometry of lines Fluency in adding, subtracting, and multiplying polynomials Fluency in transforming expressions and seeing parts of an expression as a single object
Geometry	G-SRT.B.5 G-GPE.B.4, 5, 7 C-CO.D.12	Fluency with the triangle congruence and similarity criteria Fluency with the use of coordinates Fluency with the use of construction tools
Algebra II	A-APR.D.6 A-SSE.A.2 F.IF.A.3	Divide polynomials with remainder by inspection in simple cases See structure in expressions and use this structure to rewrite expressions Fluency in translating between recursive definitions and closed forms

Appendix J
Item Types

Math Item Type

Item Type	Examples	Illustration
Equation: Students generate response. On both part I and part II. Typically worth one point (one right answer). 10-15% of questions.	**Equation Editor:** Students type in numeric answers from a palette of options.	5 The manager of a youth soccer team bought 50 packages of socks for $10 each. He estimated the total cost to be $5,000. Create an equation that shows how many times more the manager's estimate, *e*, was than the actual cost, *a*. $a \times 10 = e$ 
Graphic: Students depict graphically. On both part I and part II but more often on part I. Usually one point. 10-25% of questions, more in high school.	**Circles:** Students graph a circle by plotting the center point first and then the radius.	Graph the equation $(x-1)^2+(y-2)^2=3^2$. 

Graphic (continued)		

Graphic (continued)

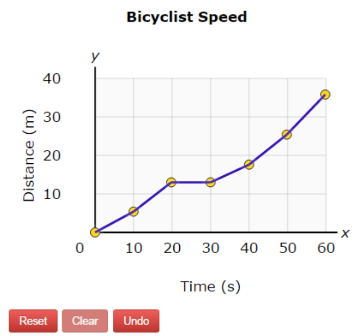
Line Graph: Students click to add a point. Adding another point will automatically connect the points to form a line graph.

The table below shows the speed of a bicyclist.

Using the data in the table, create a line on the graph showing the bicyclist's speed.

Bicyclist Speed

Time (s)	Distance (m)
0	0
10	5
20	12
30	12
40	18
50	25
60	36



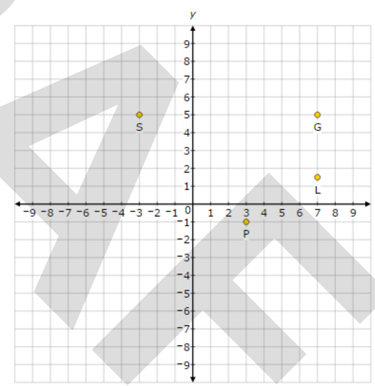
Placing Points: Students click to add a point.

The data table shows the locations of different buildings in a town.

Town Buildings

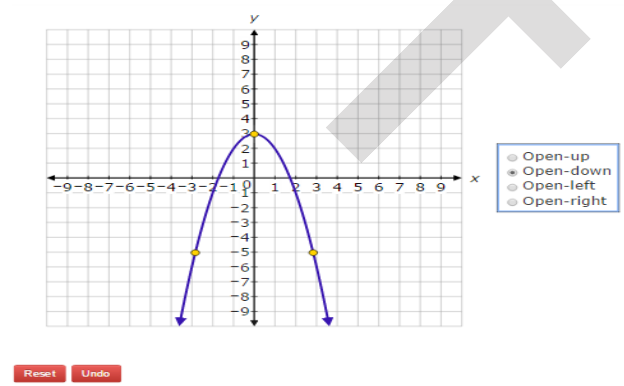
Buildings	Location
Grocery Store (G)	(7, 5)
Library (L)	(7, 1.5)
Park (P)	(3, −1)
School (S)	(−3, 5)

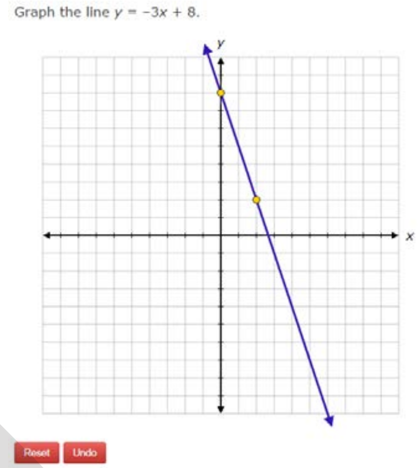
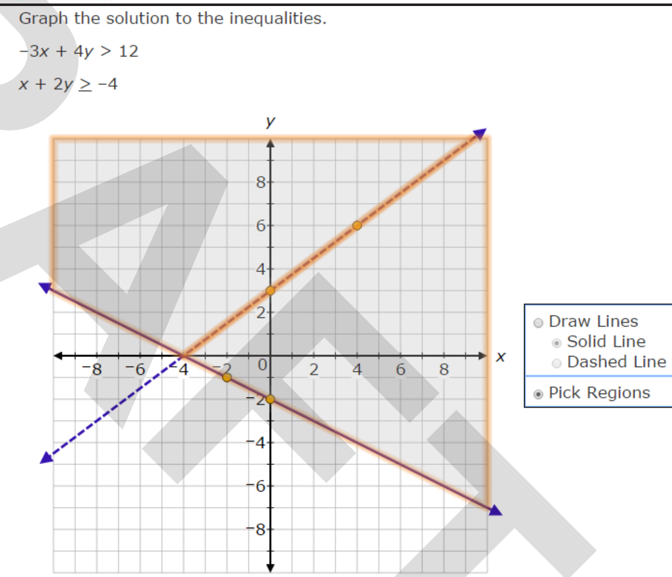
The grid shown represents a town. Plot each point on the coordinate grid. Label each point with the name of the building.

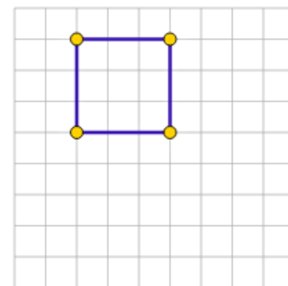


Single Parabola: Students select open-up, open-down, open-left, open-right to place a parabola on the grid. The points or the entire line can then be moved.

Graph the equation $y = -x^2 + 3$ on the coordinate grid.

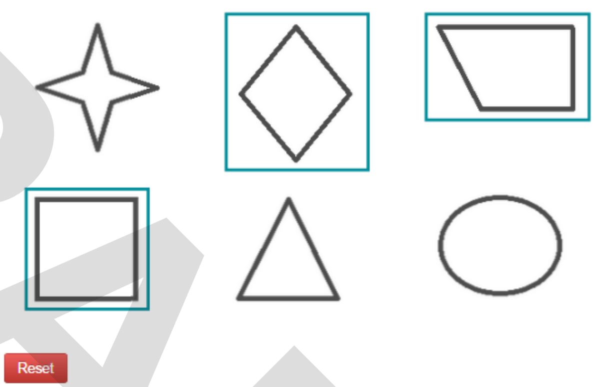


Graphic (continued)	**Straight Lines:** Students click to place the first point then clicks again to place the second point, which creates the line. It can be either a line segment or a line.	
	Straight lines then select: Students plot lines on a grid similar to "Single Line and Straight Lines" and select dotted or solid. Then students select "Pick Regions" to shade a region.	

	Vertex based Quadrilaterals: Students plot points to form different polygons. The option can be set to close the shape after four points are plotted.	Create a shape on the grid with the following: • 4 sides of equal length • 2 pairs of parallel sides • 4 right angles 
Multiple Choice: Students select one answer, only one answer is correct. On both part I and part II. Worth one point. 60-75% of questions.	**Dropdowns:** Students choose from a dropdown list.	Use this number to create a true sentence. 684.425 The value of the 4 in the tenths place is [one-tenth of ▾] the value of the 4 in the ones place.
	Multiple Choice: Student selects one correct answer from many options.	1 What is 78 rounded to the nearest ten? Ⓐ 70 Ⓑ 75 ● 80 Ⓓ 100
Multiple select: Students select multiple options, multiple correct. On both part I and part II but more on part II. Can be worth one to two points, depending on the	**Check Box:** Students select multiple correct answer choices.	3 Consider the family of quadrilaterals that includes parallelograms, rectangles, squares, and rhombuses. Select all the statements about these quadrilaterals that are true. ☑ Squares are always rectangles. ☐ Rectangles are always squares. ☐ Rhombuses are always squares. ☑ Squares are always rhombuses. ☑ Rhombuses are always parallelograms. ☑ Rhombuses are sometimes rectangles.

<table>
<tr>
<td>wording of the question.

Up to 10% of questions.</td>
<td>Matching Table: Students select multiple correct answer choices.</td>
<td>Select one phrase that describes the value of each expression.

<table>
<tr><th></th><th>Greater than 3</th><th>Equal to 3</th><th>Less than 3</th></tr>
<tr><td>$3\times\frac{1}{2}$</td><td>☐</td><td>☐</td><td>☑</td></tr>
<tr><td>$3\times1\frac{1}{4}$</td><td>☑</td><td>☐</td><td>☐</td></tr>
<tr><td>$3\times\frac{6}{6}$</td><td>☐</td><td>☑</td><td>☐</td></tr>
<tr><td>$3\times\frac{3}{2}$</td><td>☑</td><td>☐</td><td>☐</td></tr>
</table>
</td>
</tr>
<tr>
<td></td>
<td>Select Objects: Students click on objects to select. A select border color appears around the objects when selected.</td>
<td>Click on the shapes that are quadrilaterals.


</td>
</tr>
<tr>
<td>Performance Tasks: Solve multistep problems and demonstrate how solution is achieved.

1 task on part I only in grades 3-8 only.

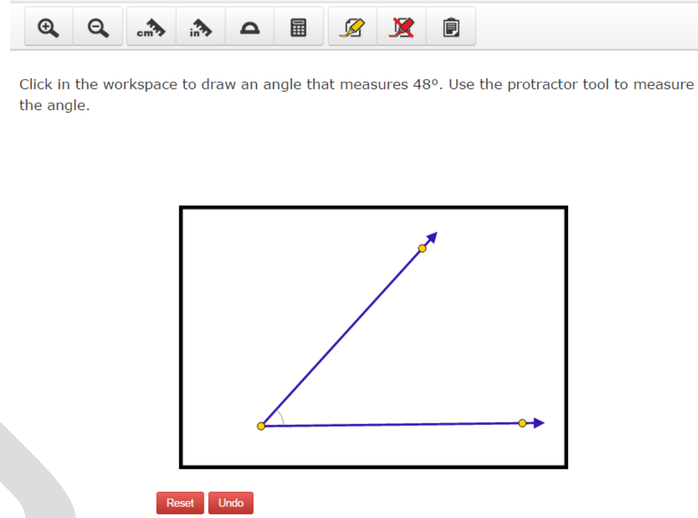
Worth 10-15 points with partial credit.</td>
<td>These will not mimic the previous CRA tasks but will require multistep problem solving and will require students to explain the problem solving approach.

Partial credit will be available.</td>
<td>We are waiting on copyright clearance to release an example performance task. A depiction of a similar type of problem will be shared in the power point.</td>
</tr>
</table>

<table>
<tr>
<td rowspan="2">TEI: Students perform an interaction to respond to the question.

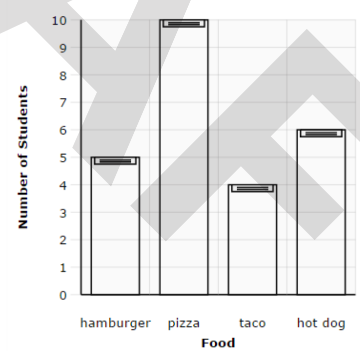
Usually worth one point. Occasionally involves two parts. With two parts, the scoring can either be worth one point or two points (with partial credit) depending on the wording of the question.

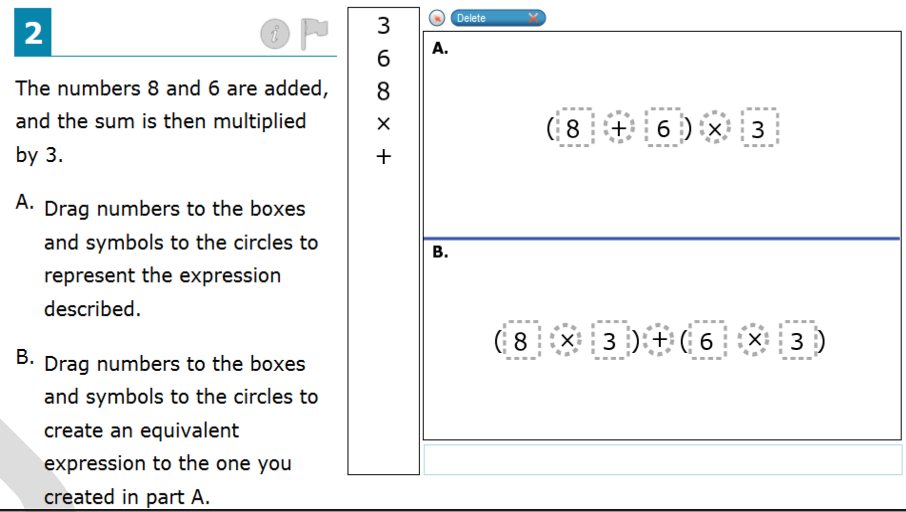
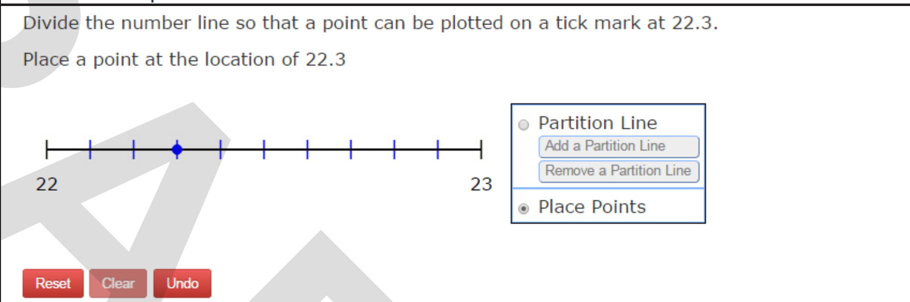
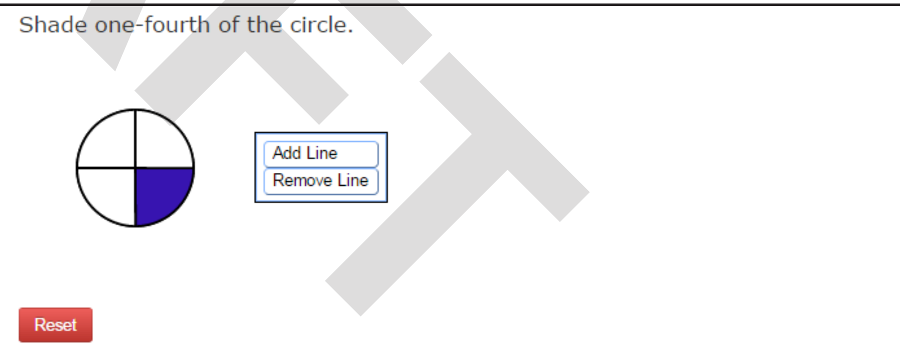
On both Part I and Part II.

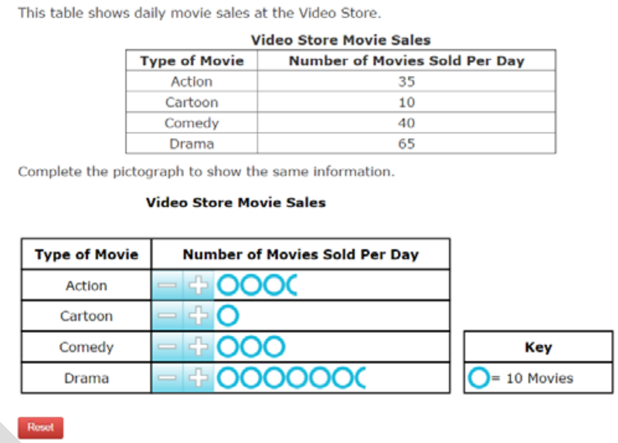
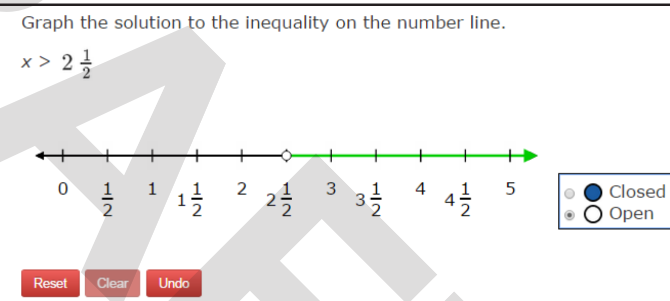
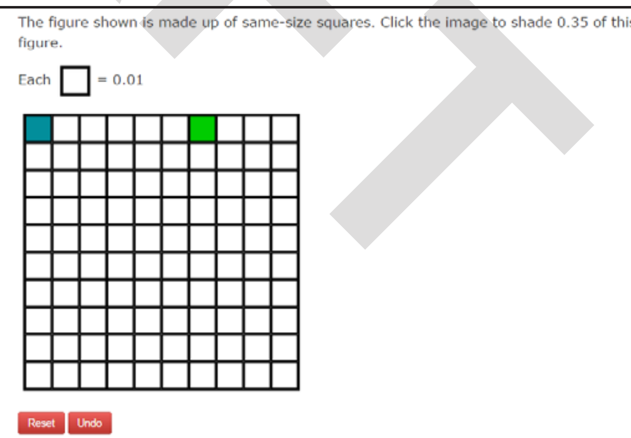
5-15%</td>
<td>Angles: Students plot 3 points to form an angle. The first point plotted is the vertex.</td>
<td>
</td>
</tr>
<tr>
<td>Bar Graph: Students drag bars up or down to place.</td>
<td>The table shows the favorite foods of students in a class.

Favorite Food

<table>
<tr><th>Food</th><th>Number of Students</th></tr>
<tr><td>hamburger</td><td>5</td></tr>
<tr><td>pizza</td><td>10</td></tr>
<tr><td>taco</td><td>4</td></tr>
<tr><td>hot dog</td><td>6</td></tr>
</table>

Complete the bar graph to show the same information.

</td>
</tr>
</table>

<table>
<tr>
<td rowspan="3">TEI (continued)</td>
<td>Classification: Students drag and drop objects to different regions.</td>
<td>
</td>
</tr>
<tr>
<td>Partition Number Lines then Place Points: Students click a button to partition a line into equal parts. They can then select "Place Points" to be able to click on the line to place a point. With Student Label, the student will add partitions to the line instead of using a button to partition equally.</td>
<td>
</td>
</tr>
<tr>
<td>Partition Object then Select: Students click a button to partition an object into equal parts. They can then select "Shade Regions" to shade sections. In Select Defined partitions, a student will not be partitioning.</td>
<td>
</td>
</tr>
</table>

<table>
<tr>
<td rowspan="4">TEI (continued)</td>
<td>Pictograph: Students click a plus or minus to add shapes to the pictograph.</td>
<td>
</td>
</tr>
<tr>
<td>Rays: Students plot 2 points to form a ray. The first point is the end point. The second point has the arrow at the end.</td>
<td></td>
</tr>
<tr>
<td>Select Points And Ranges on Number Lines: Students click on a point on the line to place an open/closed marker. After markers are placed, students can click in between points to select a region. Regions extend between two points or one point and the end of the number line.</td>
<td>
</td>
</tr>
<tr>
<td>Shade Regions: Students click to shade regions of a rectangle. Selected regions are either outlined or filled with a solid color, symbol, or custom graphic.</td>
<td>
</td>
</tr>
</table>

Appendix K
Calculator Policies

TNReady Calculator Policy for Mathematics

The TNReady Calculator Policy is based on two central beliefs:

1. Calculators are important tools and, in order to be ready for career and college, students need to understand how to use calculators effectively, and
2. In order to demonstrate mastery of the mathematics standards, students must demonstrate many skills without reliance on calculators.

Therefore, at all grade levels and in all courses, TNReady will include both calculator permitted sections and calculator prohibited sections.

- Part I will allow calculator use at all grade levels.
- Part II will include a calculator permitted section and a calculator prohibited section at all grade levels.

The following considerations will shape how items are assigned to each section:

- Questions based on standards that require students to perform calculations in order to arrive at an answer will appear on the non-calculator-permitted section of the assessment. For example, 5.NF.A.1 expects students to add/subtract fractions with unlike denominators.
- Other questions may be based on standards where a calculation is a means to demonstrating other understanding. In this case, a student's error could be based on a misconception or a miscalculation, which would color the evidence of what is intended by the assessment. For example, 6.G.A.1 expects students to find area of composite figures and the calculations performed should not be a barrier for students demonstrating understanding of how to determine the area.
- Questions based on standards like 3.G.A.1 which ask students to recognize examples of quadrilaterals may appear on either the calculator or non-calculator section.

Calculator Specifics

- It is the responsibility of the Test Administrator to ensure the regulations outlined in this policy pertaining to calculator use are followed.
- All memory and user-entered programs and documents must be cleared or removed before and after the test.
- A student may use any permitted calculator at any grade level.
- For calculator-permitted sections of TNReady, students may use the online calculator or a handheld calculator provided by the school/district or one owned personally. Students may use either or both during the test.

- Students should have access to no more than one handheld calculator device for calculator-permitted sections of TNReady.
- Students will have access to practice with the same functionalities that will be available on the operational assessment on the item sampler and the practice tests.

Calculator Types

Below are examples of calculator functionalities and calculators that are permitted on TNReady. (Note: this is not an exhaustive list and students should be familiar with particular functions at the appropriate grade level.)

Examples of Permitted Functionalities:

- Square root ($\sqrt{\ }$)/Square key (x^2 and/or x^y)
- Pi(π)
- Graphing capability
- Data entry
- Matrices
- Regression
- Trigonometric functions (sine, cosine, tangent)
- Logarithm (log and/or ln) and exponential functions (a^x and/or e^x)

Examples of permitted calculators:

- TI-30
- Casio FX260
- Sharp EL344RB
- TI-84 plus family
- TI-NSpire (non-CAS) and TI-NSpire-CX (non-CAS)

Below are calculator functionalities and examples of calculators that are not allowed on TNReady. (Students may use any four- function, scientific, or graphing calculator, which does not include any of the prohibited functionalities.)

Calculator functionalities that are prohibited:

- Any calculator with CAS (computer algebra system) capabilities (including any programs or applications)
- Wireless communication capability
- QWERTY keyboard
- Cell phones, tablets, iPods, etc.

Examples of prohibited calculators:

- TI-89
- TI-NSpire (CAS version)
- HP-40G
- Casio CFX-9970

Appendix L
Practice Tools

TNReady Practice Tools

The TDOE will make two optional tools available to educators and districts.

	TNReady *Item Sampler*		TNReady *Practice Test*
	Phase 1	**Phase 2**	
Purpose	• In Phase 1, to give educators access to questions that are reflective of the rigor and the format of questions that will be on TNReady. • In Phase 2, to give educators access to additional items and provide students a chance to practice with the same tools they will have on TNReady in an instructional setting.		• To simulate a short-form of each part of the TNReady test (Part I and Part II). • To allow students to experience a practice test with the same features as the operational assessment. • To allow teachers and systems to practice set up and administration.
Limitations	• The TNReady Item Sampler will not serve as a full set of interim or formative assessments. • The items will not be secure. (All teachers will have access at the same time.) • The items will be comparable to the items on TNReady but the test forms will not be comparable, as they are teacher-created. • The results will not necessarily be comparable to results in other classrooms because the user experience cannot be controlled. • The ability to add customized items specific to teacher, school, or district will not be available at this time.		• Results on the TNReady Practice Test will not necessarily be predictive of student performance on TNReady. • The Practice test will not reflect a full form, but it will include all major item types for each part.
Timeline	• Launch May 2015 • Continuously available • 8-12 items per grade per subject • Full range of item types • Access for teachers only	• Launch September 2015 • Continuously available • 25-40 additional items per grade per subject • Full range of standards • Access for teachers and students	• <u>Window 1:</u> September 28 – October 30, 2015 (All grades Part I & II) • <u>Window 2:</u> January 4 – February 6, 2016 (All grades Part I & II) • <u>Window 3:</u> March 7 – April 8, 2016 (All grades Part II)
User Set-Up	• All teachers will be set up to get access on May 2015. • More info about set up will be shared with testing coordinators by April 15, 2015.	• Students will be set up based on August 14 EIS pull, provided scheduling data is available. Regular EIS updates thereafter. • More info will be shared with testing coordinators by August 1, 2015.	• Same upload process as MIST practice test has been in the past.
Training	• Training on MICA will be incorporated in summer training for teachers and principals. • Web-based video training will be available. • Additional support from CORE analysts.		• Web-based video training will be available.

Reporting	• Teacher Reports
	  ○ Student, Assessment, Class
	  ○ Content strand summary
		• Teacher Reports (same as Phase I)
		• Administrator Reports
		  ○ Utilization by teacher and content strand
		  ○ "Impersonation" at teacher level
			• Student Report and Roster Report for schools and districts including:
			  ○ Overall Score and By Standard
			  ○ Raw Score (number correct)
			  ○ % correct

Platform	• MICA (Measurement Inc. Classroom Assessments) which is built to reflect MIST.		• MIST (the same platform as the operational assessment).
Accessibility Features	• Supports common web-browser text reader tools. Not integrated in MICA.		• Text reader availability by Windows 2 and 3.
Scoring	• All *machine scorable* items will be automatically scored within platform. • Teachers will be able to go into the MICA system to score student answers to open-response items using the same tools and scoring guides that will be used to score TNReady.		
Cost	• Provided to all Tennessee districts at no additional charge.		

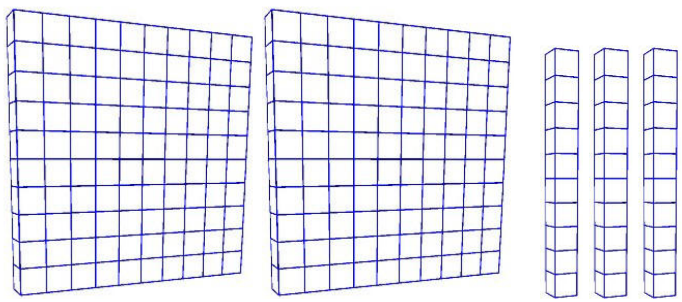
TN Task Arcs

Appendix M
Task Packet

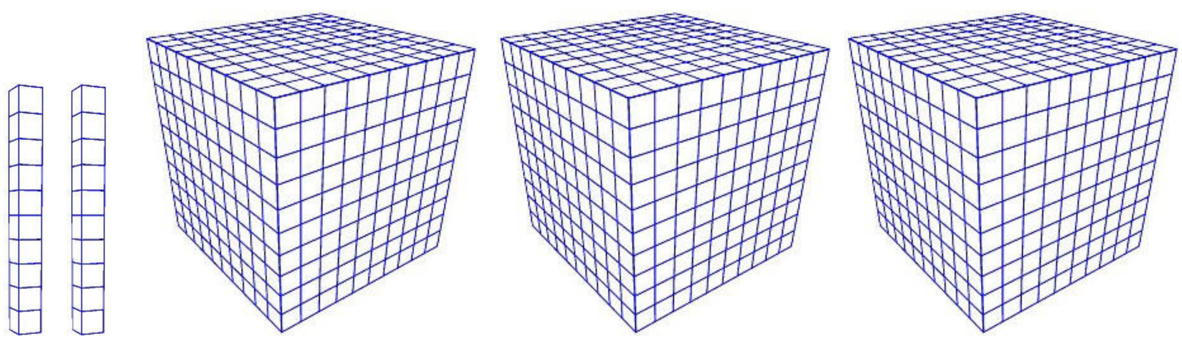
Task: Tree House Windows	**5^{th} Grade**

Tom and Gina are thinking about the number 2.3 mils, which represents the thickness of the plastic sheeting used to cover windows of their tree house to keep out the wind in the winter. The teacher asks each of them to create a representation of the number 2.3 using place value blocks.

Tom chose a flat for his unit and represented 2.3 with two flats and 3 rods.



Gina chose a cube for her unit and represented 2.3 with two rods and 3 cubes.



a. Which do you think is the better representation for the number 2.3? Explain your reasoning.
b. Consider the representation you did not choose for part a. Explain why you don't think it is the best representation.
c. Create and draw your own representation of the number 2.3 using a rod for your unit. Explain why this representation is also appropriate.

Teacher Notes:

If students do not include place value and the relationship between and among blocks in their explanations, teacher should guide students to include this level of detail. As a means of differentiation, you could change Gina's representation to 2 flats and 3 cubes because this requires only one level of place-value explanation, as opposed to the problem as written, which requires two levels of place value explanation. Any block can be chosen to represent the unit as long as there is a block that is smaller than it by 1/10 to represent the 0.3.

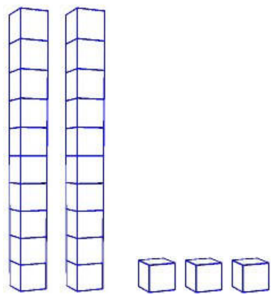
Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
5.NBT.A.1 *Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- **The concept of unit is fundamental to the interpretation of rational numbers.**
- **One interpretation of a rational number is a part-whole relationship.**
- **A rational number can be expressed as a decimal.**

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
a. Tom's representation is better because he uses a flat as the unit and a rod as a tenth of a unit. These two blocks have the appropriate relationship. The number 2.3 has 2 whole units so he uses 2 flats to represent them. The 0.3 represents 3/10 of a unit and a rod is 1/10 of a flat so the rod is the appropriate block to choose to represent the tenths place digit. b. Gina's representation is not the best because she uses a cube for the unit and uses rods with the cube. The number should include 2 units and 3 blocks that are 1/10 of that unit. However, a rod is 1/100 of a cube so these two blocks do not have the appropriate relationship to each other. OR Gina's representation is not the best because in choosing the cube as her unit, she represented 3 wholes and two hundredths rather than 2 wholes and 3 tenths. She could	Assessing (a): • Why do you think one representation is better than the other? • Which digit has the larger place value? • Should you use the larger or the smaller block to represent the larger place value? • How many place-value positions are there between 2 and 0.3? • Knowing that, how should you choose the two sizes of blocks to represent these digits? Advancing (a): • Could you use a different block than either the flat or the cube to represent the whole unit? • If so, which block would you use for your tenths and why? • How can you represent the place value of each digit using expanded notation with powers of 10? • How does this expanded notation relate the size of the blocks you used to model the number 2.3?

have used 2 cubes and 3 flats to represent 2.3 appropriately with the cube as her unit. c.  Since the rod is my unit, I need 2 of them to represent the 2 wholes in the number 2.3. Now I need 3 singles to represent the 0.3 because 0.3 represents 3/10 of a whole and a single is 1/10 of a rod.	Assessing (b): • Can the number 2.3 be represented using the flat as the unit? If so, which block should represent the tenths and why? • If not, why not? • Can the number 2.3 be represented using the cube as the unit? If so, which block should represent the tenths and why? • If not, why not? • In terms of place-value, how is the flat related to the rod? • In terms of place-value, how is the cube related to the rod? • Which block should be used to represent the larger place value and why? Advancing (b): • Use expanded form with powers of 10 to write the number represented by 3 cubes and 2 rods. • Explain, using the relative sizes of the blocks, why each block represents a particular place-value. Assessing (c): • Why did you choose that block to represent the tenths, if the rod represents the unit? Advancing (c): • Is there another block you could use to represent the unit? If so, why and which block would you need to use to represent the tenths and why?
Possible Student Misconceptions	
Students may confuse which place value blocks are 10 times larger or smaller than others.	What place value does the digit 2 in the number 2.3 represent? What place value does the 3 represent in the number 2.3? Is the place value for 0.3 larger or smaller than the place value for the 2? Which block should be larger, the one used to represent the whole or the one used to represent the tenth?
Students may choose Gina's representation over Tom's.	What did Gina choose for her unit? How many units should there be in the representation of 2.3? How many blocks unit blocks did Gina choose to represent her units?
Students may choose something other than a single to represent a tenth of a rod.	Is the place value block you chose for the tenth, larger or smaller than the rod? Which should be larger, the block that represents the unit or the one that represents the tenth of a unit?
Entry/Extensions	**Assessing and Advancing Questions**

If students can't get started….	Assessing: • If a flat represents a whole unit, what place values are represented by the other blocks? • If a cube represents a whole unit, what place values are represented by the other blocks? Advancing: • Should the block used to represent a tenth be smaller or larger than the block used to represent a whole unit? • How many different representations of 2.3 can you create using place value blocks?
If students finish early….	Assessing: • Using the cube as a unit, how would you represent the number 12? • Using a flat as a unit, how would you represent the number 4.2? Advancing: • How many different ways can you use place value blocks to represent 2.03. • How many different ways can you use place value blocks to represent 2.003? • Are there more or fewer ways to represent 2.003 than there are to represent 2.03 and why is that? • Is it possible to use place value blocks to model the number .023? If so, explain how. If not, explain why not.

Discuss/Analyze

Whole Group Questions

Key Understandings:

- Place value blocks are appropriate for modeling numbers because the single is 1/10 of a rod; a rod is 1/10 of a flat; a flat is 1/10 of a cube.
- Because of this relationship, different blocks can be chosen to represent a particular place value.

Questions:

- In the number 2.3, what can you say about the place value of the 2 in relation to the place value of the 3?
- Can you make a general statement describing the relationship between any digit in the whole unit place and the digit immediately to its right? What can you say about a number in the tenths place and the value of the digit immediately to its left?

Appendix N
Task Arc

TNCore

Mathematics Task Arcs

A task arc is a set of related lessons consisting of a series of instructional tasks and their associated lesson guides. The lessons are focused on a small number of standards within a domain of the Tennessee Academic Standards for Mathematics. In some cases a small number of related standards from more than one domain may be addressed.

Included in this task arc are a preview of the tasks and the content and practice standards associated with each task. Essential understandings which teachers strive to develop and solidify within their students across the arc are named in each lesson guide.

The tasks and lessons are sequenced in deliberate and intentional ways and are designed to be implemented consecutively and in their entirety. It is possible for students to develop a deep understanding of concepts because a small number of standards are targeted. Lesson concepts remain the same as the lessons progress; however, the context or representations may change.

Bias: Social, ethnic, racial, religious, and gender bias is best determined at the local level where educators have in-depth knowledge of the culture and values of the community in which students live. The TDOE asks local districts to review these curricular units for social, ethnic, racial, religious, and gender bias before use in local schools.

Grade 5: Place Value and Base 10

A Set of Related Tasks and Lesson Guides

Table of Contents

Introduction

Tasks and Lesson Guides

ARC OVERVIEW

This set of related tasks provides an introduction to the place value of digits to the thousandths and multiplication and division by the powers of ten. Students will solve real world problems in which they have to consider the relationships among multi-digit numbers. They will use written and physical representations and mathematical reasoning to link the concept of the powers of ten to multiplication and division.

The Arc Preview table on page 4 provides all of the task questions contained in this arc. Note that the Essential Understandings listed in each task were taken from NCTM's Developing Essential Understanding series and "Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics"[1]. Tennessee State Mathematics Standards were retrieved from http://www.tn.gov/education/standards/math.shtml.

These tasks are aligned to standards 5.NBT.1 through 5.NBT.4.

Through engaging in the lessons in this set of related lessons students will:

- solve real-life situations using a variety of strategies
- recognize that numbers can be decomposed into parts based on place value of the digits
- round numbers by comparing place values
- determine the relationship of the place value between consecutive digits read and write decimal numbers in words and expanded form to thoudandths.

By the end of these seven tasks, students will be able to answer the following overarching questions:

- What are the patterns when multiplying or dividing by powers of ten?
- How does the value change as you move "left" in place value position? As you move "right"?
- What is the relationship between multiplying and dividing with whole numbers to multiplying and dividing with decimals?

The assessing questions, advancing questions, and whole group questions provided in this guide will ensure that students are working in ways aligned to the Standards for Mathematical Practice. Although the students will not be aware that this is occurring, the teacher can guide the process so that each MP (Mathematical Practice) is covered through good explanations, understanding of context, and clarification of reasoning behind solutions.

---

[1] Charles, Randall I. "Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics." *Journal of Mathematics Education Leadership* 7.3 (2005) : 9-24. Print.

Arc Preview

<table>
<tr><td rowspan="2">Task 1: Jaden's Gym Class
Jaden, Connor, and David all ran a 3-mile timed run in gym class. Write each student's time below in words and in expanded form using fractions.
a) Jaden ran for 35.245 minutes.
b) Connor ran for 29.15 minutes.
c) David ran for 29.915 minutes.</td><td>Goals for Task 1:
• Read and write decimal numbers in words and expanded form using fractions</td></tr>
<tr><td>Standards for Task 1:
5.NBT.3.a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 ×10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).</td></tr>
<tr><td rowspan="2">Task 2: The Stopwatch
Julie and Matt got a new stopwatch from their uncle. They decided to time each other running up and down the hallway. Julie's time was 9.845 seconds. Matt's time was 9.87 seconds.
a) Who had the faster time running up and down the hallway? Explain.
b) Write a correct inequality for the two stopwatch times using the symbols >, <, or =.
c) Round each time to the nearest tenth of a second. Explain your work with words and a number line.</td><td>Goals for Task 2:
• Read and compare two decimals to thousandths
• Round decimals</td></tr>
<tr><td>Standards for Task 2:
5.NBT.3b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
5.NBT.4 Use place value understanding to round decimals to any place.</td></tr>
<tr><td rowspan="2">Task 3: Batting Averages
Sam and Nikki are having a discussion about their favorite baseball players. Sam incorrectly says that his favorite player's batting average of 0.293 is greater than Nikki's favorite player's batting average of 0.34.
a) Explain why Sam's statement is incorrect.
b) Write a correct inequality for the two batting averages using the symbols >, < , or =.
c) If a baseball announcer wants to read these averages using correct mathematical language, how would he say them?</td><td>• Goals for Task 3:
• Read and write decimal numbers to thousandths
• Compare two decimals to thousandths using symbols</td></tr>
<tr><td>Standards for Task 3:
5. NBT.3 Read, write, and compare decimals to thousandths.
5.NBT.3a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
5.NBT.3b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.</td></tr>
</table>

Task	Goals and Standards
Task 4: Dancing Digits A local organization hosted a dance competition to raise money for its new building. The dance competition turned out to be very popular! The organization raised \$374,318 for their new building. a) Look at the digits in the hundreds place and hundred thousands place. How many times greater is the 3 in the hundred thousands place than the 3 in the hundreds place? Explain how you know. b) Change the digit in the thousands place so that the value of the new digit represents $\frac{1}{10}$ of the value of the digit in the ten thousands place. Explain how you know.	**Goals for Task 4:** • Relate digits within a number to each other based on place value • Alter digits within a single number to show 1/10th of the value of the place to its left **Standards for Task 4:** **5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
Task 5: Exponent Mix-Up Holly and Marcus missed the lesson on exponents. Mr. Anderson asked Jason to discuss the exponent lesson with Holly and Marcus during homeroom to help them prepare for today's lesson. Jason talked them through what he understood and then gave them each a problem to work on separately. a) Holly wrote $10^5 = 50$ on her paper. Is Holly's answer correct? Explain why or why not. If her answer is incorrect, what would she need to do to correct it? b) Marcus thinks that since $2 \text{ X } 10^2 = 200$ that $2.5 \text{ x } 10^2 = 2.500$. Is Marcus's answer correct? Explain why or why not. If his answer is incorrect, what would he need to do to correct it?	**Goals for Task 5:** • Multiply by powers of 10 • Explain patterns when multiplying by powers of 10 **Standards for Task 5:** **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Task 6: Trendy T-Shirts

The Trendy T-Shirt Company makes $27.45 profit on each box of t-shirts. The following table shows the latest orders received.

Business Names	Boxes of t-shirts
Amanda's Apparel	10
Casey's Clothes	100
Wallie's Wardrobe	1,000

a) Write an expression using powers of 10 to show how much profit Trendy T-Shirts will make on each order. Explain how the exponents relate to the number of boxes of t-shirts.
b) Calculate the total profit Trendy T-Shirts will make on each order in the table above. What do you notice about the placement of the decimal point in the totals?
c) Will this pattern be the same when dividing a decimal by a power of 10? Explain your reasoning.

Goals for Task 6:

- Multiply by powers of 10
- Explain patterns when multiplying by powers of 10
- Explain patterns when dividing by powers of 10

Standards for Task 6:

5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Task 7: Vacation Plans

Travis, Jamie, and Lauren are all planning trips to visit grandparents during summer vacation. The table below gives the cities each person is visiting and the city's distance from Nashville.

	City Visiting	Distance from Nashville
Travis	Dickson, TN	40 miles
Jamie	Anchorage, AK	4045 miles
Lauren	Charlotte, NC	409 miles

a) Approximate each distance as the product of 4 and a power 10.
b) Jamie said, "My trip to Anchorage is about 10 times as far from Nashville as Travis's trip to Dickson." Travis said, "My trip to Dickson is about 1/10th as far from Nashville as Lauren's trip to Charlotte." Using the products from part a), are both Jamie and Travis correct? If yes, use numbers and words to prove they are correct. If no, rewrite the statements so they are correct.

Goals for Task 7:

- Multiply by powers of 10
- Explain patterns when multiplying by powers of 10
- Explain patterns when dividing by powers of 10
- Critique the reasoning of others using powers of ten

Standards for Task 7:

5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

5.NBT.4 Use place value understanding to round decimals to any place.

Tasks' Standards Alignment

Task	5.NBT.1	5.NBT.2	5.NBT.3a	5.NBT.3b	5.NBT.4	MP 1	MP 2	MP 3	MP 4	MP 5	MP 6	MP 7	MP 8
Task 1 **Jaden's Gym Class**			✔			✔			✔		✔	✔	
Task 2 **The Stopwatch**				✔	✔	✔	✔	✔			✔	✔	
Task 3 **Batting Averages** *Solidifying Understanding*			✔	✔		✔	✔	✔	✔		✔	✔	✔
Task 4 **Dancing Digits**	✔					✔	✔				✔	✔	
Task 5 **Exponent Mix-Up**		✔				✔		✔			✔		✔
Task 6 **Trendy T-Shirts**		✔				✔		✔			✔		✔
Task 7 **Vacation Plans** *Solidifying Understanding*		✔			✔	✔	✔	✔			✔	✔	

The Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Name ______________________________

Task 1: Jaden's Gym Class

Jaden, Connor, and David all ran a 3-mile timed run in gym class. Write each student's time below in words and in expanded form using fractions.

a) Jaden ran for 35.245 minutes.

b) Connor ran for 29.15 minutes.

c) David ran for 29.915 minutes.

Task 1: Jaden's Gym Class	**5th Grade**

Jaden, Connor, and David all ran a 3-mile timed run in gym class. Write each student's time below in words and in expanded form using fractions.

d) Jaden ran for 35.245 minutes.

e) Connor ran for 29.15 minutes.

f) David ran for 29.915 minutes.

Teacher Notes:

Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
5.NBT.3.a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 ×10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- Numbers can be represented using objects, words, and symbols.
- For any number, the place of a digit tells how many ones, tens, hundreds, and so forth are represented by that digit.
- Each place value to the left of another is ten times greater than the one to the right (e.g., 100 = 10 x 10).
- The value of the digits can be added together to find the value of the number.
- Decimal place value is an extension of whole number place value.

Explore Phase

Possible Solution Paths	**Assessing and Advancing Questions**
WORD FORM: a) Jaden's time in word form: thirty-five and two hundred forty-five thousandths b) Connor's time in word form: twenty-nine and fifteen hundredths	**Assessing Questions:** • How did you know where to write "and" in the number word form? • How did you know which place value word to write at the end of the number word form? **Advancing Questions:** • Would your answer change if the decimal point moved to a different position within the number?

<table>
<tr><td>c) David's time in word form:

twenty-nine and nine hundred fifteen thousandths</td><td>• Why is it important to be able to write numbers in word form?
• When is the word form of numbers used in real life?</td></tr>
<tr><td>EXPANDED FORM USING FRACTIONS:

a) Jaden's time in expanded form using fractions:

$(3 \times 10) + (5 \times 1) + (2 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

b) Connor's time in expanded form using fractions

$(2 \times 10) + (9 \times 1) + (1 \times \frac{1}{10}) + (5 \times \frac{1}{100})$

c) David's time in expanded form using fractions:

$(2 \times 10) + (9 \times 1) + (9 \times \frac{1}{10}) + (1 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$</td><td>Assessing Questions:
• How did you select the factor to multiply by each digit in the standard form number?
• Explain how the decimal form relates to the fraction form.

Advancing Questions:
• How would writing a zero at the end of the standard form number change your answer in expanded form?
• Would your answer still be correct without the plus signs?</td></tr>
<tr><td>Possible Student Misconceptions</td><td>Assessing and Advancing Questions</td></tr>
<tr><td>Students who do not use precise mathematical language when reading numbers may experience difficulty in writing numbers in word form and in expanded form.
Examples:
• 5.281 should be read as "five and two hundred eighty-one thousandths," not "five point two eight one"
• $\frac{7}{10}$ should be read as "seven tenths" not "seven over ten"
• Students should be given ample practice in reading numbers in decimal and in fraction form. Such as:
67.32, 102.825, 2.073, $\frac{3}{100}$, $\frac{89}{1000}$</td><td>• Can you tell me how you would say 5.3? What about 5.35? What about 5.351?
• Can you tell me how you would say 1/10?</td></tr>
<tr><td>Entry/Extensions</td><td>Assessing and Advancing Questions</td></tr>
<tr><td>If students can't get started….</td><td>• What do you know about this problem?
• Can you read the number to yourself and then aloud?
• Can you name the place value of each digit?
• Can you read the number before the decimal point?</td></tr>
</table>

	• Can you read the number following the decimal point? • Can you model 9.15 using base ten blocks?
If students finish early….	• Can you and a partner each think of a decimal number in the thousandths, switch numbers and write the numbers in word form as well as standard form?
Discuss/Analyze	
Whole Group Questions	
• How did you know where to write “and” in the number word form? • How did you know which fraction to use when writing expanded form? • What grouping symbols are used when writing numbers in expanded form? Why do you think they are needed? • What would happen if we did not use any grouping symbols in writing numbers in expanded form? • What operations are used in writing numbers in expanded form? Why are those operations used?	

Name ________________________________

Task 2: The Stopwatch

Julie and Matt got a new stopwatch from their uncle. They decided to time each other running up and down the hallway. Julie's time was 9.845 seconds. Matt's time was 9.87 seconds.

a) Who had the faster time running up and down the hallway? Explain.

b) Write a correct inequality for the two stopwatch times using the symbols >, <, or =.

c) Round each time to the nearest tenth of a second. Explain your work with words and a number line.

Task 2: The Stopwatch — **5th Grade**

Julie and Matt got a new stopwatch from their uncle. They decided to time each other running up and down the hallway. Julie's time was 9.845 seconds. Matt's time was 9.87 seconds.

a) Who had the faster time running up and down the hallway? Explain.

b) Write a correct inequality for the two stopwatch times using the symbols >, <, or =.

c) Round each time to the nearest tenth of a second. Explain your work with words and a number line.

Teacher Notes:

Have a variety of math manipulatives available to students. Without giving explicit instructions, allow students to choose from these math manipulatives when solving this task.

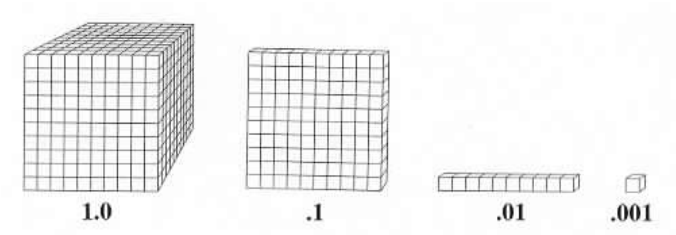
Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
5.NBT.3b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5.NBT.4** Use place value understanding to round decimals to any place.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

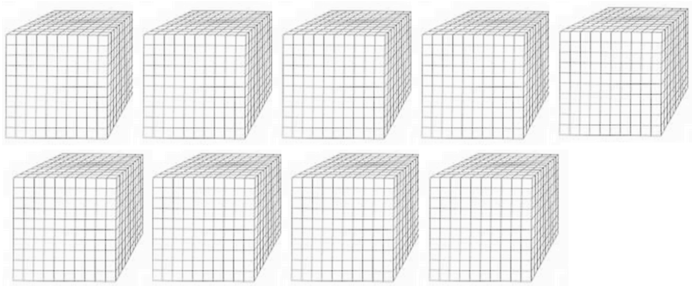
- For any number, the place of a digit tells how many ones, tens, hundreds, and so forth are represented by that digit.
- Each place value to the left of another is ten times greater than the one to the right (e.g., 100 = 10 x 10).
- Decimal place value is an extension of whole number place value.
- Whole numbers and decimals can be compared by analyzing corresponding place values.
- A number to the right of another on the number line is the greater number.
- Numbers can be compared using greater than, less than, or equal.

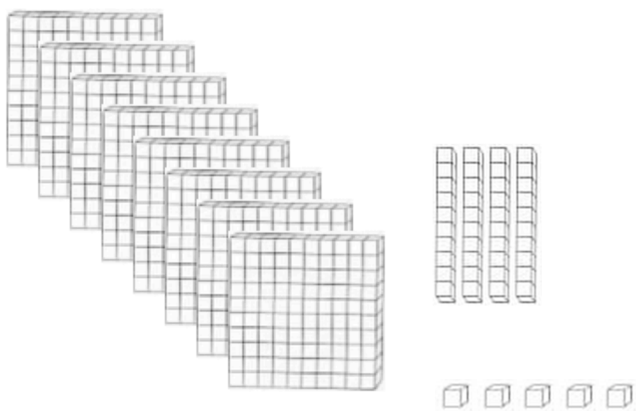
Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
For all solution paths for part a), students should realize that the faster time is the smaller number. a) *Base Ten Block Strategy* Students should use the following scale to build the stopwatch times:	**Assessing Question:** • Can you explain how you constructed the numbers using base ten blocks? • How did you know which blocks to use for each place value space?



9.845 should be represented as:

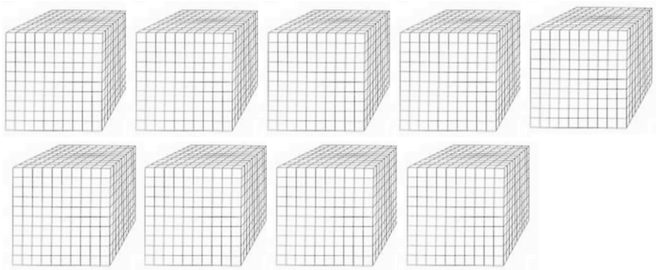
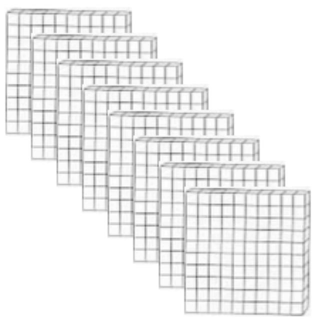
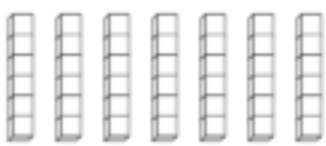




Advancing Questions:

- Could you have shown the stopwatch times without using the cube?
- Can the values of the blocks that you used be different for different problems?

Task 2

9.87 should be represented as:    Students should notice that 9.87 has the same number of ones and tenths, but has more hundredths represented by the rods than 9.845 indicating that 9.845 is the smaller number.	
a) *Number line Strategy* Students should construct a number line with a scale of 0.001 with the labels beginning at 9.840 and ending at 9.870 on the number line. The 9.870 is the original number 9.87 with an added zero in the thousandths place in order to easily compare with 9.845 on the number line. Students should recognize that 9.845 is less than 9.87 because it is located to the left on the number line.	**Assessing Question:** • How did you constructed your number line? • How did you know which number to begin and end with on the number line? • What is the scale on the number line? **Advancing Questions:** • Could you demonstrate this comparison using a number line with a scale of 0.01? Explain. • How do you know that numbers to the right on the number line have greater value?

Task 2

a) *Place Value Table Strategy*

ones	•	tenths	hundredths	thousandths
9	•	8	4	5
9	•	8	7	

Students should see that 9.845 is smaller than 9.87 because it has fewer hundredths.

Assessing Questions:

- How do you know which number is smaller?
- Does it matter that one number is written to the thousandths place but the other is not?

Advancing Questions:

- Which columns contain the same numbers?
- What is the first place value where the numbers are different?

a) *Fraction Strategy*

Students should place a 0 at the end of 9.87 to make 9.870 in order to represent both numbers using the same denomination of thousandths.

9.870 = $9\frac{870}{1000}$ and 9.845 = $9\frac{845}{1000}$

By representing the decimal values as fractions, students are able to compare the numerators 870 and 845 since they have the same denominators. 845 is less than 870, therefore 9.845 is less than 9.87.

Assessing Questions:

- How did you get your answer?
- Why did you use 1000 as a common denominator?

Advancing Questions:

- What would the common denominator be if the numbers being comparing were 9.85 and 9.845?
- How is reading decimal numbers related to fractional representations?

b) Write a correct inequality for the two stopwatch times using the symbols >, <, or =.

9.845 < 9.87 or 9.87 > 9.845

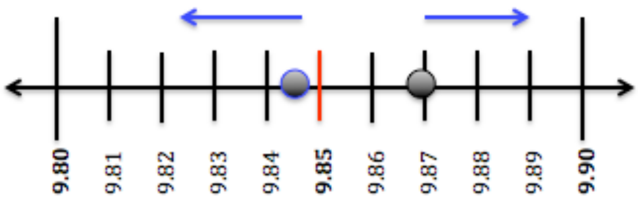
Assessing Questions:

- How did you get your answer?
- Can you explain why you chose that symbol?

Advancing Questions:

- Is there another way to write this comparison using another symbol?
- What is another way to write this comparison using another symbol? Explain.

<table>
<tr>
<td>c) 9.845 rounds to 9.8 when rounded to the nearest tenths place because the 4 in the hundredths place means the number is closer to.9.8 than 9.9. 9.87 rounds to 9.9 because the 7 in the hundredths place means the number is closer to 9.9 than 9.8.


</td>
<td>Assessing Questions:
• What are you asked to find and show?
• How did you get your answer?
• Why did you choose the scale you chose for your number line?

Advancing Questions:
• What is the smallest number on your number line that would round to 9.9? Explain.
• What is the largest number on your number line that would round to 9.8? Explain.
• When is it necessary to round numbers in the real word?</td>
</tr>
<tr>
<th>Possible Student Misconceptions</th>
<th>Assessing and Advancing Questions</th>
</tr>
<tr>
<td>Students may think the larger number is the faster time.</td>
<td>• If you want the faster time, should you choose the larger or smaller number?</td>
</tr>
<tr>
<td>Students may think that longer numbers are greater than shorter numbers.</td>
<td>• Can you use a place value chart to determine the value of each digit in the number?</td>
</tr>
<tr>
<td>Students who do not use precise mathematical language when reading numbers may experience difficulty in writing numbers in word form and in expanded form.
Examples:
• 5.281 should be read as “five and two hundred eighty-one thousandths,” not “five point two eight one”
• $\frac{7}{10}$ should be read as “seven tenths” not “seven over ten”</td>
<td>• Can you and a partner each think of a decimal number in the thousandths, switch numbers and write the numbers in word form as well as standard form?</td>
</tr>
<tr>
<td>Students may start at the rightmost place in a decimal number and keep rounding to the value of the next place to the left until reaching the place of the number to be rounded. For example, if a student with this thinking was asked to round 1.492 to the nearest unit: 1.492 ➔ 1.49 ➔1.5 ➔ 2. This may work for some numbers but not for all such as in the example. 1.492 is actually closer to 1 than to 2 because it is less than 1.5.</td>
<td>• Can you round 7.447 and 7.457 each to the nearest tenth? Can you demonstrate this on a number line?</td>
</tr>
</table>

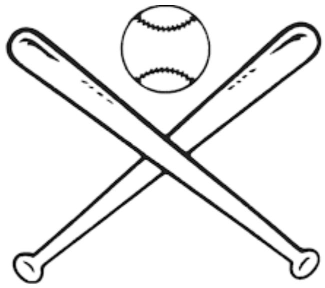
Task 2

Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• Can you read each number aloud? • What you know about each number? • Which place value space should you look at in each number to begin the comparison? • Can you build each number using base ten blocks and compare? • Are there any words used in the problem that you do not understand? • What are you asked to find or show? • How can you show the number that you are rounding on a number line? • What are the 2 possible numbers that this number could round to? • Can you underline the place to be rounded?
If students finish early….	• Imagine that Marvin joined in the race up and down the hallway with a time of 9.85 seconds. Can you compare the three numbers using >, <, or = symbols? • Which strategy seems the most efficient to you? Why? • Does the number line strategy work for rounding any number? Explain.

Discuss/Analyze

Whole Group Questions

- Pick groups to show their work. Select groups who chose different solution strategies.
- Is the faster time the larger or smaller numeric value?
- Explain the value of each digit in terms of place value.
- How does the place value chart help in determining the value of each digit?
- Looking again at your place value chart, how does each place value space relate to the space to the right?
- How does each place value space relate to the space to the left?
- How is using greater than, less than, or equal helpful in comparing numbers? What are the symbols used to represent these words?
- Give an example using each symbol correctly.
- When plotting a number on the number line, which direction would you go to find numbers that are greater? Explain. Is that always true?
- How would you round 9.85 to the nearest tenth?



Name ________________________________

Task 3: Batting Averages

Sam and Nikki are having a discussion about their favorite baseball players. Sam incorrectly says that his favorite player's batting average of 0.293 is greater than Nikki's favorite player's batting average of 0.34.

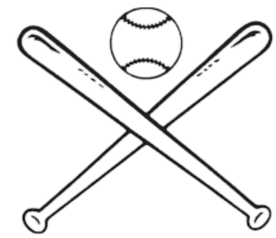
a) Explain why Sam's statement is incorrect.

b) Write a correct inequality for the two batting averages using the symbols >, < , or =.

c) If a baseball announcer wants to read these averages using correct mathematical language, how would he say them?

Task 3: Batting Averages — **5th Grade**

Sam and Nikki are having a discussion about their favorite baseball players. Sam incorrectly says that his favorite player's batting average of 0.293 is greater than Nikki's favorite player's batting average of 0.34.



d) Explain why Sam's statement is incorrect.

e) Write a correct inequality for the two batting averages using the symbols >, < , or =.

f) If a baseball announcer wants to read these averages using correct mathematical language, how would he say them?

Teacher Notes:

This task can be used to solidify understanding developed in the first two tasks in this arc. Have a variety of math manipulatives available to students. Without giving explicit instructions, allow students to choose from these math manipulatives when solving this task.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
5. NBT.3 Read, write, and compare decimals to thousandths. **5.NBT.3a** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). **5.NBT.3b** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- Numbers can be represented using objects, words, and symbols.
- Numbers can be named in equivalent ways using place value (e.g., 2 hundreds 4 tens is equivalent to 24 tens).
- For any number, the place of a digit tells how many ones, tens, hundreds, and so forth are represented by that digit.
- Each place value to the left of another is ten times greater than the one to the right (e.g., 100 = 10 x 10).
- Sets of ten, one hundred and so forth must be perceived as single entities when interpreting numbers using place value (e.g., 1 hundred is one group, it is 10 tens or 100 ones).
- Decimal place value is an extension of whole number place value.
- Whole numbers and decimals can be compared by analyzing corresponding place values.
- A number to the right of another on the number line is the greater number.
- Numbers can be compared using greater than, less than, or equal.

Explore Phase

Possible Solution Paths

a) *Place value chart strategy*

ones	•	tenths	hundredths	thousandths
0	•	2	9	3
0	•	3	4	

By looking at the tenths place (the largest of the decimal spaces to the right of the decimal), we can see that 3 is greater than 2. Therefore, 0.34 > 0.293.

Assessing and Advancing Questions

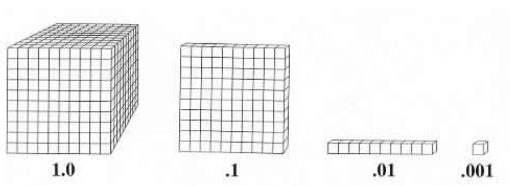
Assessing Questions:

- How do you know which number is smaller?
- How did you decide to explain it this way?
- What were you asked to find or explain in the problem?
- Does your answer fit what you were asked to find?

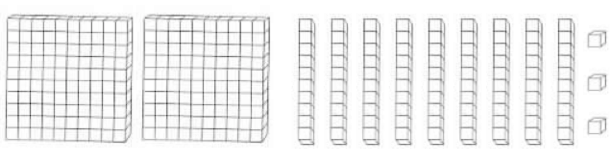
Advancing Questions:

- Are there any other tools that might help you solve this problem?
- Will this tool work on another set of numbers?

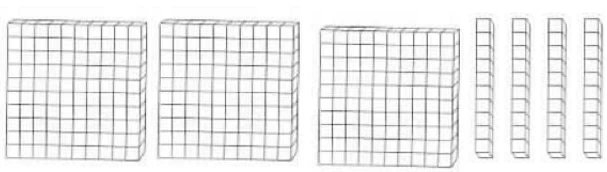
a) *Base ten block strategy*



0.293 =



0.34 =



By looking at the base ten blocks, you can see that there are more tenths (flats) in 0.34 than in 0.293. Therefore, 0.34 > 0.293.

Assessing Questions:

- How do you know which number is smaller?
- How did you decide to explain it this way?
- What were you asked to find or explain in the problem?
- Does your answer fit what you were asked to find?

Advancing Questions:

- Are there any other tools that might help you solve this problem?
- Will this tool work on another set of numbers?
- Why is it important to tell what each piece stands for?

b) 0.34 > 0.293 means that 0.34 is greater than 0.293. See reasoning in b).

or

0.293 < 0.34 means that 0.293 is less than 0.34. See reasoning in b).

Assessing Questions:

- Which symbol did you use? Why?

Advancing Questions:

- Is there more than one way to answer this question correctly?
- Which of the listed symbols could not be used to compare 0.34 and 0.293?

<table>
<tr><td>c) Sam’s favorite player: “Two-hundred ninety-three thousandths”

Nikki’s favorite player: “Thirty-four hundredths”</td><td>Assessing Questions:
How did you know which place value word to write at the end of the number word form?
Advancing Questions:
Would your answer change if the decimal point moved to a different position within the number?
Why is it important to be able to write numbers in word form?</td></tr>
<tr><th>Possible Student Misconceptions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>Students may think that longer numbers are greater than shorter numbers.</td><td>Can you use a place value chart to compare 9.42 and 9.425?
What is the thousandths place value for 9.42?</td></tr>
<tr><td>Students who do not use precise mathematical language when reading numbers may experience difficulty in writing numbers in word form and in expanded form.
Examples:
5.281 should be read as “five and two hundred eighty-one thousandths,” not “five point two eight one”
$\frac{7}{10}$ should be read as “seven tenths” not “seven over ten”</td><td>Can you and a partner each think of a decimal number in the thousandths, switch numbers and write the numbers in word form as well as standard form?</td></tr>
<tr><th>Entry/Extensions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>If students can’t get started….</td><td>Can you read each number aloud?
What you know about each number?
Which place value space should you look at in each number to begin the comparison?
Can you build each number using base ten blocks and compare?</td></tr>
<tr><td>If students finish early….</td><td>What would happen if these were whole numbers? Would Sam still be incorrect? Why or why not?
Could you use a number line to solve this problem? Are there any other strategies to solving this problem?</td></tr>
<tr><th colspan="2">Discuss/Analyze</th></tr>
<tr><th colspan="2">Whole Group Questions</th></tr>
<tr><td colspan="2">Pick groups to show their work. Select groups who chose different solution strategies.
Explain the value of each digit in terms of place value.
How does the place value chart help in determining the value of each digit?
How is using <, >, = helpful in comparing numbers?
When plotting a number on the number line, which direction would you go to find numbers that are greater? Explain. Is that always true?</td></tr>
</table>

Name ___________________________________

Task 4: Dancing Digits

A local organization hosted a dance competition to raise money for its new building. The dance competition turned out to be very popular! The organization raised $374,318 for their new building.

a) Look at the digits in the hundreds place and hundred thousands place. How many times greater is the 3 in the hundred thousands place than the 3 in the hundreds place? Explain how you know.

b) Change the digit in the thousands place so that the value of the new digit represents $\frac{1}{10}$ of the value of the digit in the ten thousands place. Explain how you know.

Task 4

Task 4: Dancing Digits	**5th Grade**

A local organization hosted a dance competition to raise money for its new building. The dance competition turned out to be very popular! The organization raised $374,318 for their new building.

c) Look at the digits in the hundreds place and hundred thousands place. How many times greater is the 3 in the hundred thousands place than the 3 in the hundreds place? Explain how you know.

d) Change the digit in the thousands place so that the value of the new digit represents $\frac{1}{10}$ of the value of the digit in the ten thousands place. Explain how you know.

Teacher Notes:

If students do not include the place value chart in their explanations, teachers should guide students to include this level of detail. As a means of differentiation, you could change the place value positions to smaller values and incorporate the use of base ten blocks among possible representations.

Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- The value of a digit in a written numeral depends on its place, or position, in a number.
- For any number, the place of a digit tells how many ones, tens, hundreds, and so forth are represented by that digit.
- Each place value to the left of another is ten times greater than the one to the right (e.g., 100 = 10 x 10).
- Decimal place value is an extension of whole number place value.
- The base-ten numeration system extends infinitely to very large and very small numbers (e.g., millions & millionths).

Explore Phase

Possible Solution Paths	**Assessing and Advancing Questions**
a) Explained with a <u>place value chart</u>: Each place to the left of the hundreds place is ten times greater than the one to the right. Therefore, the 3 in the hundred thousands place is 1,000 times	**Assessing Questions:** • Tell me about your chart. What does it show? • Why did you put each digit in its own place? • Can you generalize a place value pattern from this example?

larger than the 3 in the hundreds place.

100.000s	10,000s	1000s	,	100s	10s	1s
3	**7**	**4**	,	**3**	**1**	**8**
X 1000	X 100	X 10	,			

- Can you describe the relationship between the 3 in the hundred thousands place and the 3 in the hundreds place by going to the right in the place value chart?

Advancing Questions:

- Can you use another representation to explain your answer?
- How does this pattern of going right in the place value chart relate to going left on the place value chart?

a) Explained with <u>numbers or equations</u>:

300 x 1,000 = 300,000

Assessing Questions:

- What do the numbers represent?
- Why did you multiply 300 times 1,000?
- Can you generalize a pattern from this example?

Advancing Questions:

- Can you use another representation to explain your answer?
- How many times less than is the 3 in the hundreds place than the 3 in the hundred thousands place? Explain.
- How is this similar or different to your original problem?

b) Explained with <u>place value chart</u>:

A digit in one place represents 1/10 of what it represents in the place to its left. By changing the 4 to 7 in the thousands place is $\frac{1}{10}$ the 7 in the ten thousands place.

100,000s	10,000s	1000s	,	100s	10s	1s
3	**7**	**4**	,	**3**	**1**	**8**
		x $\frac{1}{10}$	,			
3	**7**	**7**	,	**3**	**1**	**8**

Assessing Questions:

- What does your chart show?
- Why did you have to change the 4 in the thousands place to a 7?
- Can you generalize a place value pattern from this example?

Advancing Questions:

- Can you use another representation to explain your answer?
- Which place value position would you have to change in order for it to be $\frac{1}{100}$ the value of the 7 in the ten thousands place? Explain.

b) Explained with <u>numbers or equations</u>:

70,000 x $\frac{1}{10}$ = 7,000

or

70,000 ÷ 10 = 7,000

Assessing Questions:

- What do the numbers represent?
- Why did you multiply 70,000 times $\frac{1}{10}$?
- Why did your divide 70,000 by 10?
- Can you generalize a pattern from this example?

Task 4

	Advancing Questions: • Can you use another representation to explain your answer? • How is this similar or different to your original problem?
Possible Student Misconceptions	**Assessing and Advancing Questions**
Students may confuse values for digits and values for place value positions.	• What is the value of 7 in 379? • What is the value of 7 in 718? • How can both digits be 7 but have different values?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• Is there a way to organize the number in a way to see the place value positions clearly? (Place Value Chart) • Can you name the place value of each digit in the original number? • Can you write an equation with the two values in the problem?
If students finish early….	• What is the relationship between the two 3s in the number 13.34? • What is the relationship between the two 3s in the number 310.34?

Discuss/Analyze

Whole Group Questions

- How can the same digit have different values depending on its position in a number? Give an example.
- In the number 7,301, what number would be 1/10 of the value of the digit in the thousands place?
- In the number 96,084 how many ones are represented by the 8?
- How are the values of the place value positions related?
- Describe the size of ones as compared to hundreds.
- Describe the size of thousands as compared to tens.
- In the number 12.52, how many times greater is the 2 in the ones place than the 2 in the hundredths place? Explain how you know.

Name ___________________________________

Task 5: Exponent Mix-Up

Holly and Marcus missed the lesson on exponents. Mr. Anderson asked Jason to discuss the exponent lesson with Holly and Marcus during homeroom to help them prepare for today's lesson. Jason talked them through what he understood and then gave them each a problem to work on separately.

a) Holly wrote $10^5 = 50$ on her paper. Is Holly's answer correct? Explain why or why not. If her answer is incorrect, what would she need to do to correct it?

b) Marcus thinks that since $2 \text{ X } 10^2 = 200$ that $2.5 \text{ x } 10^2 = 2.500$. Is Marcus's answer correct? Explain why or why not. If his answer is incorrect, what would he need to do to correct it?

Task 5: Exponent Mix-Up	**5th Grade**
Holly and Marcus missed the lesson on exponents. Mr. Anderson asked Jason to discuss the exponent lesson with Holly and Marcus during homeroom to help them prepare for today's lesson. Jason talked them through what he understood and then gave them each a problem to work on separately. c) Holly wrote $10^5 = 50$ on her paper. Is Holly's answer correct? Explain why or why not. If her answer is incorrect, what would she need to do to correct it? d) Marcus thinks that since $2 \times 10^2 = 200$ that $2.5 \times 10^2 = 2.500$. Is Marcus's answer correct? Explain why or why not. If his answer is incorrect, what would he need to do to correct it?	
Teacher Notes:	
The power of ten determines the number of zeros in a product. The product will increase and the decimal point will move to the right the same number of place values as the number of zeros in the power of ten (or exponents). Possible materials to assist instruction include base ten blocks and place value charts. The power of ten determines the number of zeros in a quotient. The quotient will decrease the decimal point will move to the left the same number of place values as the number of zeros in the power of ten (or exponents). Possible materials to assist instruction include base ten blocks and place value charts.	
Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings:	
• Numbers can be represented using objects, words, and symbols. • Numbers can be named in equivalent ways using place value (e.g., 2 hundreds 4 tens is equivalent to 24 tens). • Each place value to the left of another is ten times greater than the one to the right (e.g., 100 = 10 x 10). • The value of the digits can be added together to find the value of the number. • Sets of ten, one hundred and so forth must be perceived as single entities when interpreting numbers using place value (e.g., 1 hundred is one group, it is 10 tens or 100 ones). • Decimal place value is an extension of whole number place value. • The base-ten numeration system extends infinitely to very large and very small numbers (e.g., millions & millionths).	
Explore Phase	

Possible Solution Paths	Assessing and Advancing Questions
a) $10^5 = 50$ is incorrect because Holly used the exponent 5 as a factor instead of as a power of 10. 10^5 means ten multiplied by ten five times (10 x 10 x 10 x 10 x 10). Holly can work out the multiplication of 10 x 10 x 10 x 10 x 10 and she can use the value of the exponent (5) to check that she has the correct number of zeros. 10 x 10 x 10 x 10 x 10 = 100,000 and 100,000 has 5 zeros	**Assessing Question:** • How did you decide what Holly did incorrectly? • What does the exponent tell you in 10^5? • Can you explain how you found the correct answer? • Is there more than one way to find the correct answer? Explain. **Advancing Questions:** • If Holly gave the answer that $10^5 = 1{,}000{,}000$, how could you correct her reasoning? • What would the standard form be for 12^2? Explain how this problem is similar to and different from 10^5.
b) $2.5 \times 10^2 = 2.500$ is incorrect because Marcus has used the exponent ^{2} to add two decimal places to the factor 2.5. 2.5 x 10^2 means 2.5 times 100. Marcus can work out the multiplication of 2.5 x (10 x 10) and he can use the value of the exponent (2) to check that the decimal point has moved the correct number of place value spaces to the right. 2.5 x 10^2 = 250 and the decimal point has moved 2 place value spaces to the right from 2.5 to 250.	**Assessing Question:** • How did you decide what Marcus did incorrectly? • What does the exponent tell you in 10^2? • Can you explain how you found the correct answer? • Is there more than one way to find the correct answer? Explain. **Advancing Questions:** • If Marcus gave the answer that $2 \times 10^2 = 2000$, how could you correct his reasoning? • What would the product be for 2 x 20^2 =? Explain how this problem is similar to and different from 2 x 10^2.
Possible Student Misconceptions	**Assessing and Advancing Questions**
Students may think that an exponent is a factor. For example, in 10^6 a student with this misconception would think the product is 60 instead of 1,000,000.	• What does an exponent number mean in relation to the base number? • How do you know the number of times that you multiply the base number by itself? • Can you describe the pattern in the number of zeros in the standard form number and the exponent?
Students may think that when multiplying by a power of ten, the exponent represents the number of zeros to add to the right of a number. For example, in 5 x 10^4 a student with this misconception would think the product would be 5.0000 instead of 50,000.	• Does this same strategy work for whole numbers? Explain with an example. • How do you know the number of place value spaces to move the decimal?

Task 5

Entry/Extensions	Assessing and Advancing Questions
If students can't get started….	• Can you model this problem with place value chart? • What does an exponent number mean in relation to the base number?
If students finish early….	• How would you explain exponents to a new student? • Can you explain $2 \div 10^2$? How this problem is similar to and different from 2×10^2?

Discuss/Analyze

Whole Group Questions

- Can you write 100,000 using powers of ten?
- Can you write 10^4 in expanded form? Explain what the exponent means in the expanded form?
- What is the standard form of 10^3? What determines the number of zeros in the standard form?
- What happens to the values of the place value spaces as you move to the right in a place value chart?
- What happens to the values of the place value spaces as you move to the left in a place value chart?
- Calculate 4×10^3. Describe the position of the decimal point in the product in relation to the factor 4.

Name ________________________________

Task 6: Trendy T-Shirts

The Trendy T-Shirt Company makes $27.45 profit on each box of t-shirts. The following table shows the latest orders received.

Business Names	**Boxes of t-shirts**
Amanda's Apparel	10
Casey's Clothes	100
Wallie's Wardrobe	1,000

a) Write an expression using powers of 10 to show how much profit Trendy T-Shirts will make on each order. Explain how the exponents relate to the number of boxes of t-shirts.

b) Calculate the total profit Trendy T-Shirts will make on each order in the table above. What do you notice about the placement of the decimal point in the totals?

c) Will this pattern be the same when dividing a decimal by a power of 10? Explain your reasoning.

Task 6: Trendy T-Shirts **5th Grade**

The Trendy T-Shirt Company makes $27.45 profit on each box of t-shirts. The following table shows the latest orders received.

Business Names	Boxes of t-shirts
Amanda's Apparel	10
Casey's Clothes	100
Wallie's Wardrobe	1,000

a) Write an expression using powers of 10 to show how much profit Trendy T-Shirts will make on each order. Explain how the exponents relate to the number of boxes of t-shirts.

b) Calculate the total profit Trendy T-Shirts will make on each order in the table above. What do you notice about the placement of the decimal point in the totals?

c) Will this pattern be the same when dividing a decimal by a power of 10? Explain your reasoning.

Teacher Notes:

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- Numbers can be represented using objects, words, and symbols.
- For any number, the place of a digit tells how many ones, tens, hundreds, and so forth are represented by that digit.
- Decimal place value is an extension of whole number place value.
- The base-ten numeration system extends infinitely to very large and very small numbers (e.g., millions & millionths).

Explore Phase

Possible Solution Paths

a)

	Boxes of t-shirts	a) Expression using Exponents of 10
Amanda's Apparel	10	27.45×10^1
Casey's Clothes	100	27.45×10^2
Wallie's Wardrobe	1,000	27.45×10^3

The exponent corresponds with the number of zeros in the number of boxes of t-shirts (powers of ten).

Assessing and Advancing Questions

Assessing Questions:

- How did you choose the exponent in each expression?
- Can you explain how you constructed each expression?
- Can you explain how you chose the operation?

Advancing Questions:

- How would the expressions change if the businesses decided to double their orders?
- Is there more than one way to write a new expression? Explain.

b)

	Boxes of t-shirts	a) Expression using Exponents of 10	b) Total Profit
Amanda's Apparel	10	27.45×10^1	\$274.50
Casey's Clothes	100	27.45×10^2	\$2,745.00
Wallie's Wardrobe	1,000	27.45×10^3	\$27, 450.00

The decimal point moves to the right based on the number of zeros following the one or the value of the exponents.

Assessing Questions:

- How did you find the total profit from each order?
- Is there more than one way to find the total profit? Explain.
- If there is more than one way to find the total profit, is one method more efficient? Explain.

Advancing Questions:

- What happens when you multiply a whole number by 10 (or 100 or 1000)? If you put a decimal point at the end of the whole number before you multiply, how would this change the problem?
- How did you write 10 (or 100 or 1000) using exponents in part a)?

c)

10	$27.45 \div 10^1$	2.745
100	$27.45 \div 10^2$	0.2745
1,000	$27.45 \div 10^3$	0.02745

The pattern is opposite from the pattern in multiplication by powers on ten. The decimal point moves to the left based on the number of zeros following the 1 or the value of the exponents.

Assessing Questions:

- What similarities and differences do you notice about these expressions with division as compared to the same expressions with multiplication?
- How did you find the quotient?
- Is there more than one way to find the quotient? Explain.
- If there is more than one way to find the quotient, is one method more efficient? Explain.

Advancing Questions:

- Can you write an expression using powers of 10 that has 0.00002745 as the quotient?

Possible Student Misconceptions	Assessing and Advancing Questions
Students may assume that the number increases in value when moving place value spaces away from the decimal.	• What is the value of the 2 in each of these numbers (20.0 and 0.2)? Describe their values in relation to the placement of the decimal point.
Students may think that a number with more digits to the right of the decimal place is larger simply because of the number of digits. With decimals a number with one decimal place may be greater than a number with 2 or 3 decimal places. One method for comparing decimals is to make all numbers have the same number of digits to the right of the decimal place by adding zeros to right of the last digit of the number. Another method is to use a place table.	• Which is greater, 204 or 2040? Explain. • What strategies can you use for comparing values of whole numbers? • Which is greater, 0.24 or 0.024? Explain. • What strategies can you use for comparing values of decimal numbers? • How are the strategies for comparing whole numbers and decimal numbers alike and different?
If students are only focusing on counting the number of zeros, they may make errors. For example, 40 X 500 is going to have four zeros in the product not three. 40 X 500 = 20,000 because 4 x 5 = 20.	• What is the product of 4 x 5? How many zeros are in the product? • Is the product of 40 x 500 = 2,000 or 20,000? Explain your reasoning.
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• What are the important pieces of information in the problem that you need to write an expression? • What are you asked to find? • What do you already know about exponents? • What do you already know about multiplying by powers of ten?
If students finish early….	• Can you find the total profit for an order of 100,000 boxes of t-shirts? Explain your thinking. • How many boxes of t-shirts would yield a profit of $27,450,000,000.00? How do you know?

Discuss/Analyze

Whole Group Questions

- How is the exponent related to the number of zeros in part a)?
- How is the exponent related to the number of places the decimal moves in part b)?
- Calculate 4 x 10^3. Describe the position of the decimal point in the product in relation to the factor 4.
- Calculate 4 ÷ 10^3. Describe the position of the decimal point in the quotient in relation to the dividend.

Name ________________________________

Task 7: Vacation Plans

Travis, Jamie, and Lauren are all planning trips to visit grandparents during summer vacation. The table below gives the cities each person is visiting and the city's distance from Nashville.

	City Visiting	Distance from Nashville
Travis	Dickson, TN	40 miles
Jamie	Anchorage, AK	4045 miles
Lauren	Charlotte, NC	409 miles

a) Approximate each distance as the product of 4 and a power 10.

b) Jamie said, "My trip to Anchorage is about 10 times as far from Nashville as Travis's trip to Dickson." Travis said, "My trip to Dickson is about 1/10th as far from Nashville as Lauren's trip to Charlotte." Using the products from part a), are both Jamie and Travis correct? If yes, use numbers and words to prove they are correct. If no, rewrite the statements so they are correct.

Task 7: Vacation Plans — **5th Grade**

Travis, Jamie, and Lauren are all planning trips to visit grandparents during summer vacation. The table below gives the cities each person is visiting and the city's distance from Nashville.

	City Visiting	Distance from Nashville
Travis	Dickson, TN	40 miles
Jamie	Anchorage, AK	4045 miles
Lauren	Charlotte, NC	409 miles

a) Approximate each distance as the product of 4 and a power 10.

b) Jamie said, "My trip to Anchorage is about 10 times as far from Nashville as Travis's trip to Dickson." Travis said, "My trip to Dickson is about 1/10th as far from Nashville as Lauren's trip to Charlotte." Using the products from part a), are both Jamie and Travis correct? If yes, use numbers and words to prove they are correct. If no, rewrite the statements so they are correct.

Teacher Notes:

This task can be used to solidify understanding developed in tasks 4 through 6.

Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.4** Use place value understanding to round decimals to any place.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- The value of a digit in a written numeral depends on its place, or position, in a number.
- Inherent in place value are units of different size.
- Numbers can be represented using objects, words, and symbols.
- For any number, the place of a digit tells how many ones, tens, hundreds, and so forth are represented by that digit.
- Each place value to the left of another is ten times greater than the one to the right (e.g., 100 = 10 x 10).

Explore Phase

Possible Solution Paths

a)

Dickson	40 miles	4×10^1
Anchorage	4000 miles	4×10^3
Charlotte	400 miles	4×10^2

Assessing and Advancing Questions

Assessing Questions:

- How did you get your answers?
- Do your answers seem reasonable? Why or why not?
- Can you explain your method and why it works?

Advancing Questions:

- What if you had started with numbers in the hundred thousands, would your method have worked?
- Do you see a pattern? If so, can you explain the pattern?

b) Travis is the only one with a correct statement. Since Dickson is approximately 40 miles from Nashville and Anchorage is 4000 miles, 4000 is 100 times 40. When multiplying 40 by 100, the decimal would move two places to the right, becoming 4000. Also, 4×10^3 is 100 times 4×10^1 since each increase in the exponent is a multiple of 10.

One way to write Jamie's statement correctly would be: Jamie said, "My trip to Anchorage is about 100 times as far from Nashville as Travis's trip to Dickson."

Assessing Questions:

- How did you get your answers?
- Does your answers seem reasonable? Why or why not?
- Can you explain your method? Can you explain why it works?

Advancing Questions:

- Is there another strategy for finding the answer?

Possible Student Misconceptions

Students may think that an exponent is a factor. For example, in 10^6 a student with this misconception would think the product is 60 instead of 1,000,000.

Assessing and Advancing Questions

- What does an exponent mean in relation to the base number?
- How do you know the number of times that you multiply the base number by itself?
- Can you describe the pattern in the number of zeros in the standard form number and the exponent?

Students may think that when multiplying by a power of ten, the exponent represents the number of zeros to add to the right of a number. For example, in 5.9×10^4 a student with this misconception would think the product would be 5.90000 instead of 59,000.

- Does this same strategy work for whole numbers? Explain with an example.
- How do you know the number of place value spaces to move the decimal?

<table>
<tr><th>Entry/Extensions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>If students can’t get started….</td><td>What do you need to find out?
What information to do you have?
What strategies are you going to use?
Will you do it mentally? With paper and pencil? Using a number line?
Will a calculator help you?
What tools will you need?
What do think the answer or result will be?
How would you describe the problem in your own words?
What do you know that is not stated in the problem?</td></tr>
<tr><td>If students finish early….</td><td>Can you find a city that is within 10^1,10^2, etc., from Nashville?
Can you compare the distance using >, <, or = ?</td></tr>
<tr><th colspan="2">Discuss/Analyze</th></tr>
<tr><th colspan="2">Whole Group Questions</th></tr>
<tr><td colspan="2">Honolulu’s distance from Nashville is 4277.40 miles. There is a 4 in the thousands place and in the tenths place, what is the relationship between their place values? Identify the values of each digit.
If we wanted to find a city that is 1/100th as far from Nashville as Honolulu, what distance would this be?</td></tr>
</table>

Progressions

Appendix O
K-5, Number and Operations in Base Ten

Progressions for the Tennessee State Standards in Mathematics (draft)

21 April 2012

K–5, Number and Operations in Base Ten

Overview

Students' work in the base-ten system is intertwined with their work on counting and cardinality, and with the meanings and properties of addition, subtraction, multiplication, and division. Work in the base-ten system relies on these meanings and properties, but also contributes to deepening students' understanding of them.

Position The base-ten system is a remarkably efficient and uniform system for systematically representing all numbers. Using only the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, every number can be represented as a string of digits, where each digit represents a value that depends on its place in the string. The relationship between values represented by the places in the base-ten system is the same for whole numbers and decimals: the value represented by each place is always 10 times the value represented by the place to its immediate right. In other words, moving one place to the left, the value of the place is multiplied by 10. In moving one place to the right, the value of the place is divided by 10. Because of this uniformity, standard algorithms for computations within the base-ten system for whole numbers extend to decimals.

Base-ten units Each place of a base-ten numeral represents a base-ten unit: ones, tens, tenths, hundreds, hundredths, etc. The digit in the place represents 0 to 9 of those units. Because ten like units make a unit of the next highest value, only ten digits are needed to represent any quantity in base ten. The basic unit is a *one* (represented by the rightmost place for whole numbers). In learning about whole numbers, children learn that ten ones compose a new kind of unit called a *ten*. They understand two-digit numbers as composed of tens and ones, and use this understanding in computations, decomposing 1 ten into 10 ones and composing a ten from 10 ones.

The power of the base-ten system is in repeated bundling by ten: 10 tens make a unit called a hundred. Repeating this process of creating new units by bundling in groups of ten creates units called *thousand*, *ten thousand*, *hundred thousand* .... In learning about decimals, children partition a one into 10 equal-sized smaller units, each of which is a tenth. Each base-ten unit can be understood in terms of any other base-ten unit. For example, one hundred can be viewed as a tenth of a thousand, 10 tens, 100 ones, or 1,000 tenths. Algorithms for operations in base ten draw on such relationships among the base-ten units.

Computations Standard algorithms for base-ten computations with the four operations rely on decomposing numbers written in base-ten notation into base-ten units. The properties of operations then allow any multi-digit computation to be reduced to a collection of single-digit computations. These single-digit computations sometimes require the composition or decomposition of a base-ten unit.

Beginning in Kindergarten, the requisite abilities develop gradually over the grades. Experience with addition and subtraction within 20 is a Grade 1 standard[1.OA.6] and fluency is a Grade 2 standard.[2.OA.2] Computations within 20 that "cross 10," such as $9+8$ or $13-6$, are especially relevant to NBT because they afford the development of the Level 3 make-a-ten strategies for addition and subtraction described in the OA Progression. From the NBT perspective, make-a-ten strategies are (implicitly) the first instances of composing or decomposing a base-ten unit. Such strategies are a foundation for understanding in Grade 1 that addition may require composing a ten[1.NBT.4] and in Grade 2 that subtraction may involve decomposing a ten.[2.NBT.7]

Strategies and algorithms The Standards distinguish strategies from algorithms.• For example, students use strategies for addition and subtraction in Grades K-3, but are expected to fluently add and subtract whole numbers using standard algorithms by the end of Grade 4. Use of the standard algorithms can be viewed as the culmination of a long progression of reasoning about quantities, the base-ten system, and the properties of operations.

This progression distinguishes between two types of computational strategies: special strategies and general methods. For example, a special strategy for computing $398+17$ is to decompose 17 as $2+15$, and evaluate $(398+2)+15$. Special strategies either cannot be extended to all numbers represented in the base-ten system or require considerable modification in order to do so. A more readily generalizable method of computing $398+17$ is to combine like base-ten units. General methods extend to all numbers represented in the base-ten system. A general method is not necessarily efficient. For example, counting on by ones is a general method that can be easily modified for use with finite decimals. General methods based on place value, however, are more efficient and can be viewed as closely connected with standard algorithms.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8+6=8+2+4=10+4=14$); decomposing a number leading to a ten (e.g., $13-4=13-3-1=10-1=9$); using the relationship between addition and subtraction (e.g., knowing that $8+4=12$, one knows $12-8=4$); and creating equivalent but easier or known sums (e.g., adding $6+7$ by creating the known equivalent $6+6+1=12+1=13$).

2.OA.2 Fluently add and subtract within 20 using mental strategies.[1] By end of Grade 2, know from memory all sums of two one-digit numbers.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

• **Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Mathematical practices Both general methods and special strategies are opportunities to develop competencies relevant to the NBT standards. Use and discussion of both types of strategies offer opportunities for developing fluency with place value and properties of operations, and to use these in justifying the correctness of computations (MP.3). Special strategies may be advantageous in situations that require quick computation, but less so when uniformity is useful. Thus, they offer opportunities to raise the topic of using appropriate tools strategically (MP.5). Standard algorithms can be viewed as expressions of regularity in repeated reasoning (MP.8) used in general methods based on place value.

Numerical expressions and recordings of computations, whether with strategies or standard algorithms, afford opportunities for students to contextualize, probing into the referents for the symbols involved (MP.2). Representations such as bundled objects or math drawings (e.g., drawings of hundreds, tens, and ones) and diagrams (e.g., simplified renderings of arrays or area models) afford the mathematical practice of explaining correspondences among different representations (MP.1). Drawings, diagrams, and numerical recordings may raise questions related to precision (MP.6), e.g., does that 1 represent 1 one or 1 ten? This progression gives examples of representations that can be used to connect numerals with quantities and to connect numerical representations with combination, composition, and decomposition of base-ten units as students work towards computational fluency.

Kindergarten

In Kindergarten, teachers help children lay the foundation for understanding the base-ten system by drawing special attention to 10. Children learn to view the whole numbers 11 through 19 as ten ones and some more ones. They decompose 10 into pairs such as $1+9$, $2+8$, $3+7$ and find the number that makes 10 when added to a given number such as 3 (see the OA Progression for further discussion).

Work with numbers from 11 to 19 to gain foundations for place value[K.NBT.1] Children use objects, math drawings,• and equations to describe, explore, and explain how the "teen numbers," the counting numbers from 11 through 19, are ten ones and some more ones. Children can count out a given teen number of objects, e.g., 12, and group the objects to see the ten ones and the two ones. It is also helpful to structure the ten ones into patterns that can be seen as ten objects, such as two fives (see the OA Progression).

A difficulty in the English-speaking world is that the words for teen numbers do not make their base-ten meanings evident. For example, "eleven" and "twelve" do not sound like "ten and one" and "ten and two." The numbers "thirteen, fourteen, fifteen, . . . , nineteen" reverse the order of the ones and tens digits by saying the ones digit first. Also, "teen" must be interpreted as meaning "ten" and the prefixes "thir" and "fif" do not clearly say "three" and "five." In contrast, the corresponding East Asian number words are "ten one, ten two, ten three," and so on, fitting directly with the base-ten structure and drawing attention to the role of ten. Children could learn to say numbers in this East Asian way in addition to learning the standard English number names. Difficulties with number words beyond nineteen are discussed in the Grade 1 section.

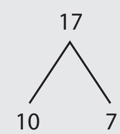
The numerals $11, 12, 13, \ldots, 19$ need special attention for children to understand them. The first nine numerals $1, 2, 3, \ldots, 9$, and 0 are essentially arbitrary marks. These same marks are used again to represent larger numbers. Children need to learn the differences in the ways these marks are used. For example, initially, a numeral such as 16 looks like "one, six," not "1 ten and 6 ones ." Layered place value cards can help children see the 0 "hiding" under the ones place and that the 1 in the tens place really is 10 (ten ones).

By working with teen numbers in this way in Kindergarten, students gain a foundation for viewing 10 ones as a new unit called a ten in Grade 1.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

• Math drawings are simple drawings that make essential mathematical features and relationships salient while suppressing details that are not relevant to the mathematical ideas.

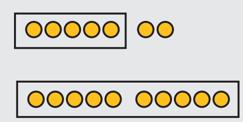
Number-bond diagram and equation



$17 = 10 + 7$

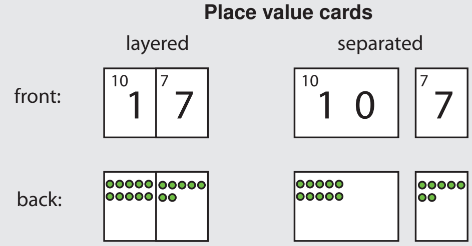
Decompositions of teen numbers can be recorded with diagrams or equations.

5- and 10-frames



Children can place small objects into 10-frames to show the ten as two rows of five and the extra ones within the next 10-frame, or work with strips that show ten ones in a column.

Place value cards



Children can use layered place value cards to see the 10 "hiding" inside any teen number. Such decompositions can be connected to numbers represented with objects and math drawings.

Grade 1

In first grade, students learn to view ten ones as a unit called a ten. The ability to compose and decompose this unit flexibly and to view the numbers 11 to 19 as composed of one ten and some ones allows development of efficient, general base-ten methods for addition and subtraction. Students see a two-digit numeral as representing some tens and they add and subtract using this understanding.

Extend the counting sequence and understand place value Through practice and structured learning time, students learn patterns in spoken number words and in written numerals, and how the two are related.

Grade 1 students take the important step of viewing ten ones as a unit called a "ten."[1.NBT.2a] They learn to view the numbers 11 through 19 as composed of 1 ten and some ones.[1.NBT.2b] They learn to view the decade numbers $10, \ldots, 90$, in written and in spoken form, as 1 ten, ..., 9 tens.[1.NBT.2c] More generally, first graders learn that the two digits of a two-digit number represent amounts of tens and ones, e.g., 67 represents 6 tens and 7 ones.

1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a 10 can be thought of as a bundle of ten ones—called a "ten."

b The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

The number words continue to require attention at first grade because of their irregularities. The decade words, "twenty," "thirty," "forty," etc., must be understood as indicating 2 tens, 3 tens, 4 tens, etc. Many decade number words sound much like teen number words. For example, "fourteen" and "forty" sound very similar, as do "fifteen" and "fifty," and so on to "nineteen" and "ninety." As discussed in the Kindergarten section, the number words from 13 to 19 give the number of ones before the number of tens. From 20 to 100, the number words switch to agreement with written numerals by giving the number of tens first. Because the decade words do not clearly indicate they mean a number of tens ("-ty" does mean tens but not clearly so) and because the number words "eleven" and "twelve" do not cue students that they mean "1 ten and 1" and "1 ten and 2," children frequently make count errors such as "twenty-nine, twenty-ten, twenty-eleven, twenty-twelve."

Grade 1 students use their base-ten work to help them recognize that the digit in the tens place is more important for determining the size of a two-digit number.[1.NBT.3] They use this understanding to compare two two-digit numbers, indicating the result with the symbols $>$, $=$, and $<$. Correctly placing the $<$ and $>$ symbols is a challenge for early learners. Accuracy can improve if students think of putting the wide part of the symbol next to the larger number.

1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$.

Use place value understanding and properties of operations to add and subtract First graders use their base-ten work to compute sums within 100 with understanding.[1.NBT.4] Concrete objects, cards, or drawings afford connections with written numerical work and discussions and explanations in terms of tens and ones. In particular, showing composition of a ten with objects or drawings

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

affords connection of the visual ten with the written numeral 1 that indicates 1 ten.

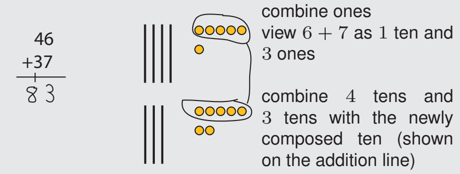
Adding tens and ones separately as illustrated in the margin is a general method that can extend to any sum of multi-digit numbers. Students may also develop sequence methods that extend their Level 2 single-digit counting on strategies (see the OA Progression) to counting on by tens and ones, or mixtures of such strategies in which they add instead of count the tens or ones. Using objects or drawings of 5-groups can support students' extension of the Level 3 make-a-ten methods discussed in the OA Progression for single-digit numbers.

First graders also engage in mental calculation, such as mentally finding 10 more or 10 less than a given two-digit number without having to count by ones.[1.NBT.5] They may explain their reasoning by saying that they have one more or one less ten than before. Drawings and layered cards can afford connections with place value and be used in explanations.

In Grade 1, children learn to compute differences of two-digit numbers for limited cases.[1.NBT.6] Differences of multiples of 10, such as $70 - 40$ can be viewed as 7 tens minus 4 tens and represented with concrete models such as objects bundled in tens or drawings. Children use the relationship between subtraction and addition when they view $80 - 70$ as an unknown addend addition problem, $70 + \square = 80$, and reason that 1 ten must be added to 70 to make 80, so $80 - 70 = 10$.

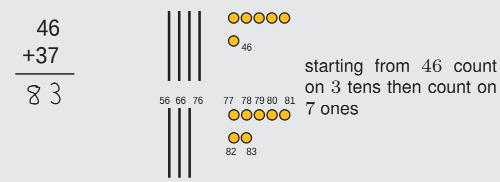
First graders are not expected to compute differences of two-digit numbers other than multiples of ten. Deferring such work until Grade 2 allows two-digit subtraction with and without decomposing to occur in close succession, highlighting the similarity between these two cases.

General method: Adding tens and ones separately



This method is an application of the associative property.

Special strategy: Counting on by tens



This strategy requires counting on by tens from 46, beginning 56, 66, 76, then counting on by ones.

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Grade 2

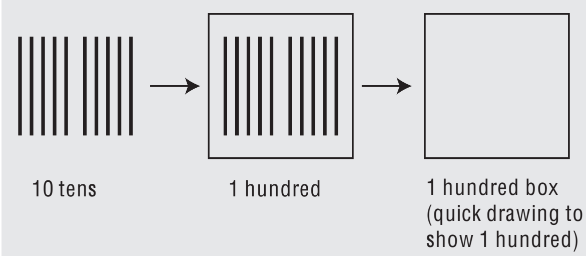
At Grade 2, students extend their base-ten understanding to hundreds. They now add and subtract within 1000, with composing and decomposing, and they understand and explain the reasoning of the processes they use. They become fluent with addition and subtraction within 100.

Understand place value In Grade 2, students extend their understanding of the base-ten system by viewing 10 tens as forming a new unit called a "hundred."[2.NBT.1a] This lays the groundwork for understanding the structure of the base-ten system as based in repeated bundling in groups of 10 and understanding that the unit associated with each place is 10 of the unit associated with the place to its right.

2.NBT.1a Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a 100 can be thought of as a bundle of ten tens—called a "hundred."

Drawings to support seeing 10 tens as 1 hundred



Representations such as manipulative materials, math drawings and layered three-digit place value cards afford connections between written three-digit numbers and hundreds, tens, and ones. Number words and numbers written in base-ten numerals and as sums of their base-ten units can be connected with representations in drawings and place value cards, and by saying numbers aloud and in terms of their base-ten units, e.g., 456 is "Four hundred fifty six" and "four hundreds five tens six ones."[2.NBT.3]

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Unlike the decade words, the hundred words indicate base-ten units. For example, it takes interpretation to understand that "fifty" means five tens, but "five hundred" means almost what it says ("five hundred" rather than "five hundreds"). Even so, this doesn't mean that students automatically understand 500 as 5 hundreds; they may still only think of it as the number reached after 500 counts of 1.

Students begin to work towards multiplication when they skip count by 5s, by 10s, and by 100s. This skip counting is not yet true multiplication because students don't keep track of the number of groups they have counted.[2.NBT.2]

2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

Comparing magnitudes of two-digit numbers draws on the understanding that 1 ten is greater than any amount of ones represented by a one-digit number. Comparing magnitudes of three-digit numbers draws on the understanding that 1 hundred (the smallest three-digit number) is greater than any amount of tens and ones represented by a two-digit number. For this reason, three-digit numbers are compared by first inspecting the hundreds place (e.g. $845 > 799$; $849 < 855$).[2.NBT.4]

2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, =, and $<$ symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract Students become fluent in two-digit addition and subtraction.[2.NBT.5, 2.NBT.6] Representations such as manipulative materials and drawings may be used to support reasoning and explanations about addition and subtraction with three-digit numbers.[2.NBT.7] When students add ones to ones, tens to tens, and hundreds to hundreds they are implicitly using a general method based on place

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

value and the associative and commutative properties of addition. Examples of how general methods can be represented in numerical work and composition and decomposition can be represented in math drawings are shown in the margin.

Drawings and diagrams can illustrate the reasoning repeated in general methods for computation that are based on place value. These provide an opportunity for students to observe this regularity and build toward understanding the standard addition and subtraction algorithms required in Grade 4 as expressions of repeated reasoning (MP.8).

At Grade 2, composing and decomposing involves an extra layer of complexity beyond that of Grade 1. This complexity manifests itself in two ways. First, students must understand that a hundred is a unit composed of 100 ones, but also that it is composed of 10 tens. Second, there is the possibility that both a ten and a hundred are composed or decomposed. For example, in computing $398 + 7$ a new ten and a new hundred are composed. In computing $302 - 184$, a ten and a hundred are decomposed.

Students may continue to develop and use special strategies for particular numerical cases or particular problem situations such as Unknown Addend. For example, instead of using a general method to add $398 + 7$, students could reason mentally by decomposing the 7 ones as $2 + 5$, adding 2 ones to 398 to make 400, then adding the remaining 5 ones to make 405. This method uses the associative property of addition and extends the make-a-ten strategy described in the OA Progression. Or students could reason that 398 is close to 400, so the sum is close to $400 + 7$, which is 407, but this must be 2 too much because 400 is 2 more than 398, so the actual sum is 2 less than 407, which is 405. Both of these strategies make use of place value understanding and are practical in limited cases.

Subtractions such as $302 - 184$ can be computed using a general method by decomposing a hundred into 10 tens, then decomposing one of those tens into 10 ones. Students could also view it as an unknown addend problem $184 + \square = 302$, thus drawing on the relationship between subtraction and addition. With this view, students can solve the problem by adding on to 184: first add 6 to make 190, then add 10 to make 200, next add 100 to make 300, and finally add 2 to make 302. They can then combine what they added on to find the answer to the subtraction problem: $6 + 10 + 100 + 2 = 118$. This strategy is especially useful in unknown addend situations. It can be carried out more easily in writing because one does not have to keep track of everything mentally. This is a Level 3 strategy, and is easier than the Level 3 strategy illustrated below that requires keeping track of how much of the second addend has been added on. (See the OA Progression for further discussion of levels.)

When computing sums of three-digit numbers, students might use strategies based on a flexible combination of Level 3 composition and decomposition and Level 2 counting-on strategies when finding the value of an expression such as $148 + 473$. For exam-

Addition: Recording combined hundreds, tens, and ones on separate lines

456 +167 ———	456 +167 ——— 500	456 +167 ——— 500 110	456 +167 ——— 500 110 13 ——— 623

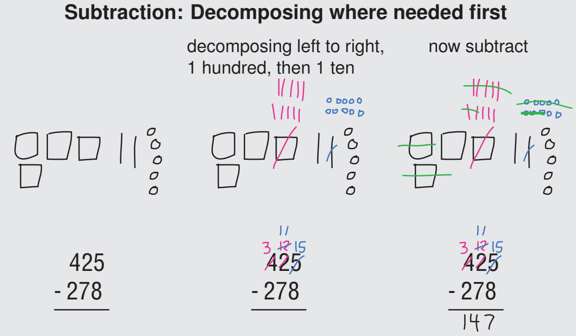
Addition proceeds from left to right, but could also have gone from right to left. There are two advantages of working left to right: Many students prefer it because they read from left to right, and working first with the largest units yields a closer approximation earlier.

Addition: Recording newly composed units on the same line

456 +167 ———	456 +167 1 ——— 3	456 +167 1 1 ——— 23	456 +167 1 1 ——— 623
	Add the ones, $6 + 7$, and record these 13 ones with 3 in the ones place and 1 on the line under the tens column.	Add the tens, $5 + 6 + 1$, and record these 12 tens with 2 in the tens place and 1 on the line under the hundreds column.	Add the hundreds, $4 + 1 + 1$ and record these 6 hundreds in the hundreds column.

Digits representing newly composed units are placed below the addends. This placement has several advantages. Each two-digit partial sum (e.g., "13") is written with the digits close to each other, suggesting their origin. In "adding from the top down," usually sums of larger digits are computed first, and the easy-to-add "1" is added to that sum, freeing students from holding an altered digit in memory. The original numbers are not changed by adding numbers to the first addend; three multi-digit numbers (the addends and the total) can be seen clearly. It is easier to write teen numbers in their usual order (e.g., as 1 then 3) rather than "write the 3 and carry the 1" (write 3, then 1).

Subtraction: Decomposing where needed first



All necessary decomposing is done first, then the subtractions are carried out. This highlights the two major steps involved and can help to inhibit the common error of subtracting a smaller digit on the top from a larger digit. Decomposing and subtracting can start from the left (as shown) or the right.

ple, they might say, "100 and 400 is 500. And 70 and 30 is another hundred, so 600. Then 8, 9, 10, 11 ...and the other 10 is 21. So, 621." Keeping track of what is being added is easier using a written form of such reasoning and makes it easier to discuss. There are two kinds of decompositions in this strategy. Both addends are decomposed into hundreds, tens, and ones, and the first addend is decomposed successively into the part already added and the part still to add.

Students should continue to develop proficiency with mental computation. They mentally add 10 or 100 to a given number between 100 and 900, and mentally subtract 10 or 100 from a given number between 100 and 900.[2.NBT.8]

[2.NBT.8] Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

Grade 3

At Grade 3, the major focus is multiplication,• so students' work with addition and subtraction is limited to maintenance of fluency within 1000 for some students and building fluency to within 1000 for others.

• See the progression on Operations and Algebraic Thinking.

Use place value understanding and properties of operations to perform multi-digit arithmetic Students continue adding and subtracting within 1000.[3.NBT.2] They achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction. Such fluency can serve as preparation for learning standard algorithms in Grade 4, if the computational methods used can be connected with those algorithms.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Students use their place value understanding to round numbers to the nearest 10 or 100.[3.NBT.1] They need to understand that when moving to the right across the places in a number (e.g., 456), the digits represent smaller units. When rounding to the nearest 10 or 100, the goal is to approximate the number by the closest number with no ones or no tens and ones (e.g., so 456 to the nearest ten is 460; and to the nearest hundred is 500). Rounding to the unit represented by the leftmost place is typically the sort of estimate that is easiest for students. Rounding to the unit represented by a place in the middle of a number may be more difficult for students (the surrounding digits are sometimes distracting). Rounding two numbers before computing can take as long as just computing their sum or difference.

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10.[3.NBT.3] For example, the product 3×50 can be represented as 3 groups of 5 tens, which is 15 tens, which is 150. This reasoning relies on the associative property of multiplication: $3 \times 50 = 3 \times (5 \times 10) = (3 \times 5) \times 10 = 15 \times 10 = 150$. It is an example of how to explain an instance of a calculation pattern for these products: calculate the product of the non-zero digits, then shift the product one place to the left to make the result ten times as large.•

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

• **Grade 3 explanations for "15 tens is 150"**

- Skip-counting by 50. 5 tens is 50, 100, 150.
- Counting on by 5 tens. 5 tens is 50, 5 more tens is 100, 5 more tens is 150.
- Decomposing 15 tens. 15 tens is 10 tens and 5 tens. 10 tens is 100. 5 tens is 50. So 15 tens is 100 and 50, or 150.
- Decomposing 15.

$$\begin{aligned} 15 \times 10 &= (10+5) \times 10 \\ &= (10 \times 10) + (5 \times 10) \\ &= 100 + 50 \\ &= 150 \end{aligned}$$

All of these explanations are correct. However, skip-counting and counting on become more difficult to use accurately as numbers become larger, e.g., in computing 5×90 or explaining why 45 tens is 450, and needs modification for products such as 4×90. The first does not indicate any place value understanding.

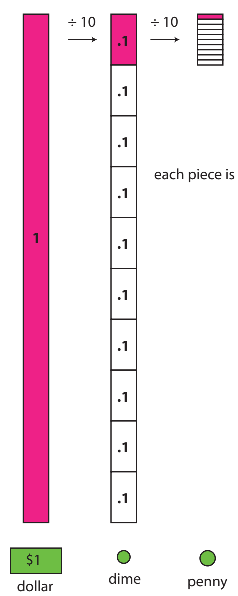
Grade 4

At Grade 4, students extend their work in the base-ten system. They use standard algorithms to fluently add and subtract. They use methods based on place value and properties of operations supported by suitable representations to multiply and divide with multi-digit numbers.

Generalize place value understanding for multi-digit whole numbers In the base-ten system, the value of each place is 10 times the value of the place to the immediate right.[4.NBT.1] Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.

To read numerals between $1{,}000$ and $1{,}000{,}000$, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, $457{,}000$ is read "four hundred fifty seven thousand."[4.NBT.2] The same methods students used for comparing and rounding numbers in previous grades apply to these numbers, because of the uniformity of the base-ten system.

Decimal notation and fractions Students in Grade 4 work with fractions having denominators 10 and 100.[4.NF.5] Because it involves partitioning into 10 equal parts and treating the parts as numbers called one tenth and one hundredth, work with these fractions can be used as preparation to extend the base-ten system to non-whole numbers.

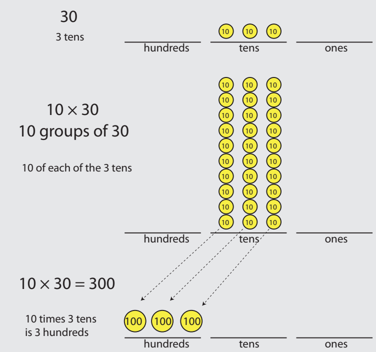


Using the unit fractions $\frac{1}{10}$ and $\frac{1}{100}$, non-whole numbers like $23\frac{7}{10}$ can be written in an expanded form that extends the form used with whole numbers: $2 \times 10 + 3 \times 1 + 7 \times \frac{1}{10}$.[4.NF.4b] As with whole-number expansions in the base-ten system, each unit in this decomposition is ten times the unit to its right. This can be connected with the use of base-ten notation to represent $2 \times 10 + 3 \times 1 + 7 \times \frac{1}{10}$ as 23.7. Using decimals allows students to apply familiar place value reasoning to fractional quantities.[4.NF.6] The Number and Operations—Fractions Progression discusses decimals to hundredths and comparison of decimals[4.NF.7] in more detail.

The decimal point is used to signify the location of the ones place, but its location may suggest there should be a "oneths" place to its right in order to create symmetry with respect to the decimal point.

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

10×30 represented as 3 tens each taken 10 times

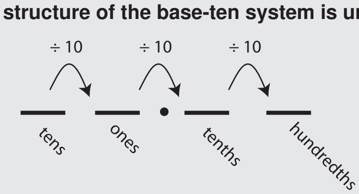


Each of the 3 tens becomes a hundred and moves to the left. In the product, the 3 in the tens place of 30 is shifted one place to the left to represent 3 hundreds. In 300 divided by 10 the 3 is shifted one place to the right in the quotient to represent 3 tens.

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, =, and $<$ symbols to record the results of comparisons.

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[2]

The structure of the base-ten system is uniform



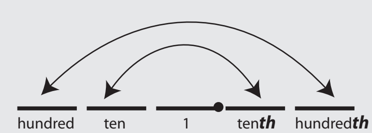
4.NF.4b Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

b Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100.

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, =, or $<$, and justify the conclusions, e.g., by using a visual model.

Symmetry with respect to the ones place



However, because one is the basic unit from which the other base-ten units are derived, the symmetry occurs instead with respect to the ones place.

Ways of reading decimals aloud vary. Mathematicians and scientists often read 0.15 aloud as "zero point one five" or "point one five." (Decimals smaller than one may be written with or without a zero before the decimal point.) Decimals with many non-zero digits are more easily read aloud in this manner. (For example, the number π, which has infinitely many non-zero digits, begins $3.1415\ldots$.)

Other ways to read 0.15 aloud are "1 tenth and 5 hundredths" and "15 hundredths," just as 1,500 is sometimes read "15 hundred" or "1 thousand, 5 hundred." Similarly, 150 is read "one hundred and fifty" or "a hundred fifty" and understood as 15 tens, as 10 tens and 5 tens, and as $100 + 50$.

Just as 15 is understood as 15 ones and as 1 ten and 5 ones in computations with whole numbers, 0.15 is viewed as 15 hundredths and as 1 tenth and 5 hundredths in computations with decimals.

It takes time to develop understanding and fluency with the different forms. Layered cards for decimals can help students become fluent with decimal equivalencies such as three tenths is thirty hundredths.

Use place value understanding and properties of operations to perform multi-digit arithmetic At Grade 4, students become fluent with the standard addition and subtraction algorithms.4.NBT.4 As discussed at the beginning of this progression, these algorithms rely on adding or subtracting like base-ten units (ones with ones, tens with tens, hundreds with hundreds, and so on) and composing or decomposing base-ten units as needed (such as composing 10 ones to make 1 ten or decomposing 1 hundred to make 10 tens). In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable.

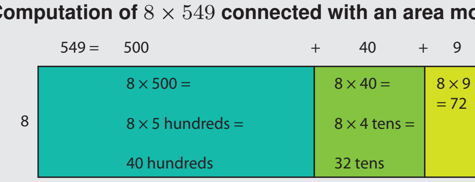
4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

In fourth grade, students compute products of one-digit numbers and multi-digit numbers (up to four digits) and products of two two-digit numbers.4.NBT.5 They divide multi-digit numbers (up to four digits) by one-digit numbers. As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities.

Students can invent and use fast special strategies while also working towards understanding general methods and the standard algorithm.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Computation of 8×549 connected with an area model



Each part of the region above corresponds to one of the terms in the computation below.

$$\begin{aligned} 8 \times 549 &= 8 \times (500 + 40 + 9) \\ &= 8 \times 500 + 8 \times 40 + 8 \times 9. \end{aligned}$$

This can also viewed as finding how many objects are in 8 groups of 549 objects, by finding the cardinalities of 8 groups of 500, 8 groups of 40, and 8 groups of 9, then adding them.

One component of understanding general methods for multiplication is understanding how to compute products of one-digit numbers and multiples of 10, 100, and 1000. This extends work in Grade 3 on products of one-digit numbers and multiples of 10. We can calculate 6×700 by calculating 6×7 and then shifting the result to the left two places (by placing two zeros at the end to show that these are hundreds) because 6 groups of 7 hundred is 6×7 hundreds, which is 42 hundreds, or 4,200. Students can use this place value reasoning, which can also be supported with diagrams of arrays or areas, as they develop and practice using the patterns in relationships among products such as 6×7, 6×70, 6×700, and 6×7000. Products of 5 and even numbers, such as 5×4, 5×40, 5×400, 5×4000 and 4×5, 4×50, 4×500, 4×5000 might be discussed and practiced separately afterwards because they may seem at first to violate the patterns by having an "extra" 0 that comes from the one-digit product.

Another part of understanding general base-ten methods for multi-digit multiplication is understanding the role played by the distributive property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, then combined. By decomposing the factors into like base-ten units and applying the distributive property, multiplication computations are reduced to single-digit multiplications and products of numbers with multiples of 10, of 100, and of 1000. Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods.

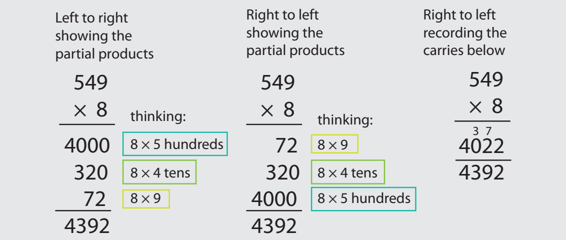
Computing products of two two-digit numbers requires using the distributive property several times when the factors are decomposed into base-ten units. For example,

$$\begin{aligned} 36 \times 94 &= (30+6) \times (90+4) \\ &= (30+6) \times 90 + (30+6) \times 4 \\ &= 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4. \end{aligned}$$

General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but cast in terms of division.[4.NBT.6] One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, $42 \div 6$ is related to $420 \div 6$ and $4200 \div 6$. Students can draw on their work with multiplication and they can also reason that $4200 \div 6$ means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group.

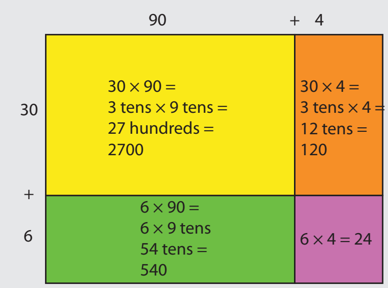
Another component of understanding general methods for multi-digit division computation is the idea of decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this relies on the distributive property. This can be viewed as finding the side length of a rectangle (the divisor is the length of the other side) or as allocating objects (the divisor is the number of groups). See the figures on the next page for examples.

Computation of 8×549: Ways to record general methods



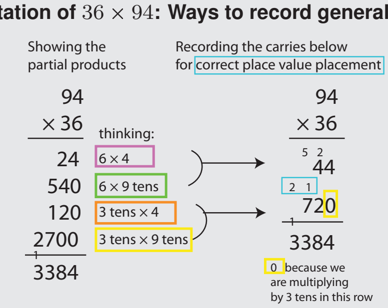
The first method proceeds from left to right, and the others from right to left. In the third method, the digits representing new units are written below the line rather than above 549*, thus keeping the digits of the products close to each other, e.g., the* 7 *from* $8 \times 9 = 72$ *is written diagonally to the left of the* 2 *rather than above the* 4 *in* 549*.*

Computation of 36×94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

Computation of 36×94: Ways to record general methods



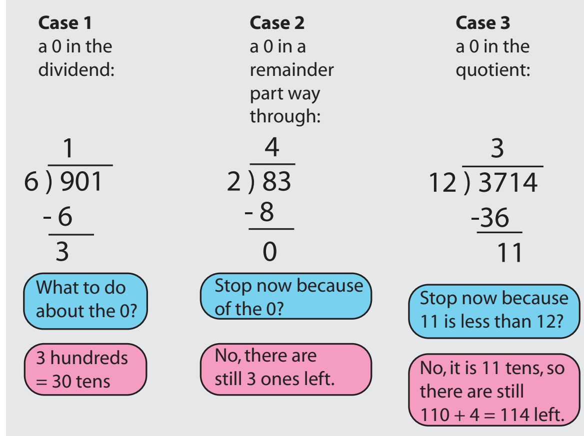
These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94*. The digits* 2 *and* 1 *are surrounded by a blue box. The* 1 *from* $30 \times 4 = 120$ *is placed correctly in the hundreds place and the digit* 2 *from* $30 \times 90 = 2700$ *is placed correctly in the thousands place. If these digits had been placed above* 94*, they would be in incorrect places. Note that the* 0 *(surrounded by a yellow box) in the ones place of the second line of the method on the right is there because the whole line of digits is produced by multiplying by* 30 *(not* 3*).*

[4.NBT.6] Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

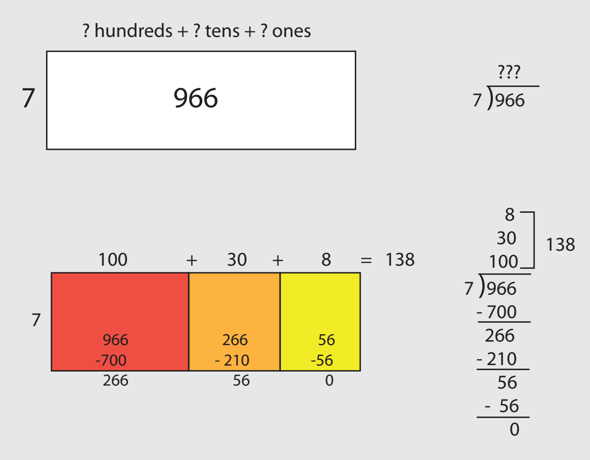
Multi-digit division requires working with remainders. In preparation for working with remainders, students can compute sums of a product and a number, such as $4 \times 8 + 3$. In multi-digit division, students will need to find the greatest multiple less than a given number. For example, when dividing by 6, the greatest multiple of 6 less than 50 is $6 \times 8 = 48$. Students can think of these "greatest multiples" in terms of putting objects into groups. For example, when 50 objects are shared among 6 groups, the largest whole number of objects that can be put in each group is 8, and 2 objects are left over. (Or when 50 objects are allocated into groups of 6, the largest whole number of groups that can be made is 8, and 2 objects are left over.) The equation $6 \times 8 + 2 = 50$ (or $8 \times 6 + 2 = 50$) corresponds with this situation.

Cases involving 0 in division may require special attention.

Cases involving 0 in division

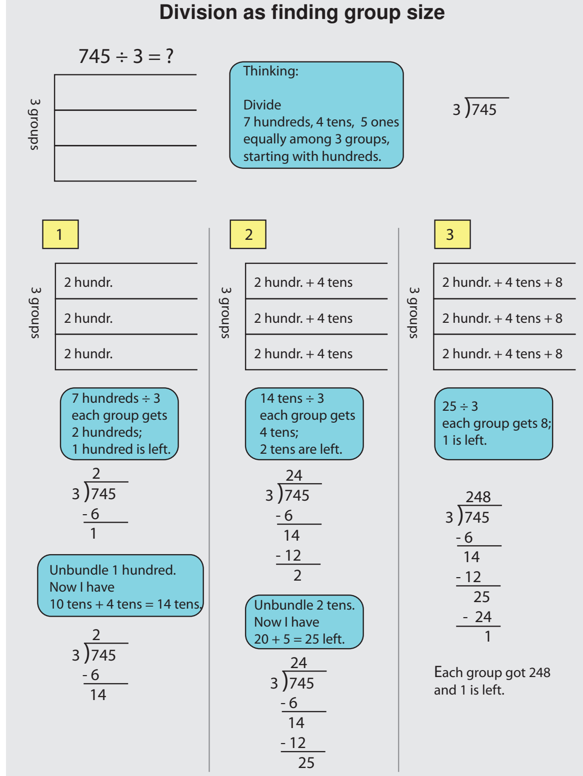


Division as finding side length



$966 \div 7$ is viewed as finding the unknown side length of a rectangular region with area 966 square units and a side of length 7 units. The amount of hundreds is found, then tens, then ones. This yields a decomposition into three regions of dimensions 7 by 100, 7 by 30, and 7 by 8. It can be connected with the decomposition of 966 as $7 \times 100 + 7 \times 30 + 7 \times 8$. By the distributive property, this is $7 \times (100 + 30 + 8)$, so the unknown side length is 138. In the recording on the right, amounts of hundreds, tens, and ones are represented by numbers rather than by digits, e.g., 700 instead of 7.

Division as finding group size



$745 \div 3$ can be viewed as allocating 745 objects bundled in 7 hundreds, 4 tens, and 3 ones equally among 3 groups. In Step 1, the 2 indicates that each group got 2 hundreds, the 6 is the number of hundreds allocated, and the 1 is the number of hundreds not allocated. After Step 1, the remaining hundred is decomposed as 10 tens and combined with the 4 tens (in 745) to make 14 tens.

Grade 5

In Grade 5, students extend their understanding of the base-ten system to decimals to the thousandths place, building on their Grade 4 work with tenths and hundredths. They become fluent with the standard multiplication algorithm with multi-digit whole numbers. They reason about dividing whole numbers with two-digit divisors, and reason about adding, subtracting, multiplying, and dividing decimals to hundredths.

Understand the place value system Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths.

New at Grade 5 is the use of whole number exponents to denote powers of 10.[5.NBT.2] Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. For example, multiplying by 10^4 is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit 4 places to the left. Patterns in the number of 0s in products of a whole numbers and a power of 10 and the location of the decimal point in products of decimals with powers of 10 can be explained in terms of place value. Because students have developed their understandings of and computations with decimals in terms of multiples (consistent with 4.OA.4) rather than powers, connecting the terminology of multiples with that of powers affords connections between understanding of multiplication and exponentiation.

5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Perform operations with multi-digit whole numbers and with decimals to hundredths At Grade 5, students fluently compute products of whole numbers using the standard algorithm.[5.NBT.5] Underlying this algorithm are the properties of operations and the base-ten system (see the Grade 4 section).

Division strategies in Grade 5 involve breaking the dividend apart into like base-ten units and applying the distributive property to find the quotient place by place, starting from the highest place. (Division can also be viewed as finding an unknown factor: the dividend is the product, the divisor is the known factor, and the quotient is the unknown factor.) Students continue their fourth grade work on division, extending it to computation of whole number quotients with dividends of up to four digits and two-digit divisors. Estimation becomes relevant when extending to two-digit divisors. Even if students round appropriately, the resulting estimate may need to be adjusted.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

Recording division after an underestimate

1655 ÷ 27	1	
	10	61
Rounding 27	(30) 50	
to 30 produces	27) 1655	
the underestimate	-1350	
50 at the first step	305	
but this method	-270	
allows the division	35	
process to be	-27	
continued	8	

Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers.[5.NBT.7] Like base-ten units must be added and subtracted, so students need to attend to aligning the corresponding places correctly (this also aligns the decimal points). It can help to put 0s in places so that all numbers show the same number of places to the right of the decimal point. Although whole numbers are not usually written with a decimal point, but that a decimal point with 0s on its right can be inserted (e.g., 16 can also be written as 16.0 or 16.00). The process of composing and decomposing a base-ten unit is the same for decimals as for whole numbers and the same methods of recording numerical work can be used with decimals as with whole numbers. For example, students can write digits representing new units below on the addition or subtraction line, and they can decompose units wherever needed before subtracting.

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. Before students consider decimal multiplication more generally, they can study the effect of multiplying by 0.1 and by 0.01 to explain why the product is ten or a hundred times as small as the multiplicand (moves one or two places to the right). They can then extend their reasoning to multipliers that are single-digit multiples of 0.1 and 0.01 (e.g., 0.2 and 0.02, etc.).

There are several lines of reasoning that students can use to explain the placement of the decimal point in other products of decimals. Students can think about the product of the smallest base-ten units of each factor. For example, a tenth times a tenth is a hundredth, so 3.2×7.1 will have an entry in the hundredth place. Note, however, that students might place the decimal point incorrectly for 3.2×8.5 unless they take into account the 0 in the ones place of 32×85. (Or they can think of 0.2×0.5 as 10 hundredths.) They can also think of the decimals as fractions or as whole numbers divided by 10 or 100.[5.NF.3] When they place the decimal point in the product, they have to divide by a 10 from each factor or 100 from one factor. For example, to see that $0.6 \times 0.8 = 0.48$, students can use fractions: $\frac{6}{10} \times \frac{8}{10} = \frac{48}{100}$.[5.NF.4] Students can also reason that when they carry out the multiplication without the decimal point, they have multiplied each decimal factor by 10 or 100, so they will need to divide by those numbers in the end to get the correct answer. Also, students can use reasoning about the sizes of numbers to determine the placement of the decimal point. For example, 3.2×8.5 should be close to 3×9, so 27.2 is a more reasonable product for 3.2×8.5 than 2.72 or 272. This estimation-based method is not reliable in all cases, however, especially in cases students will encounter in later grades. For example, it is not easy to decide where to place

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

the decimal point in 0.023×0.0045 based on estimation. Students can summarize the results of their reasoning such as those above as specific numerical patterns and then as one general overall pattern such as "the number of decimal places in the product is the sum of the number of decimal places in each factor."

General methods used for computing quotients of whole numbers extend to decimals with the additional issue of placing the decimal point in the quotient. As with decimal multiplication, students can first examine the cases of dividing by 0.1 and 0.01 to see that the quotient becomes 10 times or 100 times as large as the dividend (see also the Number and Operations—Fractions Progression). For example, students can view $7 \div 0.1 = \square$ as asking how many tenths are in 7.[5.NF.7b] Because it takes 10 tenths make 1, it takes 7 times as many tenths to make 7, so $7 \div 0.1 = 7 \times 10 = 70$. Or students could note that 7 is 70 tenths, so asking how many tenths are in 7 is the same as asking how many tenths are in 70 tenths, which is 70. In other words, $7 \div 0.1$ is the same as $70 \div 1$. So dividing by 0.1 moves the number 7 one place to the left, the quotient is ten times as big as the dividend. As with decimal multiplication, students can then proceed to more general cases. For example, to calculate $7 \div 0.2$, students can reason that 0.2 is 2 tenths and 7 is 70 tenths, so asking how many 2 tenths are in 7 is the same as asking how many 2 tenths are in 70 tenths. In other words, $7 \div 0.2$ is the same as $70 \div 2$; multiplying both the 7 and the 0.2 by 10 results in the same quotient. Or students could calculate $7 \div 0.2$ by viewing 0.2 as 2×0.1, so they can first divide 7 by 2, which is 3.5, and then divide that result by 0.1, which makes 3.5 ten times as large, namely 35. Dividing by a decimal less than 1 results in a quotient larger than the dividend[5.NF.5] and moves the digits of the dividend one place to the left. Students can summarize the results of their reasoning as specific numerical patterns then as one general overall pattern such as "when the decimal point in the divisor is moved to make a whole number, the decimal point in the dividend should be moved the same number of places."

5.NF.7b Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[3]

b Interpret division of a whole number by a unit fraction, and compute such quotients.

5.NF.5 Interpret multiplication as scaling (resizing), by:

a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Extending beyond Grade 5

At Grade 6, students extend their fluency with the standard algorithms, using these for all four operations with decimals and to compute quotients of multi-digit numbers. At Grade 6 and beyond, students may occasionally compute with numbers larger than those specified in earlier grades as required for solving problems, but the Standards do not specify mastery with such numbers.

In Grade 6, students extend the base-ten system to negative numbers. In Grade 7, they begin to do arithmetic with such numbers.

By reasoning about the standard division algorithm, students learn in Grade 7 that every fraction can be represented with a decimal that either terminates or repeats. In Grade 8, students learn informally that every number has a decimal expansion, and that those with a terminating or repeating decimal representation are rational numbers (i.e., can be represented as a quotient of integers). There are numbers that are not rational (irrational numbers), such as the square root of 2. (It is not obvious that the square root of 2 is not rational, but this can be proved.) In fact, surprisingly, it turns out that most numbers are not rational. Irrational numbers can always be approximated by rational numbers.

In Grade 8, students build on their work with rounding and exponents when they begin working with scientific notation. This allows them to express approximations of very large and very small numbers compactly by using exponents and generally only approximately by showing only the most significant digits. For example, the Earth's circumference is approximately $40{,}000{,}000$ m. In scientific notation, this is 4×10^7 m.

The Common Core Standards are designed so that ideas used in base-ten computation, as well as in other domains, can support later learning. For example, use of the distributive property occurs together with the idea of combining like units in the NBT and NF standards. Students use these ideas again when they calculate with polynomials in high school.

The distributive property and like units: Multiplication of whole numbers and polynomials

52×73	$(5x+2)(7x+3)$	
$= (5 \times 10 + 2)(7 \times 10 + 3)$	$= (5x+2)(7x+3)$	decomposing as like units (powers of 10 or powers of x)
$= 5 \times 10(7 \times 10 + 3) + 2 \times (7 \times 10 + 3)$	$= 5x(7x+3) + 2(7x+3)$	using the distributive property
$= 35 \times 10^2 + 15 \times 10 + 14 \times 10 + 2 \times 3$	$= 35x^2 + 15x + 14x + 2 \times 3$	using the distributive property again
$= 35 \times 10^2 + 29 \times 10 + 6$	$= 35x^2 + 29x + 6$	combining like units (powers of 10 or powers of x)

kidcentral tn
KIDCENTRALTN.COM

Department of Education May 2015; Publication Authorization No. 331996; 2976 copies.
This public document was promulgated at a cost of $10.30 per copy.